**SIMULATION OF LCC RESONANT CIRCUITS**
POWER ELECTRONICS ECE562
COLORADO STATE UNIVERSITY
Modified December 2008

**PURPOSE**: The purpose of this lab is to simulate the LCC circuit using CAPTURE CIS and MATLAB® to better familiarize the student with some of its operating characteristics. This lab will explore some of the following aspects of the LCC resonant converter:
_ Input impedance
_ Input current
_ Output voltage
_ Output current
_ Output power
_ Poles and Zeros
_ Magnitude and Phase of the Transfer Function
_Real versus Apparent Power

**Simulation of the LCC Resonant Circuit Using CAPTURE CIS**
**NOTE**: The simulations that follow are intended to be completed with ORCAD 16.0 CAPTURE CIS®. It is assumed that the student has a fundamental understanding of the operation of CAPTURE CIS®. CAPTURE CIS® provides tutorials for users that are not experienced with its functions.

To start CIS do:

Start->All Programs->ORCAD16.0->ORCAD CAPTURE (and not CIS)

Select OrCAD_Capture_CIS_option with OrCAD PCB Designer_PSpice

To start a new project do:

File -> New ->Project name project and check button for “Analog or Mixed A/D” OK

Check “Create a blank project” OK

When opening a previous project do:

File -> Open ->Project Pick the <my_project_name>.opj file

**PROCEDURE**:

**Part 1: Input Impedance**

Build the schematic shown in Figure 1 below.
**V1** is an AC voltage source (VAC) from the source library. Set to 1Vac, 0Vdc.
**L** is an ideal inductor from the Analog Library. Set to 25µH.
**R** is an ideal resistor from the Analog Library. Set to 25 (ohms)
**$C_s$** is an ideal capacitor from the Analog library. Set to 200nF.
**$C_p$** is an ideal capacitor from the Analog library. Set to 66nF.
The GND is 0/CAPSYM from the “Place Gnd” function.

Note: to change a device value, select (Left Mouse) just the value(not the entire device), then (Right Mouse) Edit Properties. Do not leave a space between the value and the unit, for instance “66n”, not “66 n” for the 66nF value of **$C_p$**. Use “p” for pico, “n” for nano, “u” for micro, “m” or “M” for milli, “k” for kilo, “Meg” for mega, and “G” for giga. The units are ignored for R, L, and C, although F can be added for capacitance in Farads, H can be added for inductance in Henries, and O can be added for resistance in Ohms, to enhance the human readability of the schematic. Don’t use the “O” unit for Ohms because it looks too much like a zero.

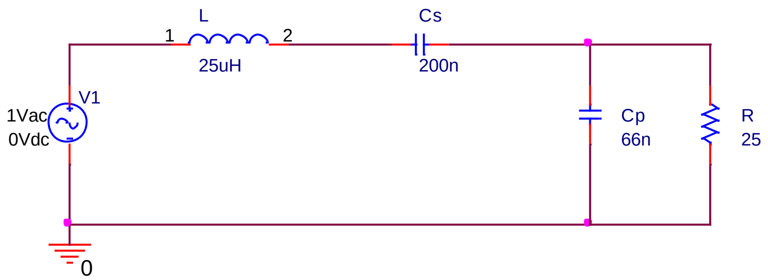


Figure 1 - PSPICE LCC Tank Circuit Schematic

Question: What is the input impedance of this LCC circuit?

Answer:
From Laplace transform theory, we know that the complex input impedance is equal to the impedance of the series inductor L, plus the impedance of the series capacitor $C_s$, plus the impedance of the parallel $C_p$ and R.

First, find the impedance of the parallel $C_p$ and R:

$$Ypar = \frac{1}{Zpar} = sCp \; + \frac{1}{R}$$

Therefore : $Ypar = Cp\left(s + \frac{1}{RCp}\right)$ and $Zpar \; = R\,\frac{\left(\frac{1}{RCp}\right)}{s + \frac{1}{RCp}}$

The $Z_{par}$ numerator and denominator are both of order $s^1$, so the fraction is dimensionless. Therefore, $Z_{par}$ has units of resistance, as expected.

Notice that this is first order, since there is only one energy storage element, $C_p$. There is a real axis pole at $s = -\frac{1}{RCp}$ and a zero at $s = \infty$.

Therefore, at DC (s=0), $Z_{par}$ = R, or purely resistive.
At frequencies of $s = j\omega \gg \frac{1}{RCp}$ , $Z_{par}$ = 1/s$C_p$ , or purely capacitive.

Now for $Z_{in}$: $Zin = sL + \frac{1}{sCs} + \frac{\frac{1}{Cp}}{s+\frac{1}{RCp}}$

Combining terms into a common denominator:

$$Z_{in} = L\frac{\left[s^3 + s^2\left(\frac{1}{RCp}\right) + s\left(\frac{1}{LCs} + \frac{1}{LCp}\right) + \frac{1}{RLCsCp}\right]}{\left[s\left(s + \frac{1}{RCp}\right)\right]}$$

All the terms in the numerator are of order $s^3$, and the denominator is order $s^2$, therefore the fraction is of order $s^1$. Therefore $Z_{in}$ has units of sL, or impedance, as expected.

Question: Why is this a cubic equation?
Answer: Because there are three unique energy storage elements, L, $C_s$, and $C_p$ .

$Z_{in}$ has a pole at s=0, due to the series $C_s$ , and at $s = -\frac{1}{RCp}$ due to the parallel R and $C_p$ having infinite impedance at resonance, i.e. the pole of $Z_{par}$. $Z_{in}$ also has a pole at s=∞ since the numerator is of higher order than the denominator. This is due to the series L.

$Z_{in}$ has zeros at the roots of the cubic numerator. Two of the roots are complex, and should be in the vicinity of $\omega 0 = 1/(\sqrt{LCs}$ , since the series L and $C_s$ are resonant at this frequency, and have zero impedance.

$$\omega 0 = \frac{1}{\sqrt{LCs}} = \frac{1}{\sqrt{(200\, nF * 25\, uH)}} = 4.47x\, 10^5 \left(\frac{rad}{\sec}\right) = 71.2\, kH$$

The effect of $C_p$ and R will cause $Z_{in}$ to have a minimum at a higher frequency.

Use the student version of Capture CIS that is installed in the CSU computer lab to simulate the circuit  above.

Set up the Simulation Settings for AC Sweep/Noise with a start frequency of 100 Hz to 10 MHz, using the pulldown menu “ Pspice->Edit Simulation Profile’ as shown below in Figure 2.

