

Fig 3.8 P279

Inductor voltage and capacitor current waveforms

Two steady state equations

(1) Average inductor voltage:

$$\begin{aligned}\langle v_L(t) \rangle &= \frac{1}{T_s} \int_0^{T_s} v_L(t) dt \\ &= D(V_g - I R_L) + D'(V_g - I R_L - V)\end{aligned}$$

Inductor volt-second balance:

$$0 = V_g - I R_L - D'V$$

(2) Average capacitor current:

$$\langle i_C(t) \rangle = D(-V/R) + D'(I - V/R)$$

Capacitor charge balance:

$$0 = D'I - V/R$$

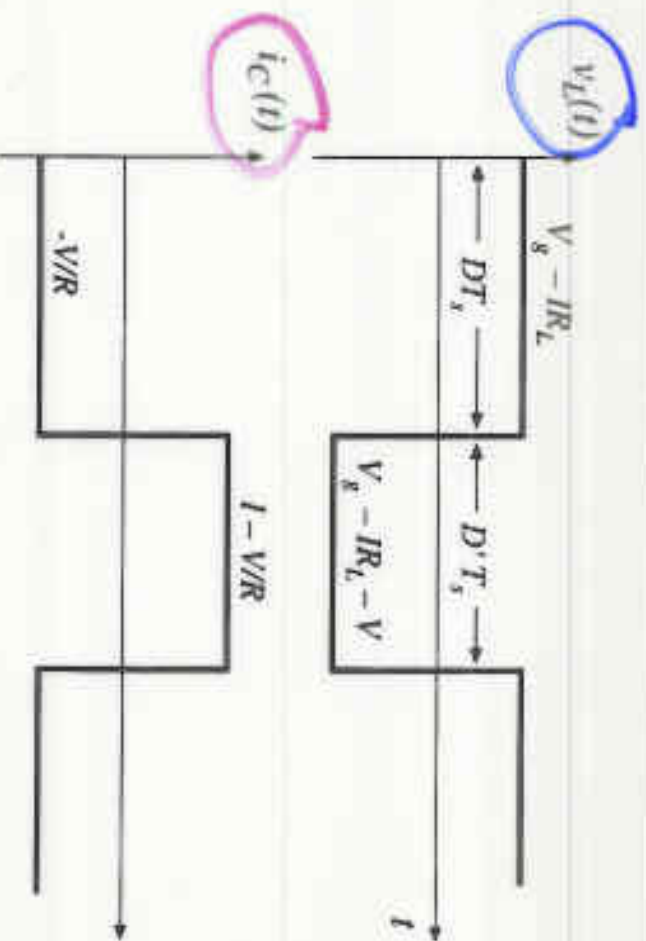


Fig 3.9 945

lossy buck

Solution for output voltage

10V

We now have two equations and two unknowns:

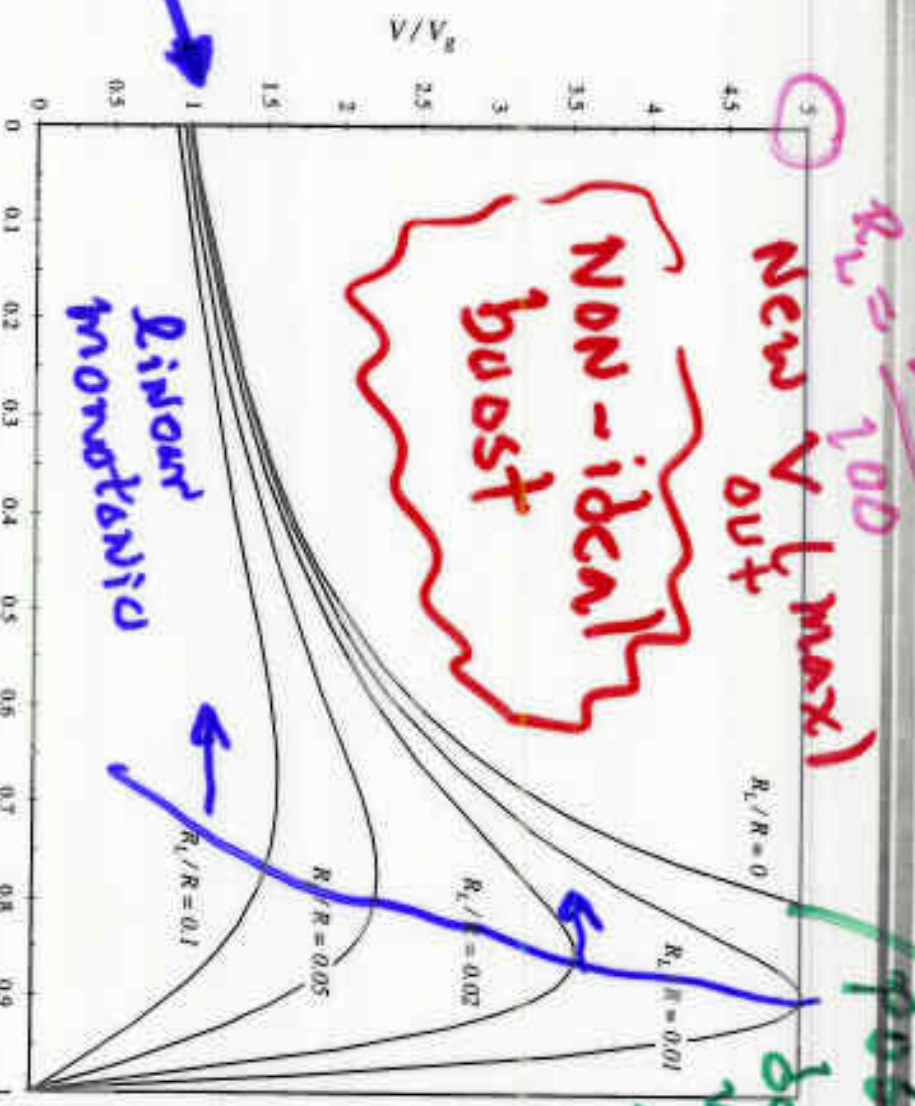
$$0 = V_g - I R_L - D' V$$

$$0 = D' I - V / R$$

Eliminate I and solve for V :

$$\frac{V}{V_g} = \frac{1}{D'} \frac{1}{(1 + R_L / D'^2 R)}$$

ideal ✓
Ch2
Usual boost power limit



$V(D)$ is double value

HW 3.8 What D is highest efficiency

Solution for output voltage

We now have two equations and two unknowns:

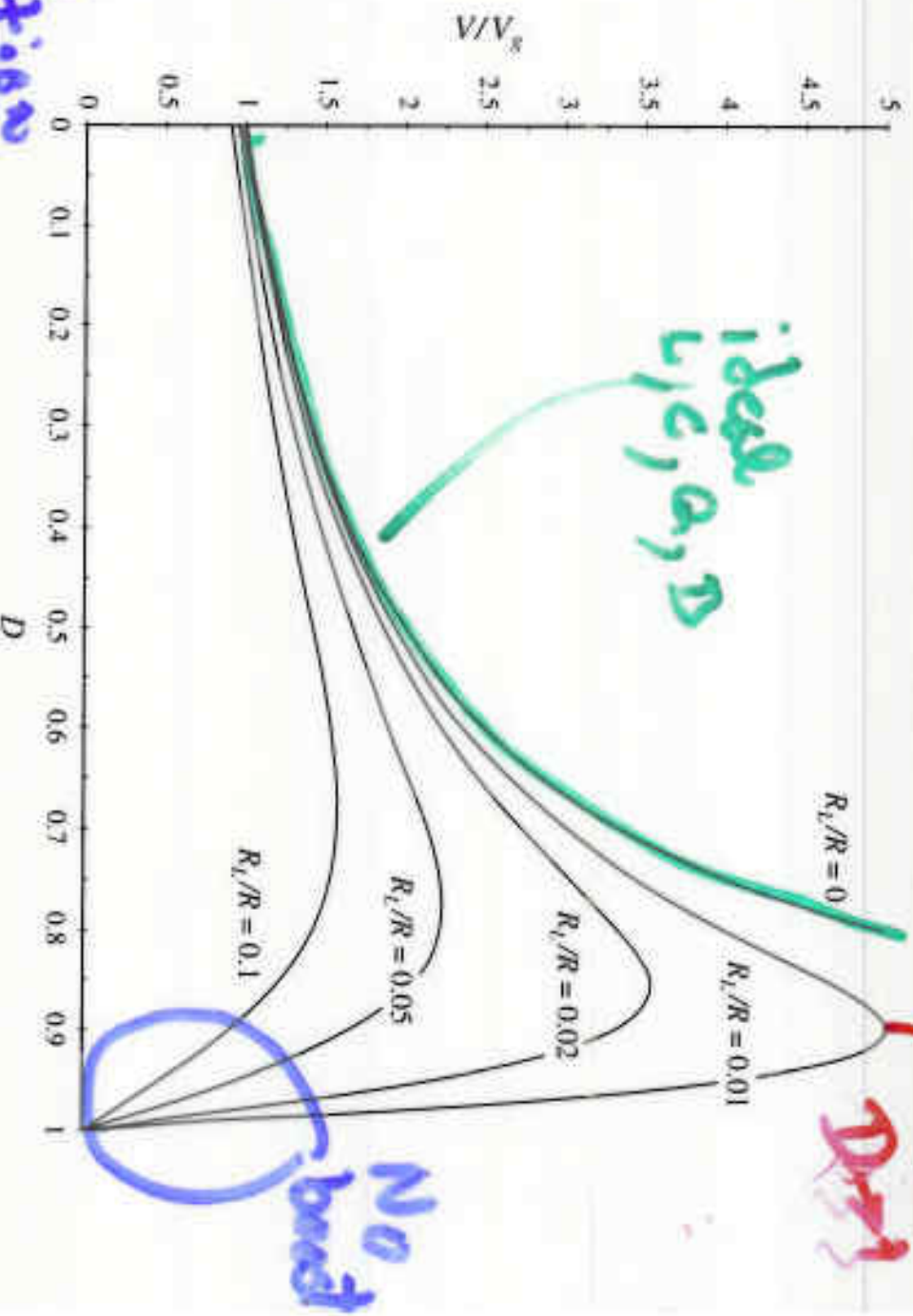
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Eliminate I and solve for V :

$$\frac{V}{V_s} = \frac{1}{D'} \frac{1}{(1 + R_L/D'^2 R)}$$

old new
ideal real
reduction



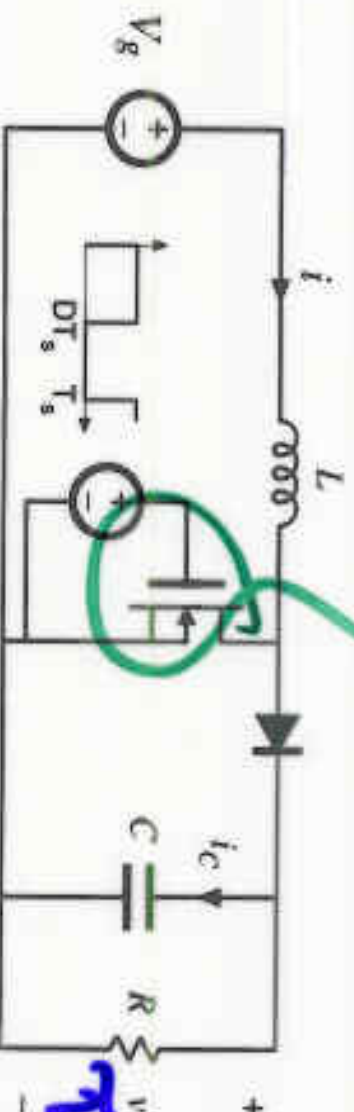
9552

3.5. Example: inclusion of semiconductor conduction losses in the boost converter model

Losses are 0

Boost converter example

Fig 3.12
9552



Typical FET:

Models of on-state semiconductor devices:

① MOSFET: on-resistance R_{on}

② Diode: constant forward voltage V_D plus on-resistance R_D

Insert these models into subinterval circuits



the components

R_{ON} (lowest) $30 m\Omega$
 $100 \mu\Omega \Rightarrow V_{ON} = 3V$

Analysis of nonideal boost converter

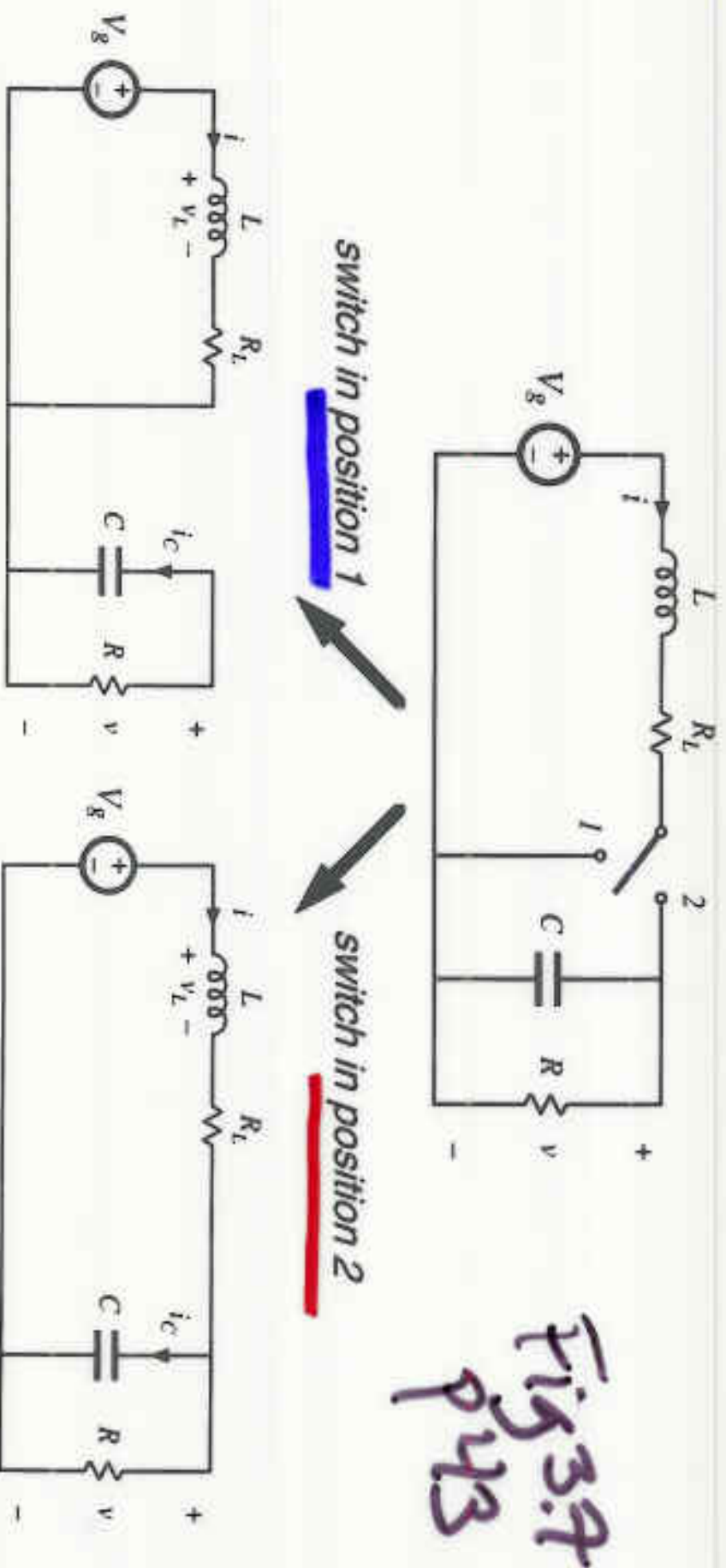


Fig 3.8 P479

Inductor voltage and capacitor current waveforms

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Inductor volt-second balance:

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Capacitor charge balance:

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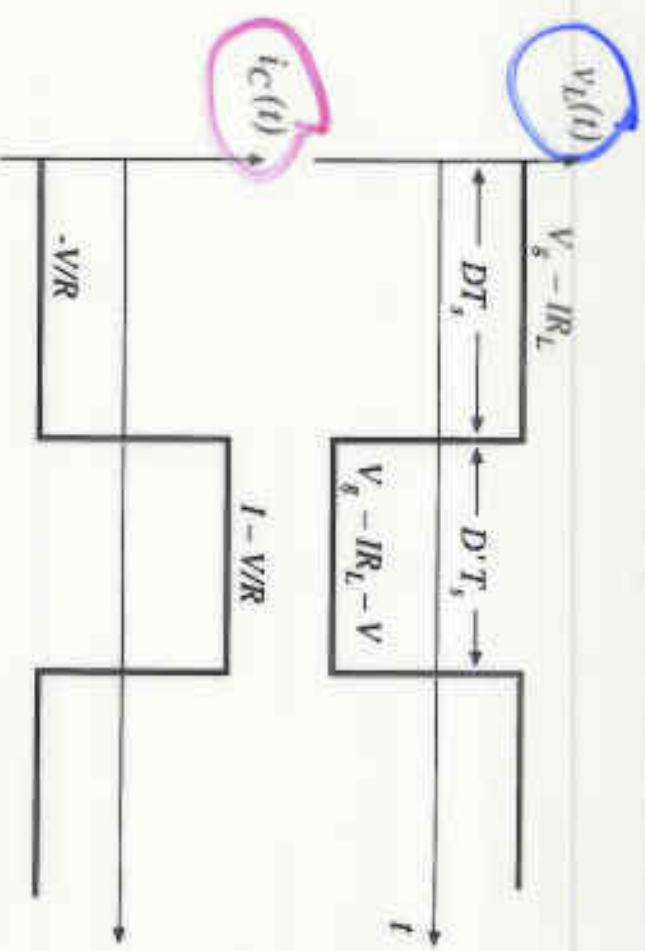


Fig 3.9 945

Solution for output voltage

lossy buck

no

possible

We now have two equations and two unknowns:

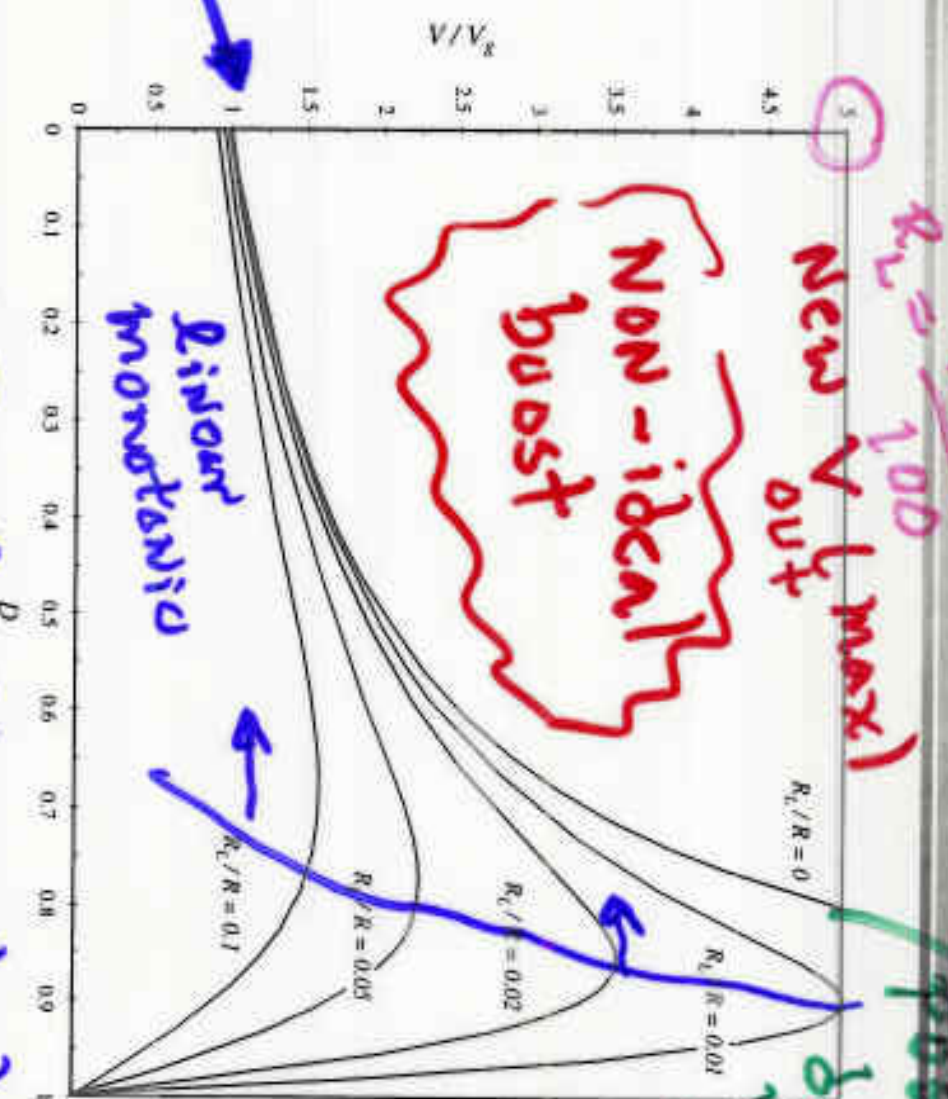
$$0 = V_g - I R_L - D'V$$

$$0 = D'I - V/R$$

Eliminate I and solve for V :