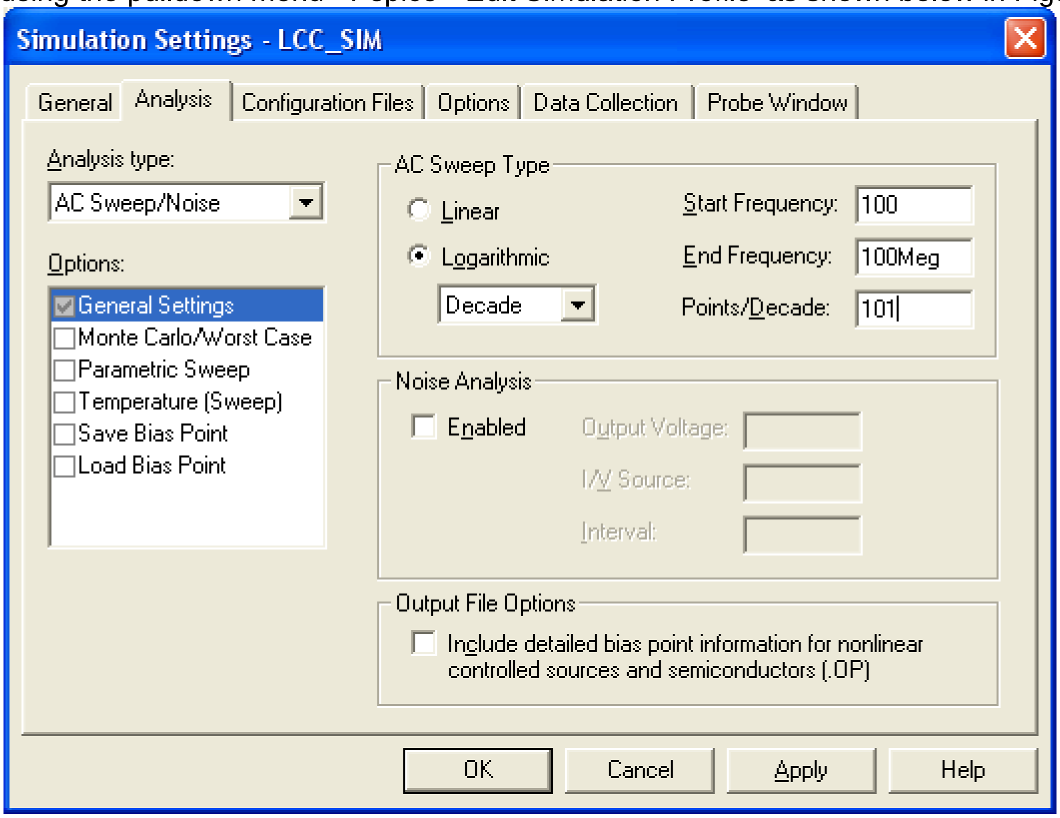


Figure 2 - Simulation Settings

Click OK

Do Pspice->Run (F11)

Figure 3 below is the result of the input impedance simulation of the LCC tank circuit.

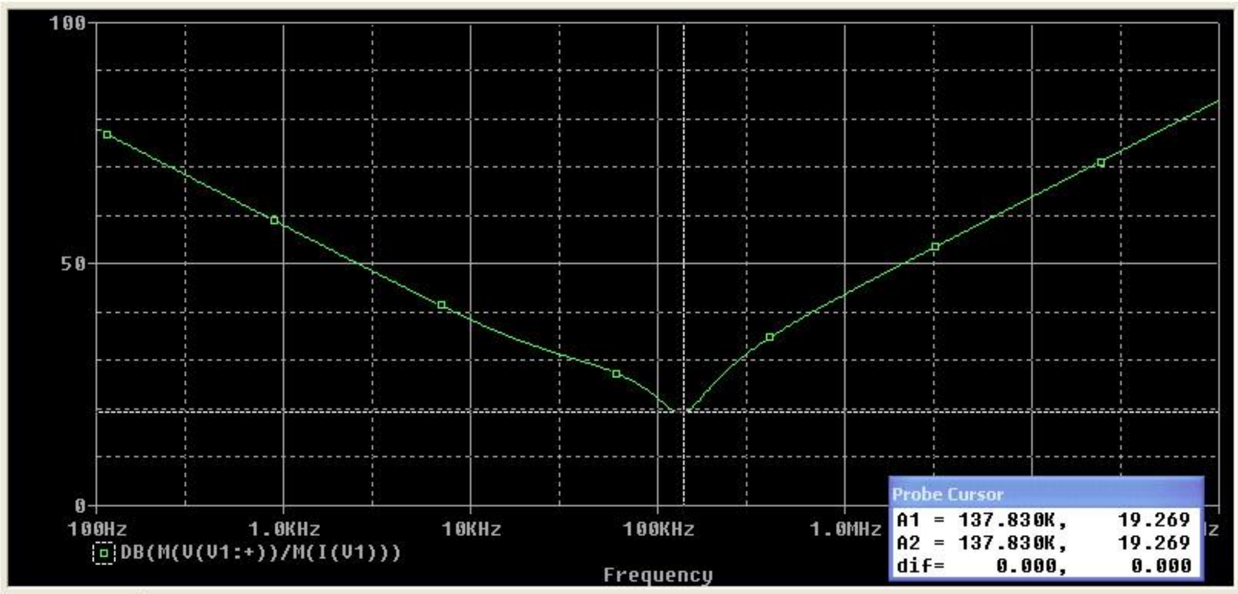


Figure 3 - Input Impedance Trace

Set up a $Z_{in}$ trace by doing Trace-> Add Trace and defining Zin(dB) as:
DB(M(V(V1:+))/M(I(V1))) , as shown in the lower left corner above.
Notice how the vertical scale has no units for computed functions, such as $Z_{in.}$

Notice that the resonant frequency is 137.8 kHz, and the minimum magnitude is 19.3 dB(Ohms) = 9.2 Ω.
This can be found by doing: Trace->Cursor->Display
And Trace->Cursor->Trough
Or by clicking the icons for each function.

### Part 2: Output Voltage

Next, we want to run the simulation of the output voltage of the LCC circuit. Use the same circuit as above, and place a **voltage marker** as shown in Figure 4 below.
Hint: With the marker highlighted, hitting the “r” key will rotate it 90º counterclockwise. Hit the “Esc” key to stop the generation of markers.

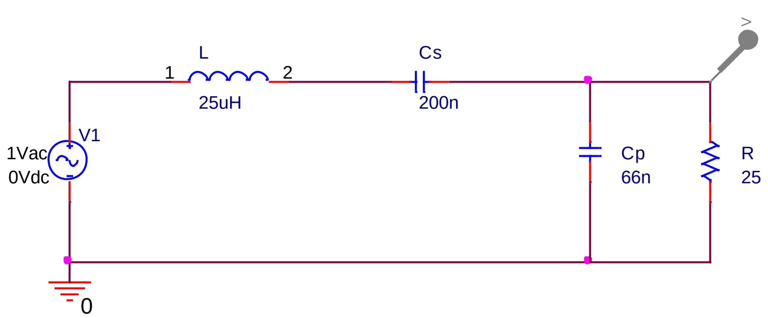


Figure 4 - Output Voltage Probe

*Figure 5 shows the output voltage of the LCC circuit.*

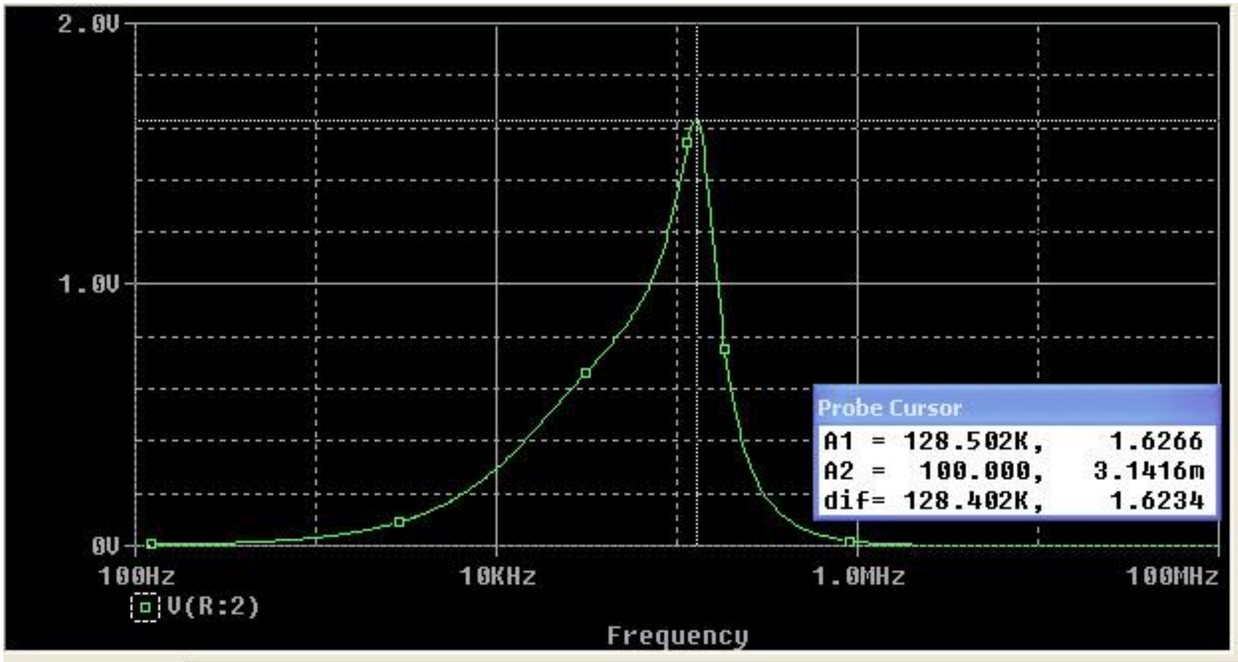


Figure 5 - LCC Output Voltage

*Question: What is the output voltage peak and resonant frequency of the LCC circuit?*
Hint: This can be found by doing: Trace->Cursor->Display
And Trace->Cursor->Peak
Or by clicking the icons for each function. Notice how the vertical scale units of V.

Note that the resonant frequency is nearly the same as the Zin peak. This makes sense, since the output voltage is the input current times Zpar, and the input current is a maximum when Zin is a minimum.

Note that the output voltage peaks at 1.6266 times the input voltage.
Question: How can the output voltage be greater than the input in a passive circuit such as this?

Answer: This ratio is proportional to the "Q" of the circuit. We'll see later that the output power is still equal to the real input power at all frequencies, including at resonance.

Now remove **voltage markers** from the circuit, and create a Bode plot by placing the "**db magnitude of voltage"** marker and the "**phase of voltage**" marker on the output resistor as shown in Figure 6. These are found in the pull-down menu at:

PSpice->Markers->Advanced->dB Magnitude of Voltage.
PSpice->Markers->Advanced->Phase of Voltage.

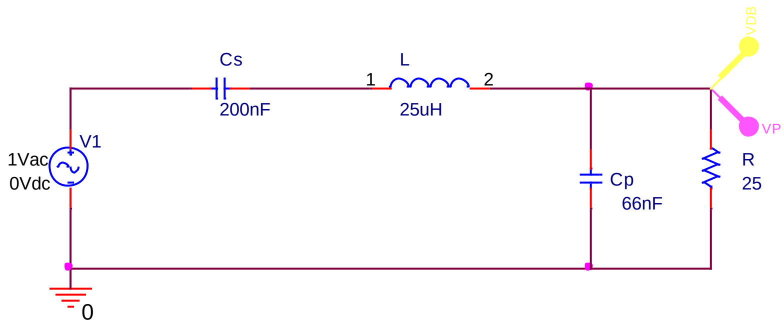


Figure 6 - Output Bode Plot Configuration

Figure 7 below is the result of the output voltage of the LCC tank circuit.

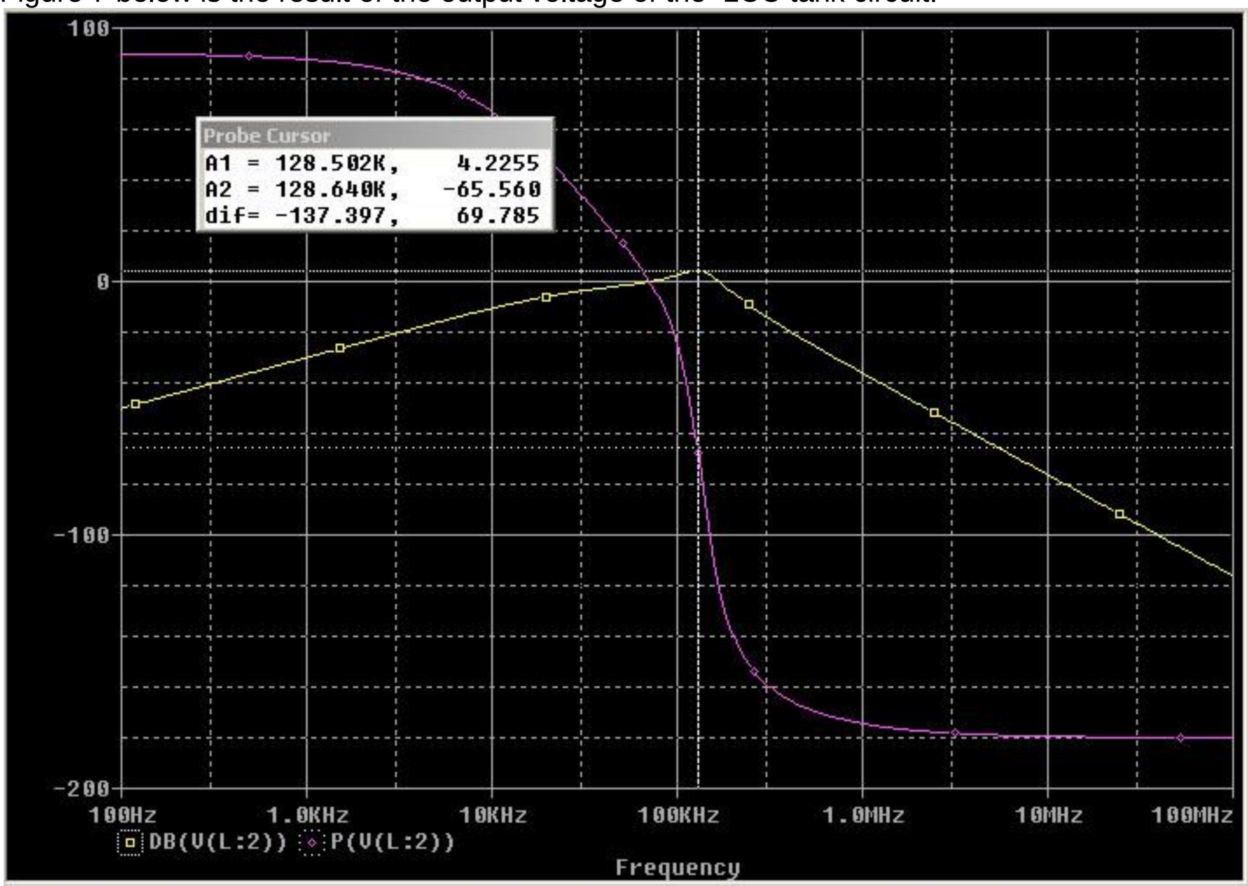


Figure 7 - Output Voltage Bode Plot: Yellow = log magnitude(A1 marker0; Magenta = Phase Angle(A2 marker)

Notice how the vertical scale has no units when more than one signal type is displayed. In this case, the vertical scale is in degrees and dB(V). A marker can be slaved to a given signal by clicking the small icon to the left of the signal definition at the lower left corner of the display. The LMB slaves the A1 marker, and the RMB slaves A2. The A1 marker shows as a square icon with 6 dots/side, and the A2 marker has 3 dots/side.

Question: What is the peak output voltage of the LCC circuit?
Answer: The voltage peaks at +4.22 dBV at 128 kHz(marker A1).

Note that $4.22\ dBV\ =\ 20log\left(\frac{1.6255V}{1V}\right)$ from Figure 5

Question: What is the phase angle of the output voltage of the LCC circuit?

Answer: At low frequencies, the phase angle is +90º.
At high frequencies, the phase angle is -180º.
At the resonance, the phase angle is in between, or -65º (marker A2).