$$\frac{V}{V_g} = \frac{1}{D'} \frac{1}{(1 + R_L / D'^2 R)}$$

ideal ✓
Ch2
Usual boost lower limit
Ch3



$V(D)$ is double value

HW 3.8 What D is highest efficiency

Solution for output voltage

We now have two equations and two unknowns:

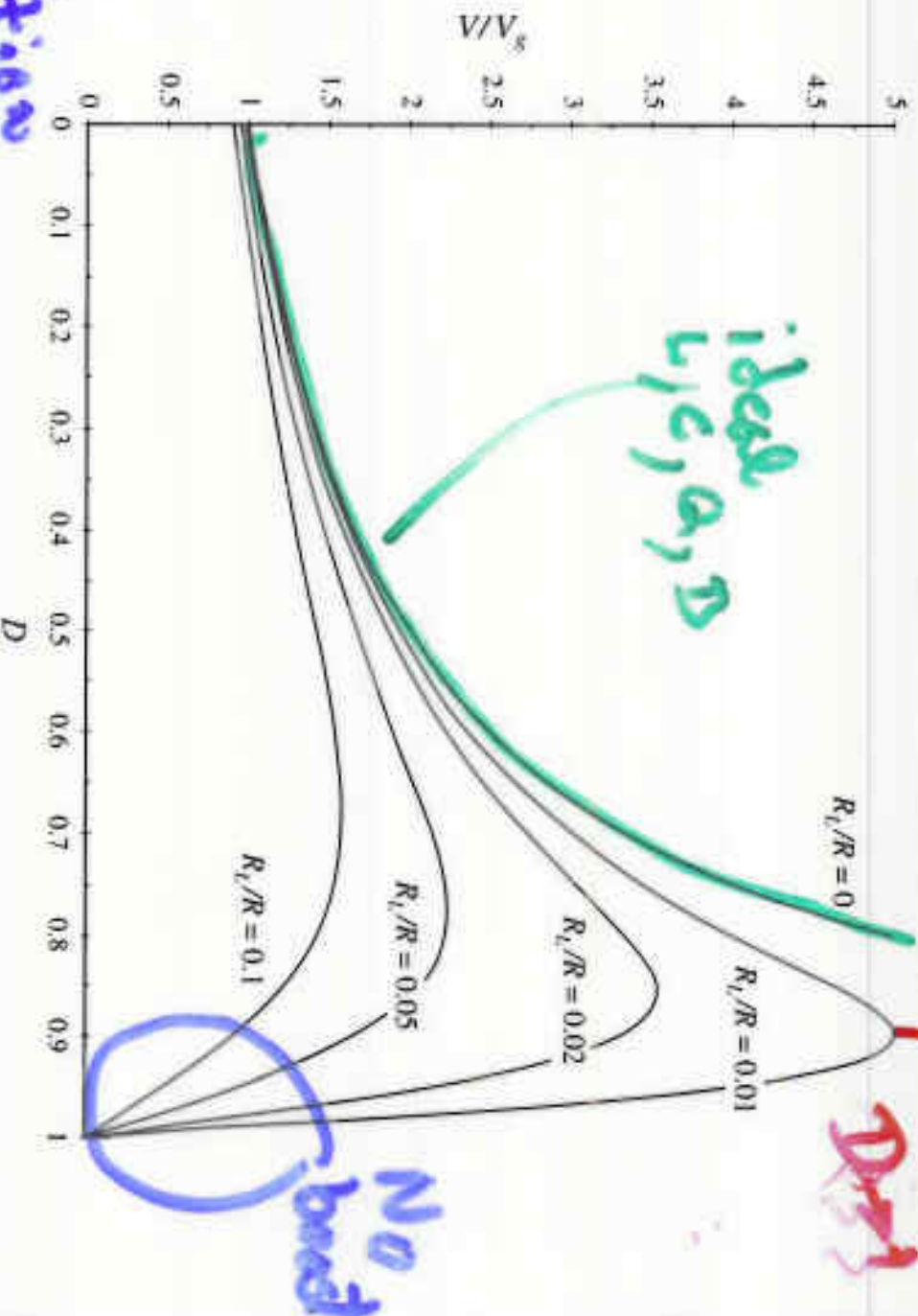
$$0 = V_s - I R_L - D'V$$

$$0 = D'I - V/R$$

Eliminate I and solve for V :

$$\frac{V}{V_s} = \frac{1}{D'} \frac{1}{(1 + R_L/D'^2 R)}$$

old ideal
new real
reduction



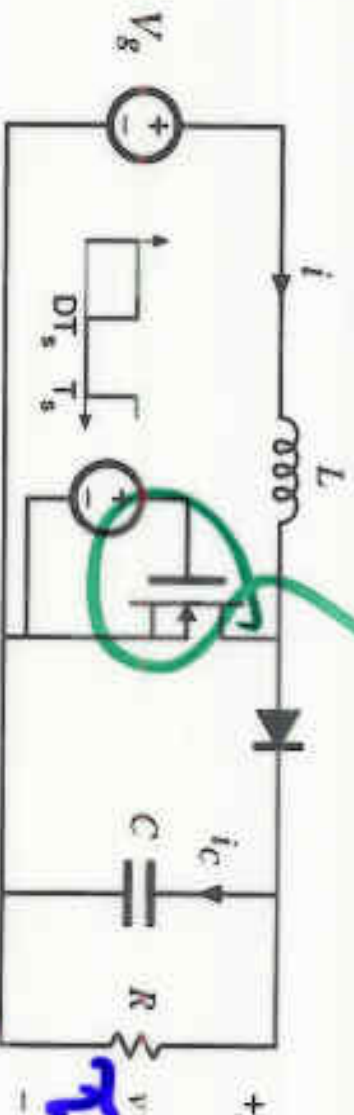
9552

3.5. Example: inclusion of semiconductor conduction losses in the boost converter model

losses are 0

Boost converter example

Fig 3.12
9552



Typical FET:

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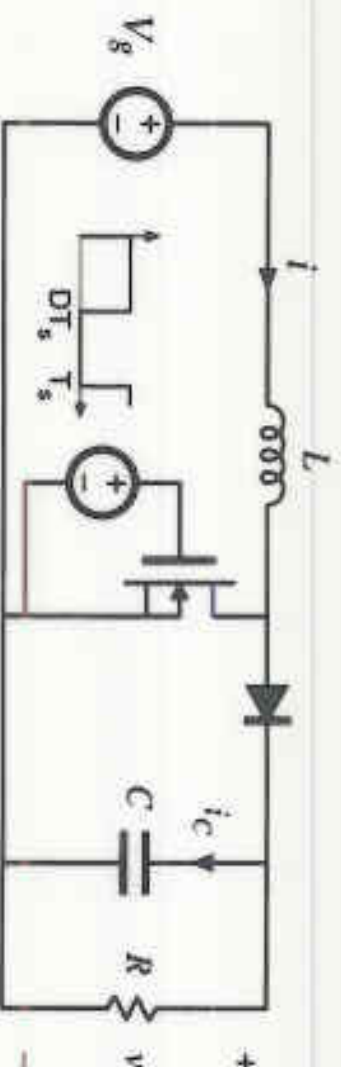
Insert these models into subinterval circuits



Two components

$$V_D R_{on} = f(D)$$

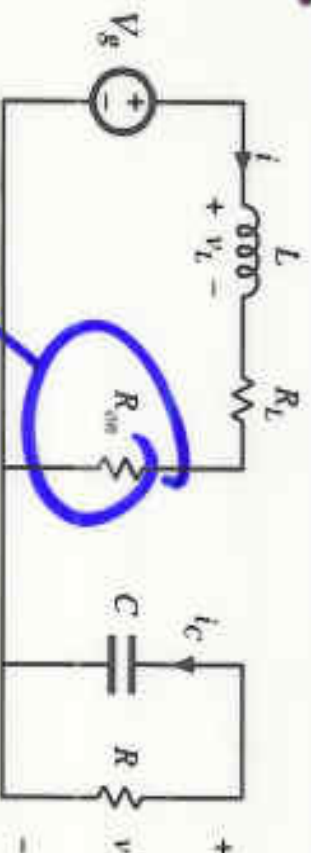
Boost converter example: circuits during subintervals 1 and 2



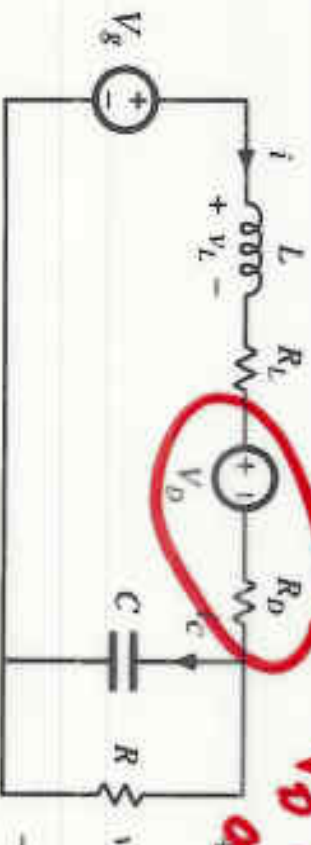
high β_{gs}

switch in position 1

switch in position 2



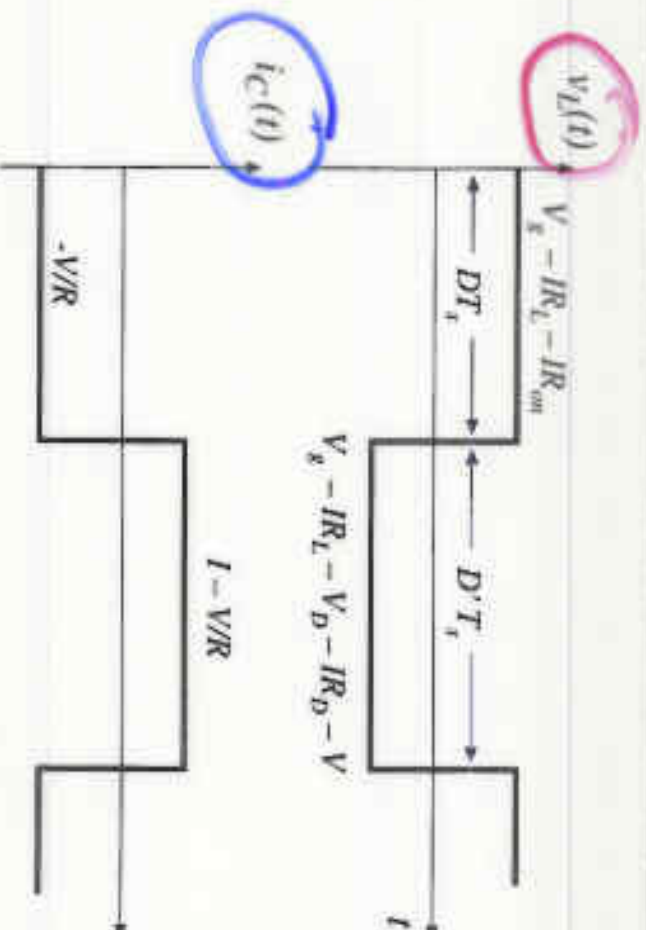
FET



diode V_o for all I

Average inductor voltage and capacitor current

443.24
443.53
9



$$\langle v_L \rangle = D(V_g - IR_L - IR_{on}) + D'(V_g - IR_L - V_D - IR_D - V) = 0$$

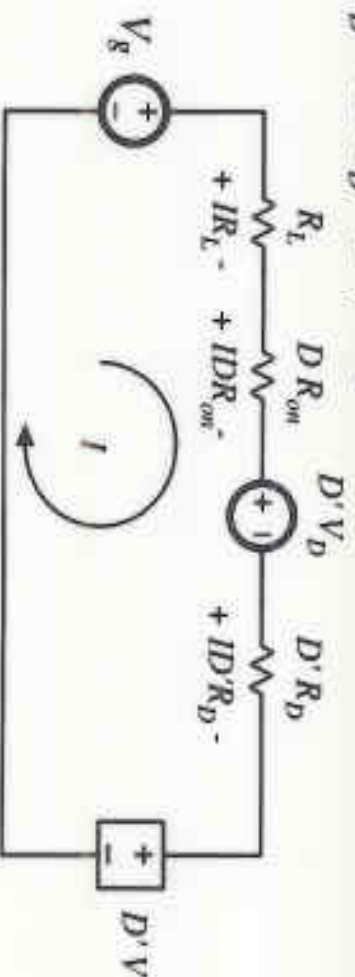
$$\langle i_C \rangle = D(-V/R) + D'(I - V/R) = 0$$