Question: Why is there a -270º phase shift from low to high frequencies?
Answer: We'll learn why in the MATLAB 2nd half of this lab.

Question: At the 128 kHz resonance Vout peak, why is the phase -65 º, instead of halfway in between +90º and -180º, or -45º?.
Answer: We'll learn why in the MATLAB 2nd half of this lab.

If you prefer to see Bode plots with separate magnitude and phase plots, do:

Plot->Add Plot to Window …. To create a new blank plot window
Trace->Add Trace …. To add a new trace to this window

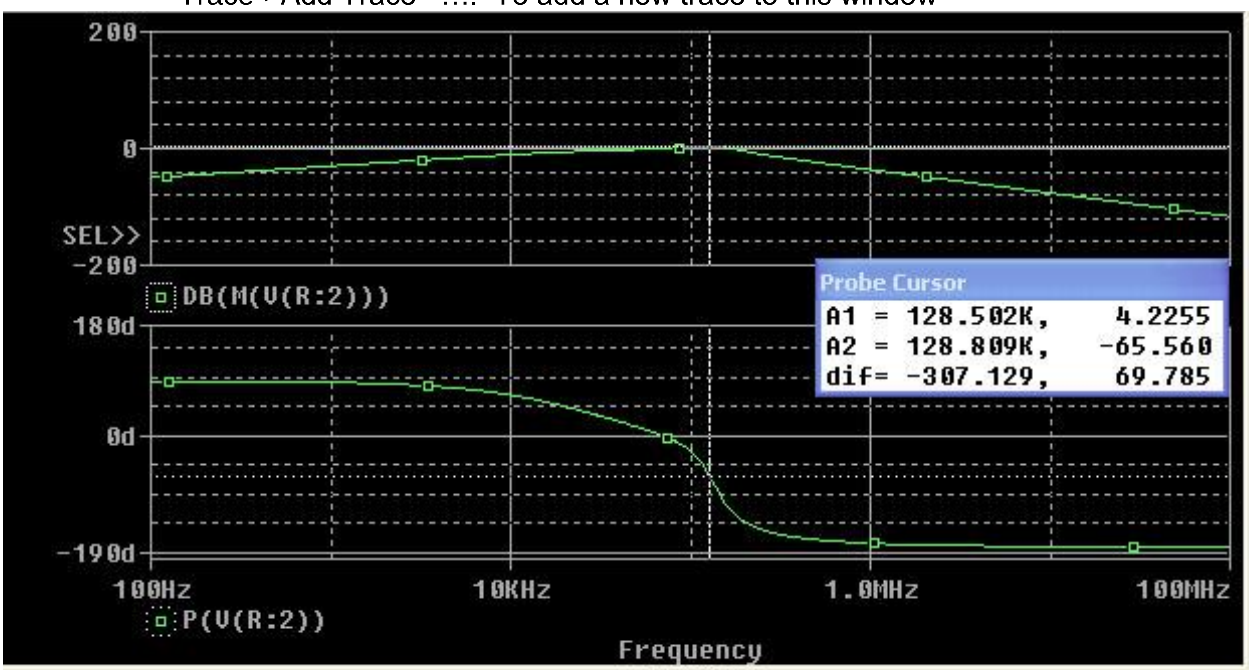


Figure 8 – Output Voltage Bode Plot with separate plots

**Part 3: Input or Inductor Current:**

Use the same circuit as above and remove the "**dB Magnitude of Voltage"** marker from the circuit. Place the "**dB Magnitude of Current** " and the "**Phase of Current**" markers next to L, as shown in Figure 9.

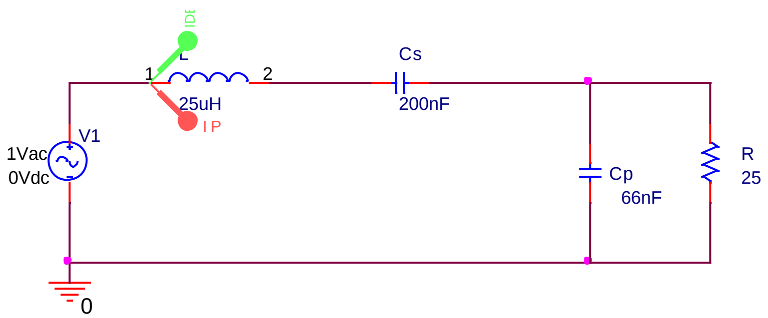


Figure 9 - Inductor Current Bode Plot Configuration

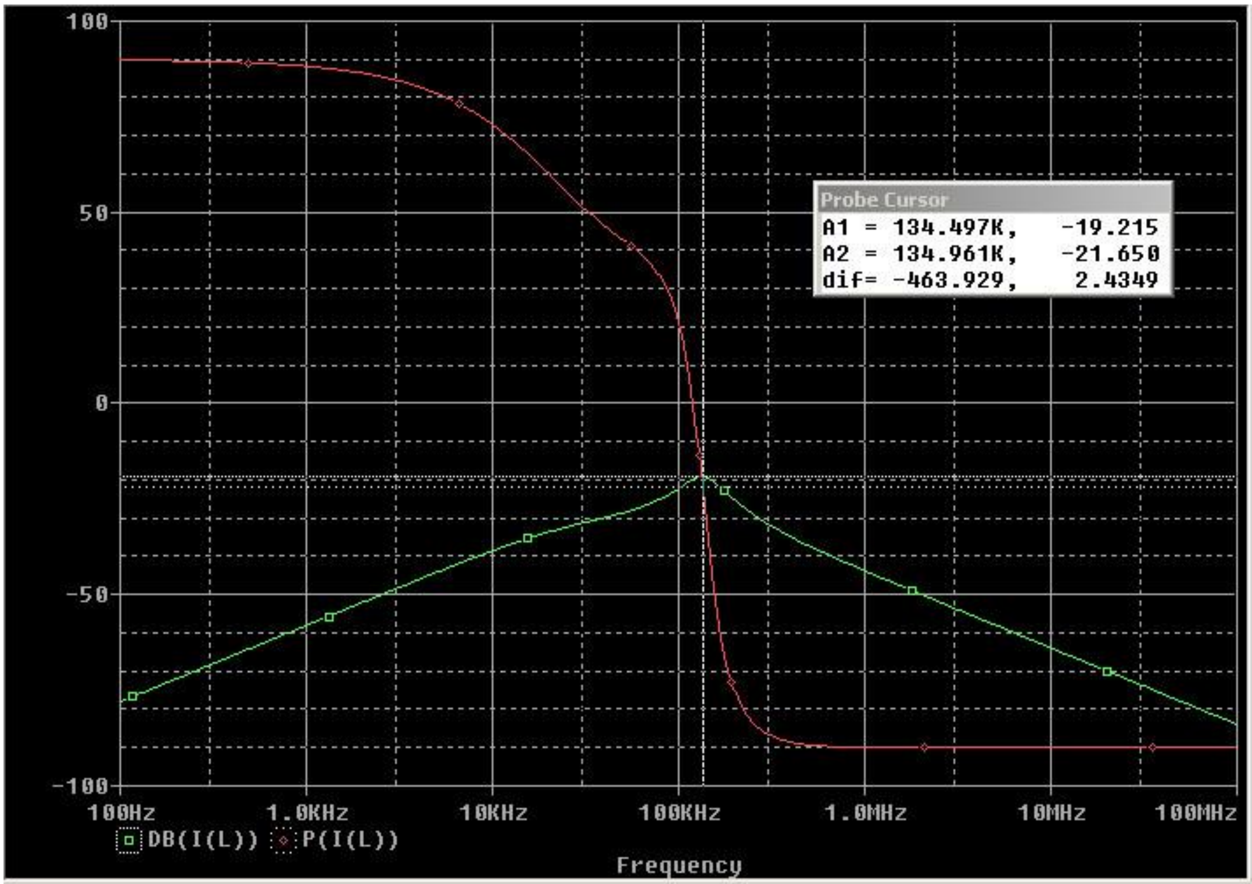


Figure 10 - Inductor (input) Current Bode Plot – Green = log magnitude; Red = phase angle

Question: What is the value of the inductor current of the LCC circuit?
Answer: It peaks at -19.21 dB(Amp) at 134 kHz Note that this magnitude is basically the inversion of Figure 3, the $Z_{in}$ plot. This is because $I_{in} = V_{in} / Z_{in}$ and $V_{in} = 1V$.

Question: What is the phase value of the inductor current of the LCC circuit?
Answer: At low frequencies, the phase angle is +90º.
At high frequencies, the phase angle is -90º.
At resonance, it is in between at -21º.

Question: Why is this?

Answer:  At low frequencies, the input impedance is dominated by the series $C_S$, so the current leads the voltage by 90º.  At high frequencies, the input impedance is dominated by the inductor L, so the input current lags the voltage by 90º.

Question: Why does the input current go through -180º of phase shift from low to high frequencies, when the output voltage went through -270º?

Answer:  We'll learn why in the MATLAB 2nd half of this lab.

Question:  How would this input current simulation be different if the current magnitude and phase probes were moved to the right of the inductor L, or moved to the right of $C_s$ ?

Answer:  None.  The input current is the same through V1, L and $C_s$ . However, the Phase of Current marker indicates the phase of current going into the device at the node it is placed on. Therefore, it will indicate an inversion, or 180º phase shift from a probe on the current output terminal of the device. See Figures 11 and 12 below.

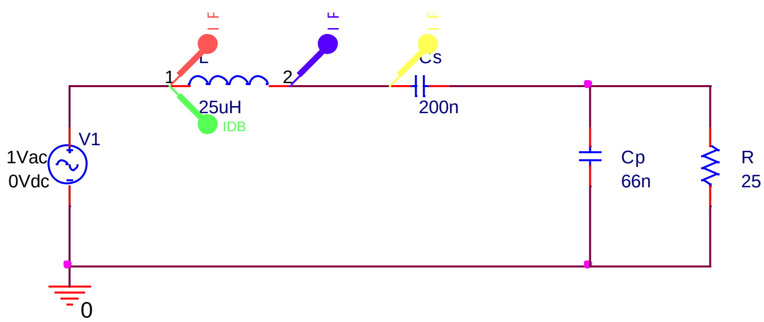


Figure 11 - Input Current with phase probes on input and output terminals

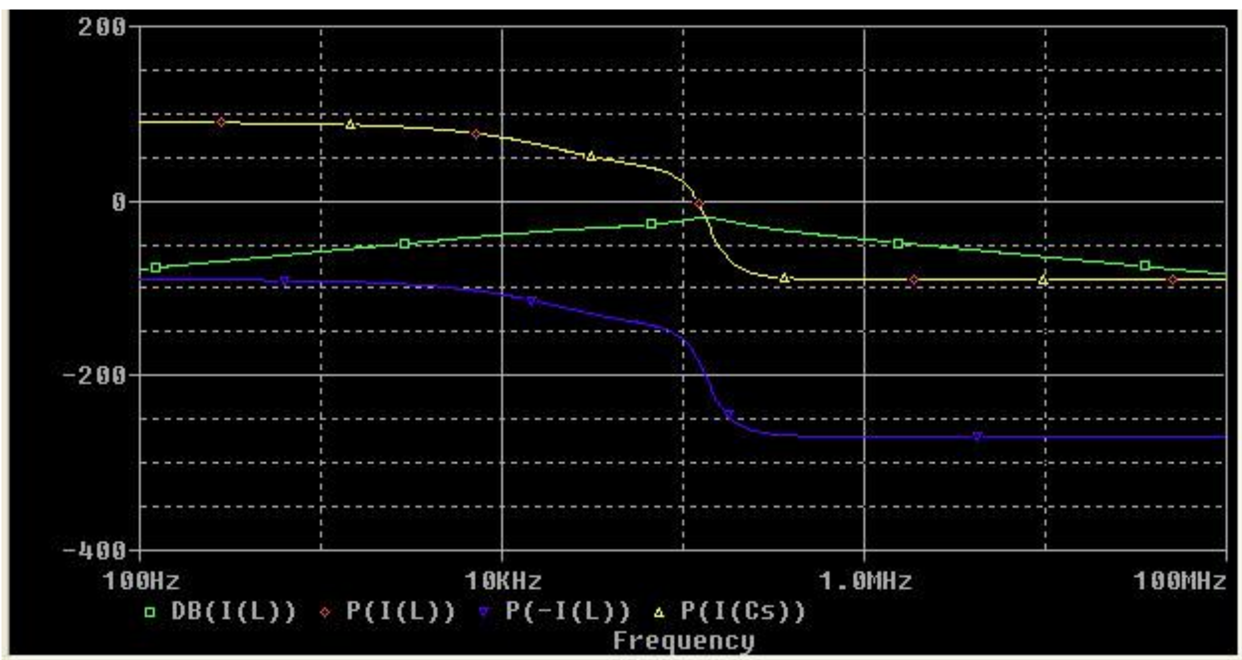


Figure 12 - Blue phase probe has -180º phase shift (opposite polarity) from red and yellow probes

**Part 4: Current Division in the Output**

Let's see how the input current splits between the parallel $C_s$ and R, looking first at the dB magnitudes in Figures 13 and 14.