Fig 3.25 / 3.26 Pg 54

Construction of equivalent circuits

$\angle V_L > 0$ Input Circuit Loop

$$V_g - IR_L - IDR_{on} - D'V_d - ID'R_d - D'V = 0$$



$$\angle i_L > 0$$

$$D'I - V/R = 0$$

Output Loop

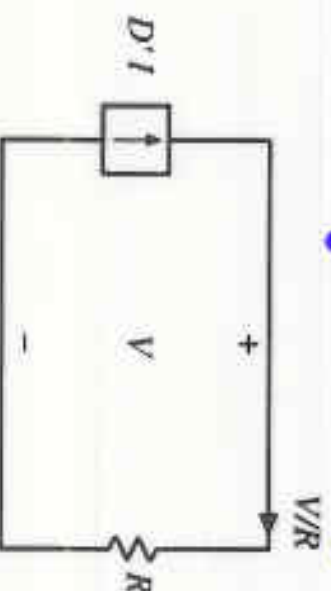
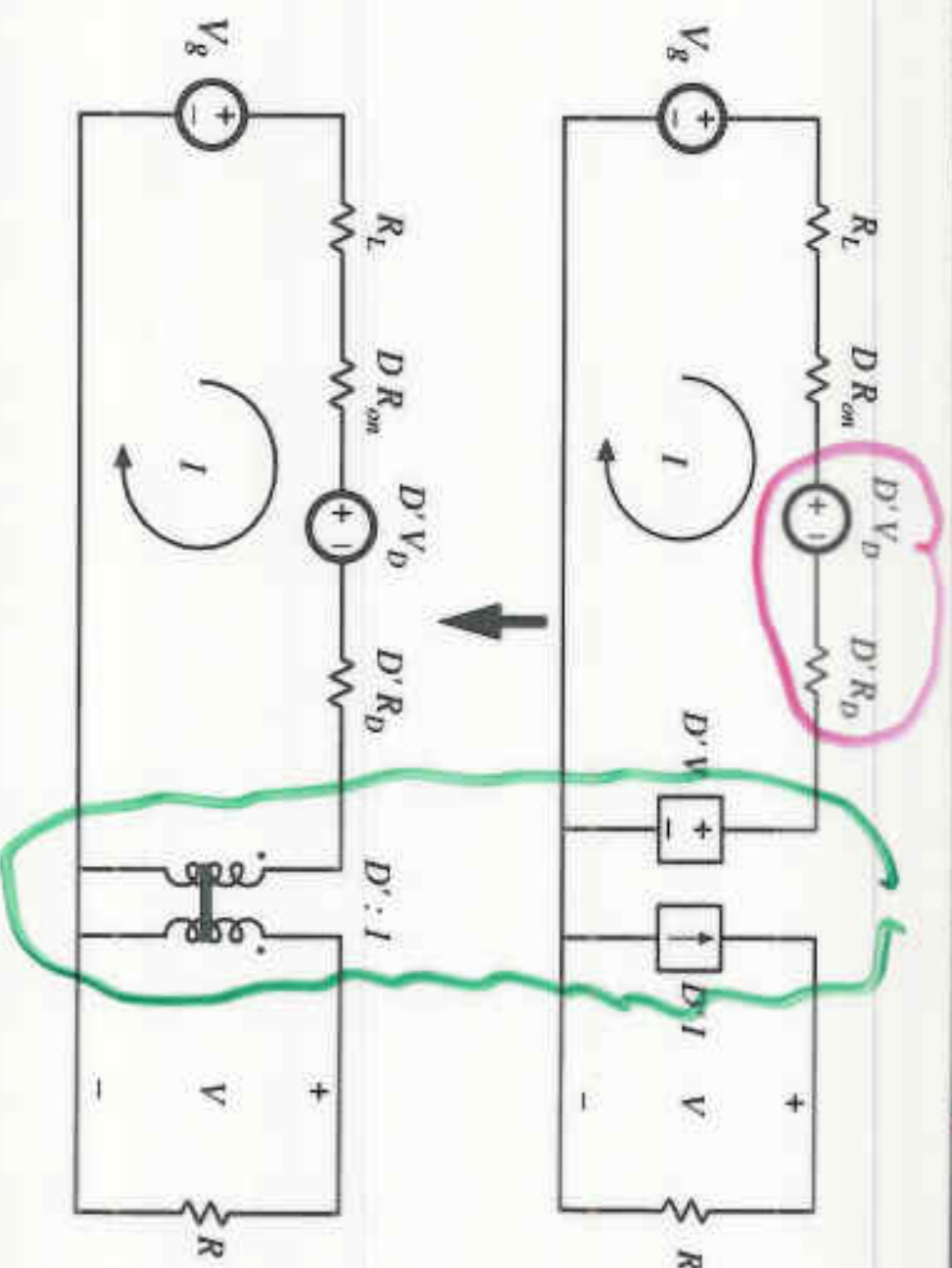


Fig 8.24 13.28

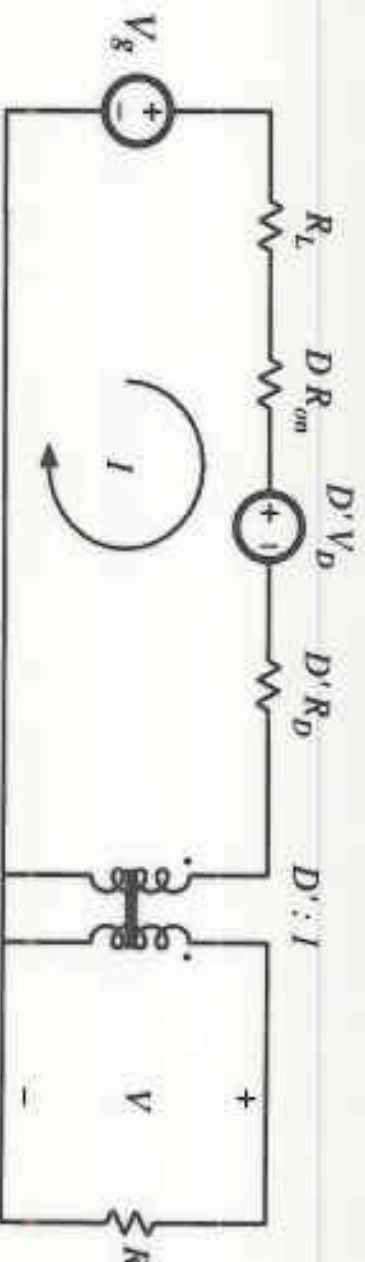
PS5

Complete equivalent circuit
Combine both loops together



Solution for output voltage

Fig 3.28



diode
losses

$$V = \left(\frac{1}{D} \right) \left(V_g - D' V_d \right) \left(\frac{D'^2 R}{D'^2 R + R_L + D R_{on} + D' R_D} \right)$$

$$\frac{V}{V_g} = \left(\frac{1}{D} \right) \left(1 - \frac{D' V_d}{V_g} \right) \left(\frac{1}{1 + \frac{R_L + D R_{on} + D' R_D}{D'^2 R}} \right)$$