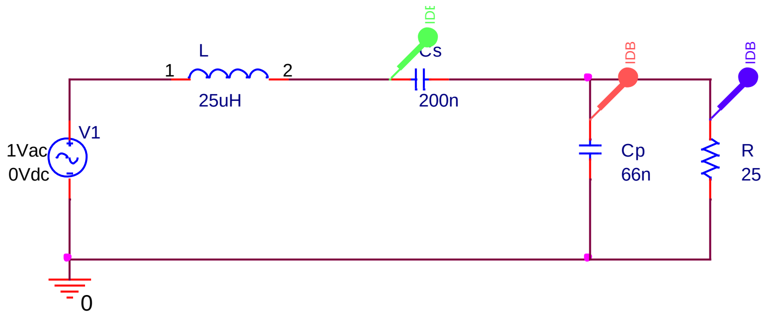


Figure 13 - Input Current Division - dB Magnitude

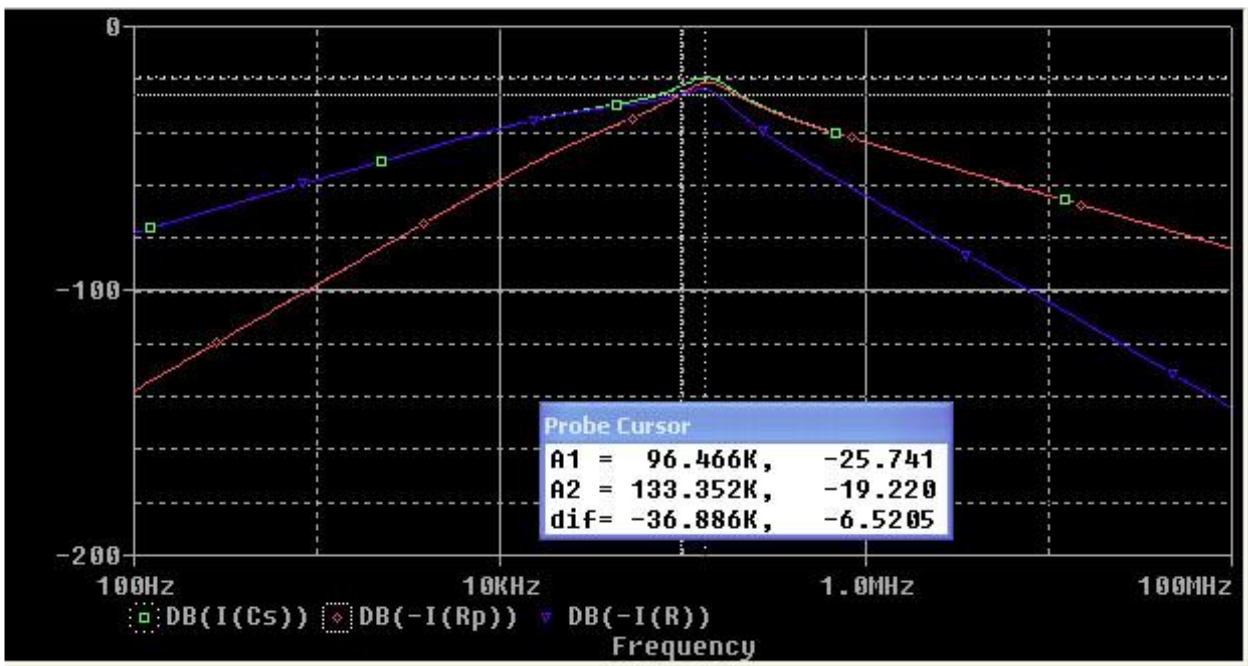


Figure 14 - Input Current Division - dB Magnitude

Notice how the input current through Cs (green) goes predominately through R (blue) at low frequencies, below resonance. At high frequencies, it goes predominately through $C_p$ (red).

Question: Why is this:
Answer:
At low frequencies, the $C_p$ impedance is high, so the input current goes entirely through R.
At high frequencies, the $C_p$ impedance is low, so the input current goes entirely through $C_p$.

Notice how the $C_s$ and R currents cross over at 96.5 kHz(marker A1), which is less than the peak input current at 133 kHz(marker A2).

Question: Why are the $C_p$ and R currents equal at 96.5 kHz?

Answer: Because $C_p$ and R are resonant at :

$$s = -\frac{1}{RCp} \text{ because } Zpar = R\frac{\left(\frac{1}{RCp}\right)}{s+\frac{1}{RCp}}$$

$$\omega_0 = \frac{1}{25\ ohms * 66\ nF} = 6.06\ x\ 10^5 \left(\frac{rad}{\sec}\right) = 96.5\ kHz$$

Now, determine how the phase of the input current divides between $C_p$ and R, by placing Phase of Current markers on the input of $C_s$, $C_p$, and R, as shown in Figure 15 and 16.

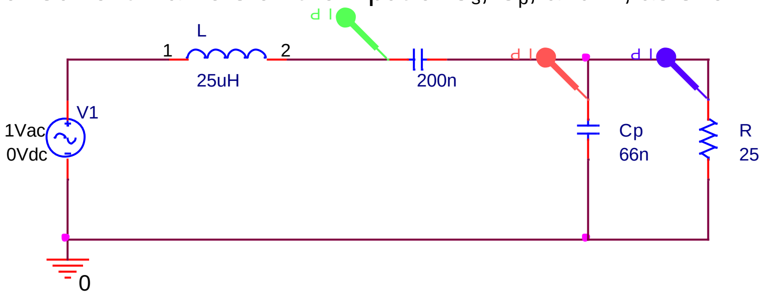


Figure 15 - Input and Output Current Phase Markers

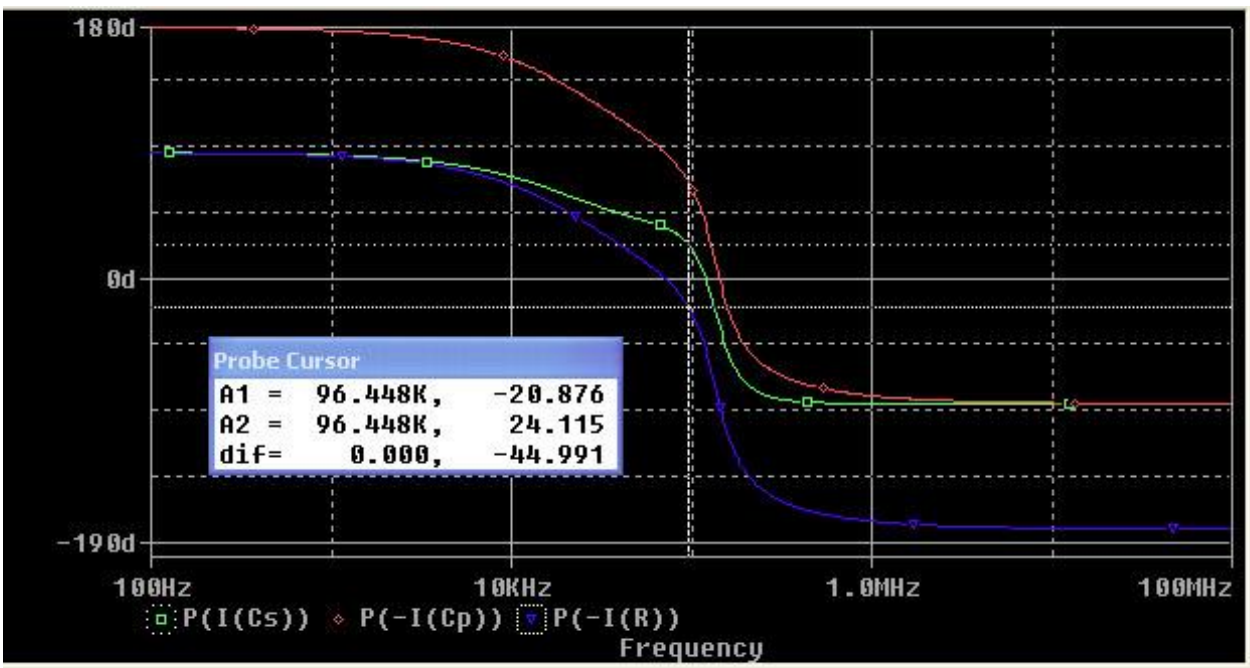


Figure 16 - Phases of Input Current Division into Output Current

Notice how the input current through $C_s$ (green) is in phase with the R current (blue) at low frequencies, and in phase with the $C_p$ current (red) at high frequencies.
Question: Why is this?
Answer:
At low frequencies, the $C_p$ impedance is high, so the input current goes entirely through R.
At high frequencies, the $C_p$ impedance is low, so the input current goes entirely through $C_p$.

Notice how the phase of the $C_p$ current (red) is always 90º more than the phase of the R current(blue).
Question: Why is this?
Answer: R and $C_p$ are in parallel, with the same voltage across each. Their impedances are at right angles, i.e., R vs. $-j/\omega C_p$ , therefore, the $C_p$ current will <u>always</u> lead the R current by 90º, whether below, at, or above resonance.

Notice that at the R-$C_p$ resonance of 96.5 kHz, the $C_s$ current (green) (A2 marker) leads the R current (blue) (A1 marker) by 45º.
Question: Why is this?
Answer: Because the $C_p$ and R currents magnitudes are equal, but their phases are 90º apart at 96.5 kHz. Therefore, the phase of the input current is exactly in between the phase of the $C_p$ and R currents at 96.5 kHz.

**Part 5: Output and Input Power Calculation**

Output Power: It's not necessary to make another simulation to compute the output power.

Make a plot of the output power by doing: Trace->Delete All Traces
And Trace->Add Trace
Create the Trace Expression: V(R:2) * I(R) to compute the output power shown in Figure 17.

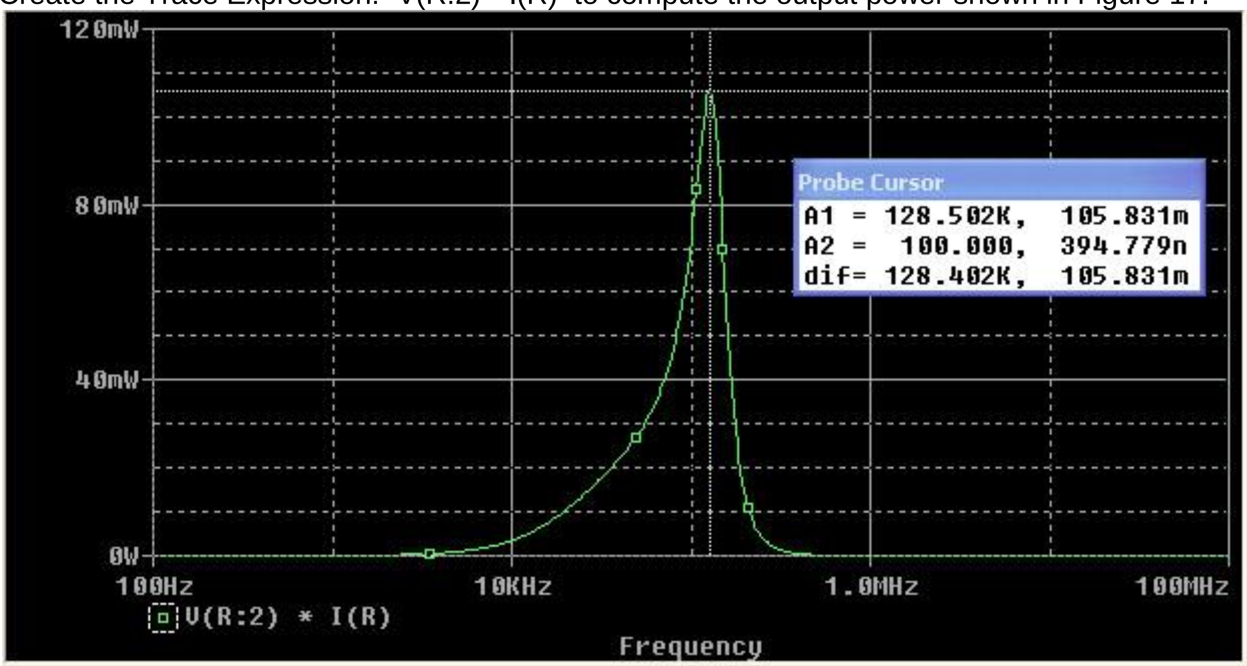

Figure 17 - Output Power

Question: What is the peak output power, and at what frequency?

Answer: Peak output power is 105 mW at the 128 kHz resonance (marker A1).
Since the output voltage, and load current, both peak at the 128 kHz resonance, it is not surprising that the output power is a peak at that frequency also.

Question: How does this output power compare to the previous measurements made with the output voltage and current.

Answer: In Figure 5, peak $V_o$ = 1.6266V, therefore peak output power:

$$P_o = \frac{V_o^2}{R} = \frac{(1.6266V)^2}{25Ohms} = 105\ mW$$

Note: The input voltage V1 and voltage markers are in $V_{rms}$ , and not $V_{peak}$ .

The peak output current at this frequency would be:

$$Io\ = \frac{Po}{Vo} = \frac{105mW}{1.6255V} = \ 64.6\ mA = \ -23.8\ dB(Amps)$$

Question: How does the output power compare to the input power?
Doesn't this violate the "Conservation of Energy" concept?

Answer: No, it doesn't. Power can only be dissipated in R, since L, $C_p$ and $C_s$ are considered ideal. In Figure 10, the input current peaks at -19.215 dB(Amps) = 109.5 mA, so input power would be 109.5 mW. This makes it look like more power is being put into the circuit, than is being dissipated in the load resistance. But remember, the input voltage and current may not be in phase, so this is just the apparent power (S, in mVA), and possibly not the real power (P, in mW).

The power factor (PF) is defined as:

PF = P/S = cosθ , where θ is the phase angle between the real and apparent power.

$$cos\theta\ =\ P/S\ =\ 105mVA/109.5\ mW\ =\ 0.9589$$

θ = $\cos^{-1}$(0.9589) = +/- 16.5º

Let's see if we can simulate this result.   Put a power marker on the input:

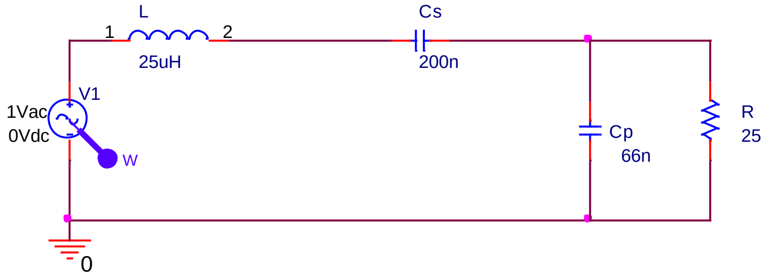


Figure 18 - Input Power Configuration

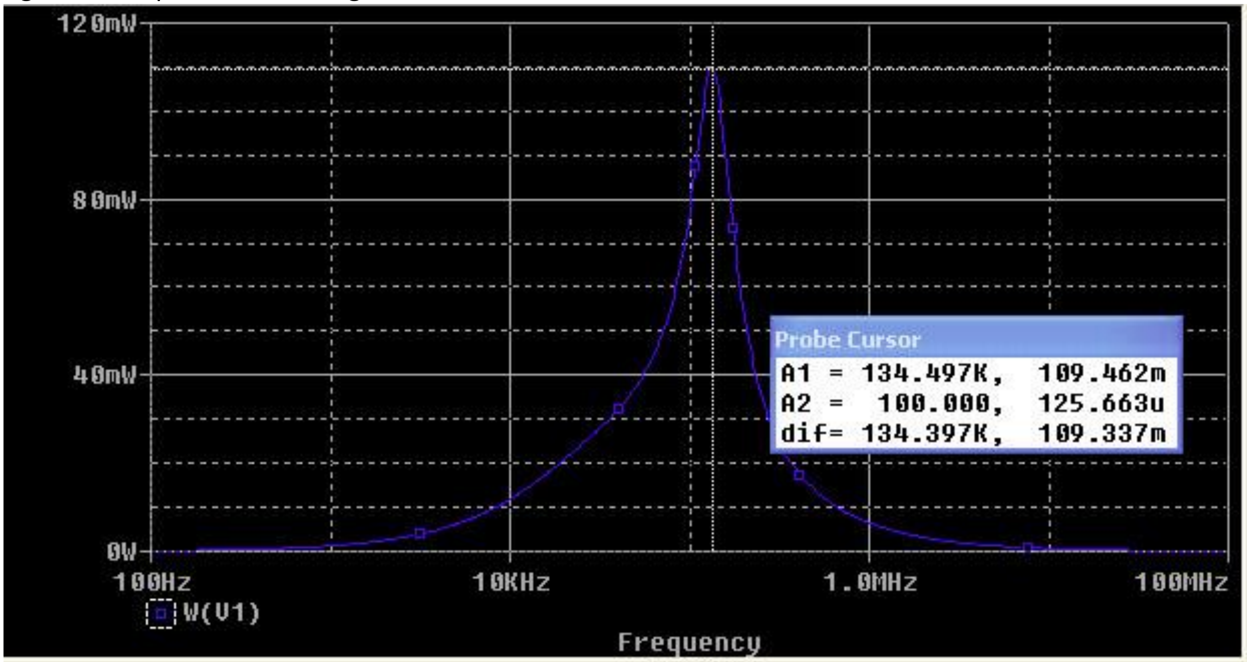


**Figure 19 - Input Power Measurement**

Notice that the peak power of 109.5 mVA(marker A1) matches what we calculated above. Now add a phase marker to the left side of the inductor, in order to avoid the unwanted inversion, or 180º phase shift, to read the phase of the current leaving the source.