ideal

boost

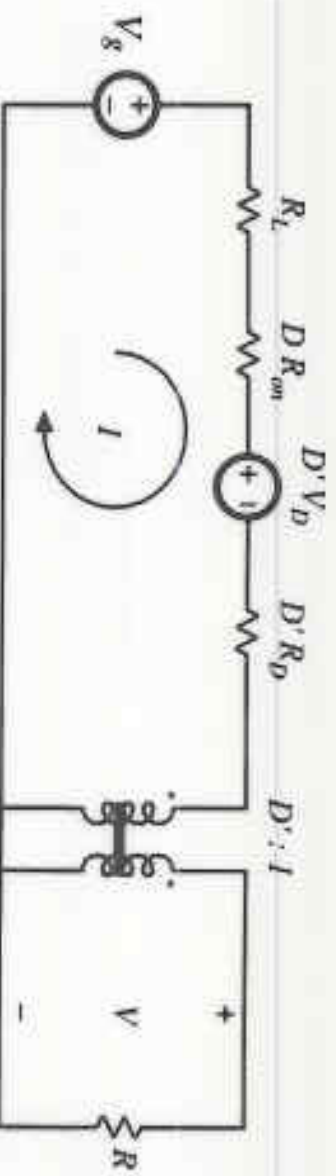
switch loss
corrections

Eq 3.34

Solution for converter efficiency

$$P_{in} = (V_g)(I)$$

$$P_{out} = (V)(D'I)$$



$$\eta = D' \frac{V}{V_g} = \frac{\left(1 - \frac{D' V_D}{V_g}\right)}{\left(1 + \frac{R_L + D R_{on} + D' R_D}{D'^2 R}\right)}$$

Conditions for high efficiency:

→ **Battery Operated Boost**

$$V_g / D' \gg V_D$$

$$\text{and } D'^2 R \gg R_L + D R_{on} + D' R_D$$

9945 Boost Analysis via

3.3 Construction of equivalent circuit model Solve for $\eta(D)$

Results of previous section (derived via inductor volt-sec balance and capacitor charge balance):

$$\langle v_L \rangle = 0 = V_g - I R_L - D'V$$

$$\langle i_C \rangle = 0 = D'I - V/R$$

View these as loop and node equations of the equivalent circuit.
Reconstruct an equivalent circuit satisfying these equations

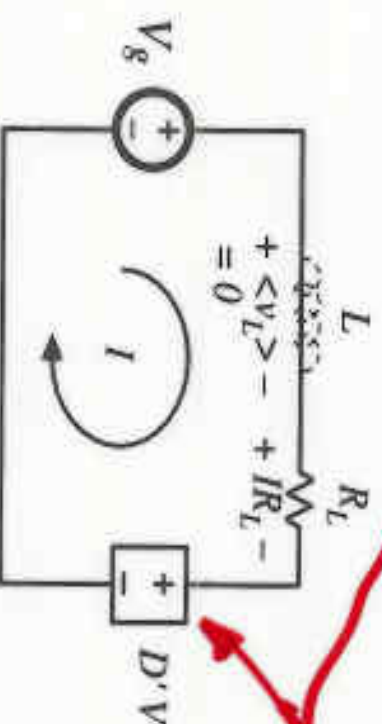
Inductor voltage equation

Equation $\rightarrow$ circuit loop

$$\langle v_L \rangle = 0 = V_g - I R_L - D'V$$

Fig 3.10
p 216

- Derived via Kirchhoff's voltage law, to find the inductor voltage during each subinterval
- Average inductor voltage then set to zero
- This is a loop equation: the dc components of voltage around a loop containing the inductor sum to zero

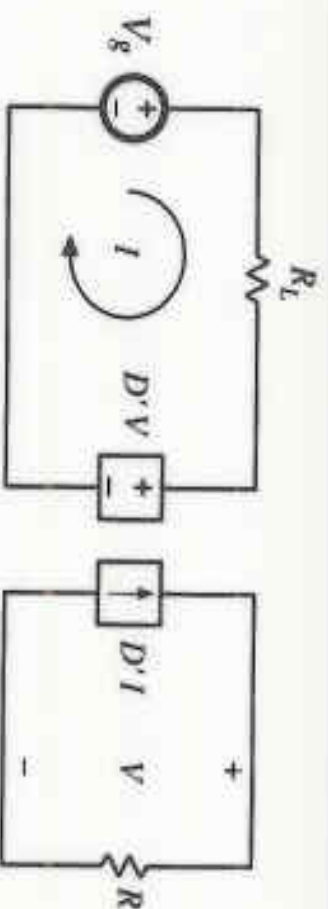


- $I R_L$ term: voltage across resistor of value R_L having current I
- $D'V$ term: for now, leave as dependent source

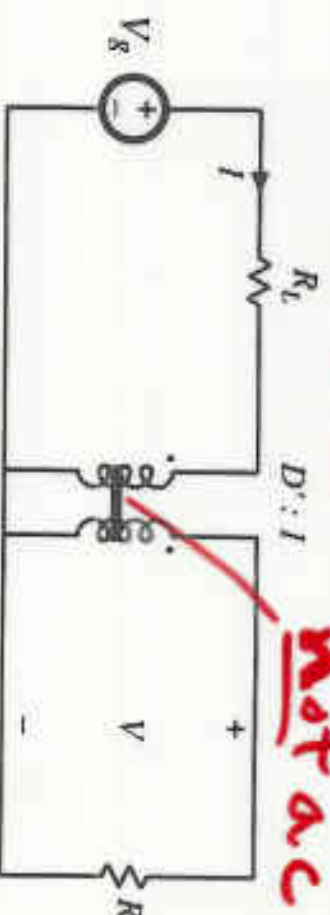
Complete equivalent circuit Both circuits together

Figs 3.12
3.13
Pg 47

The two circuits, drawn together:

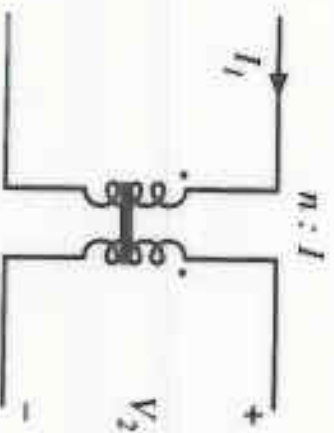
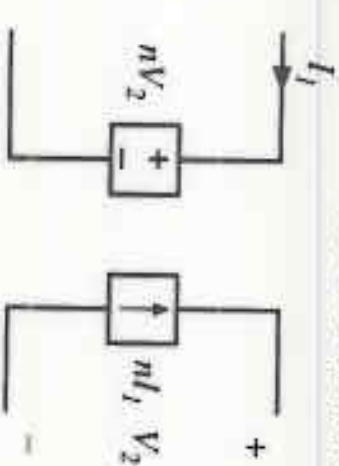


The dependent sources are equivalent to a $D' : I$ transformer:



DC T+
not ac

Dependent sources and transformers

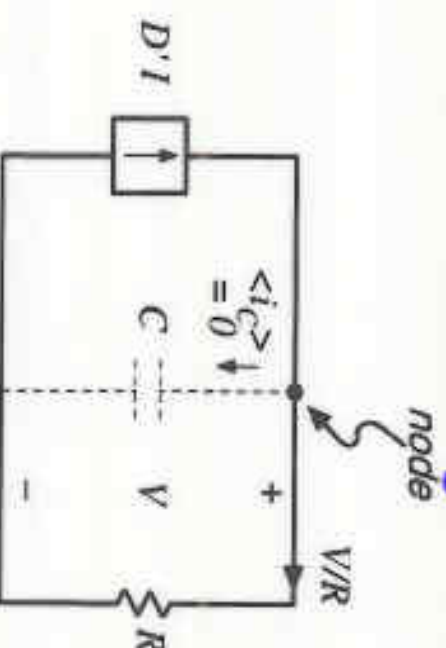


- sources have same coefficient
- reciprocal voltage/current dependence

Capacitor current equation

$$\langle i_c \rangle = 0 = D'I - V/R$$

- Derived via Kirchoff's current law, to find the capacitor current during each subinterval
- Average capacitor current then set to zero
- This is a node equation: the dc components of current flowing into a node connected to the capacitor sum to zero



2nd Eq. → 2nd circuit

Fig 3.11
P416

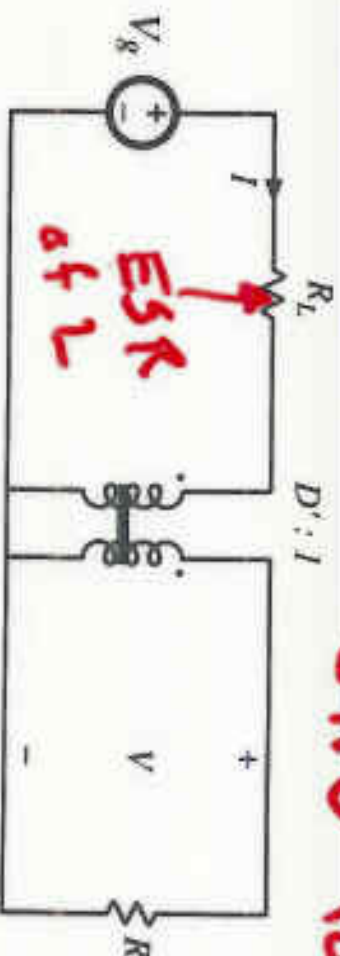
- V/R term: current through load resistor of value R having voltage V
- $D'I$ term: for now, leave as dependent source

Solution of equivalent circuit

Use trf rules to:

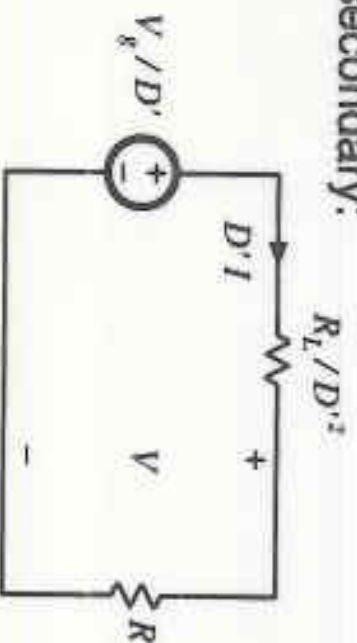
Reduce two loops to one loop

Converter equivalent circuit



Circuits are more insightful?
 than equations

Refer all elements to transformer secondary:

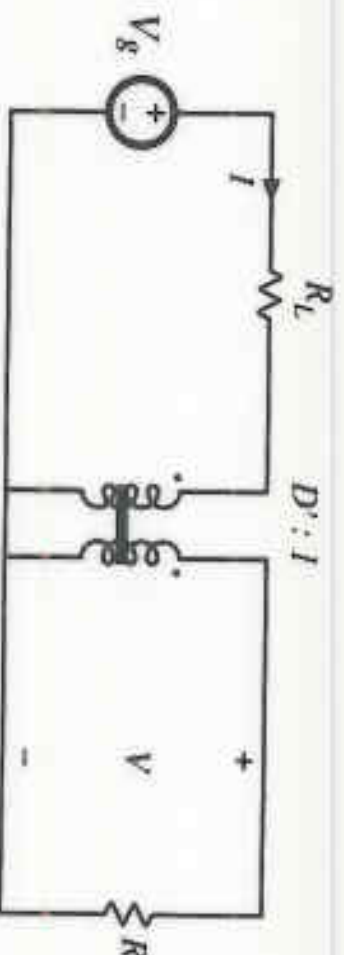


Solution for output voltage using voltage divider formula:

$$V = \frac{V_g}{D'} \frac{R}{R + \frac{R_L}{D'^2}} = \frac{V_g}{D'} \frac{1}{1 + \frac{R_L}{D'^2 R}}$$

Buck

Solution for input (inductor) current



"Average"
inductor
current

$$I = \frac{V_g}{D'^2 R + R_L}$$

$$= \frac{V_g}{D'^2} \frac{1}{1 + \frac{R_L}{D'^2 R}}$$

ideal
boost

correction
factor

Eg 3.19
Pg 48

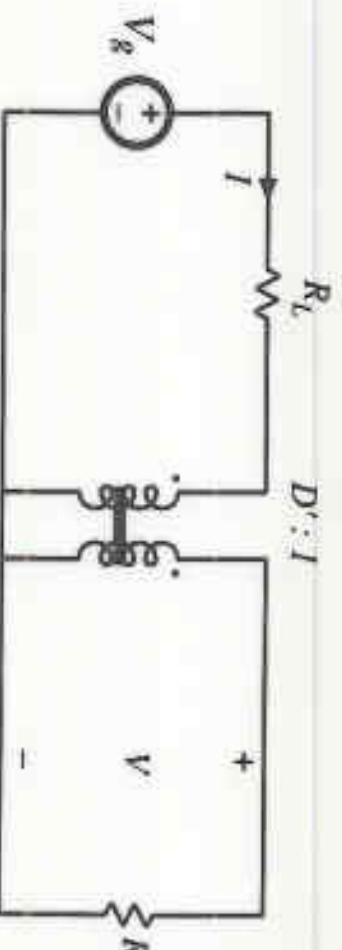
Solution for converter efficiency

Now dc efficiency is f(D)

$$P_{in} = (V_g)(I)$$

$$P_{out} = (V)(D'I)$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{(V)(D'I)}{(V_g)(I)} = \frac{V}{V_g} D'$$



$$\eta = \frac{1}{1 + \frac{R_L}{D'^2 R}}$$

Eq 3.23
Pg 48

Could/show you run boost at high D

Efficiency, for various values of R_L

Assume Inductor Losses Efficiency

$$\eta = \frac{1}{1 + \frac{R_L}{D^2 R}}$$

Fig 3.15
95 49