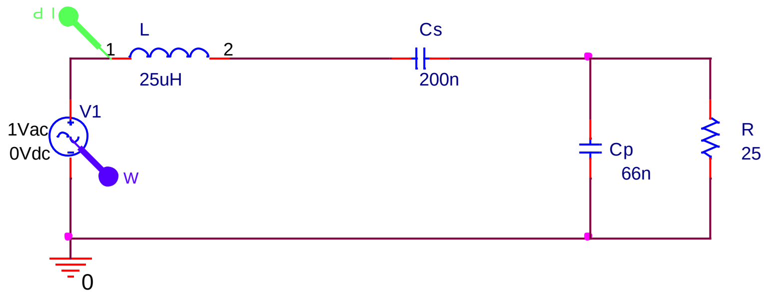


**Figure 20 - Input Current Phase Measurement**

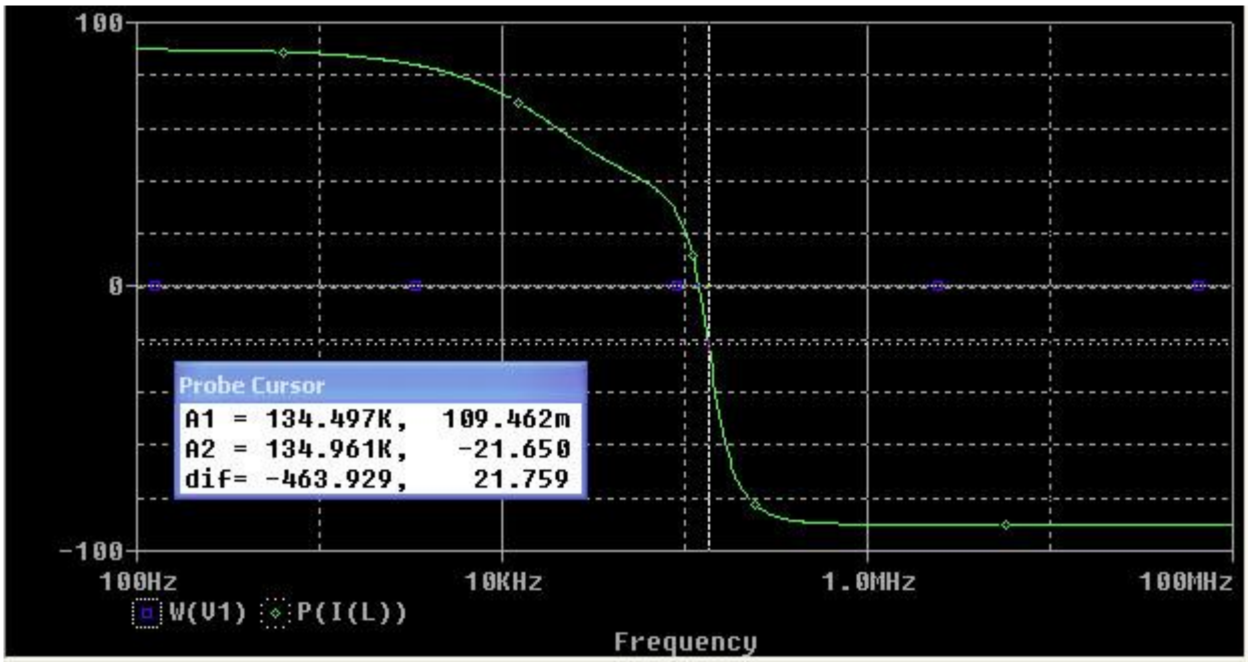


Figure 21 - Input Current Phase Measurement

Rescaling causes the input power to be compressed near zero, in order to scale the +90º to -90º phase range. However, we can still use the A1 marker to find the peak power of 109.5 mW at the 134.5 kHz resonance. The A2 marker reads the -21.65º phase angle at this frequency.

This calculated power factor is:

$$cos\theta \;=\; cos(-21.65^{\text{o}}) \;=\; 0.93$$

This is very close to the power factor calculated previously.

One final real vs. apparent power comparison:  Put a Power marker on the input and output R.

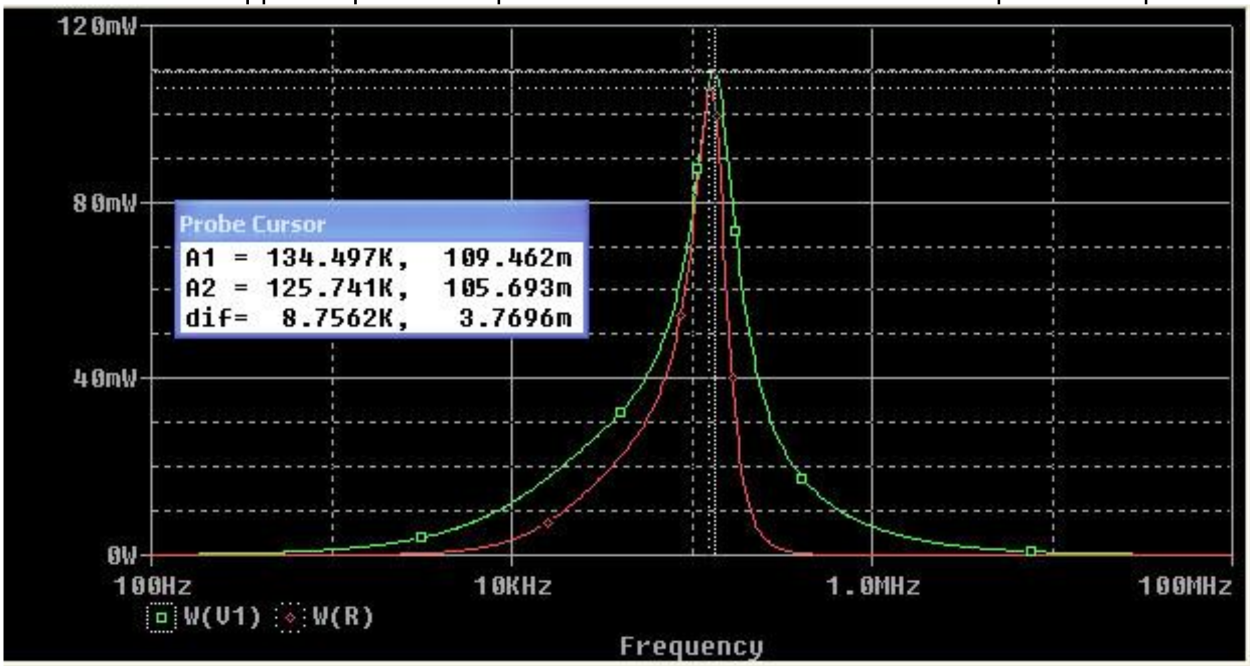


**Figure 22 - Input Apparent Power (green) vs. Output Real Power (red)**

Notice that the input apparent power (green) is always greater than the output real power (red), with the greatest differences below resonance, where the input impedance is capacitive, and above resonance, where the input impedance is inductive.

The peak input apparent power is 109.5 mVA (marker A1).
The peak input apparent power is 105.7 mW  (marker A2).

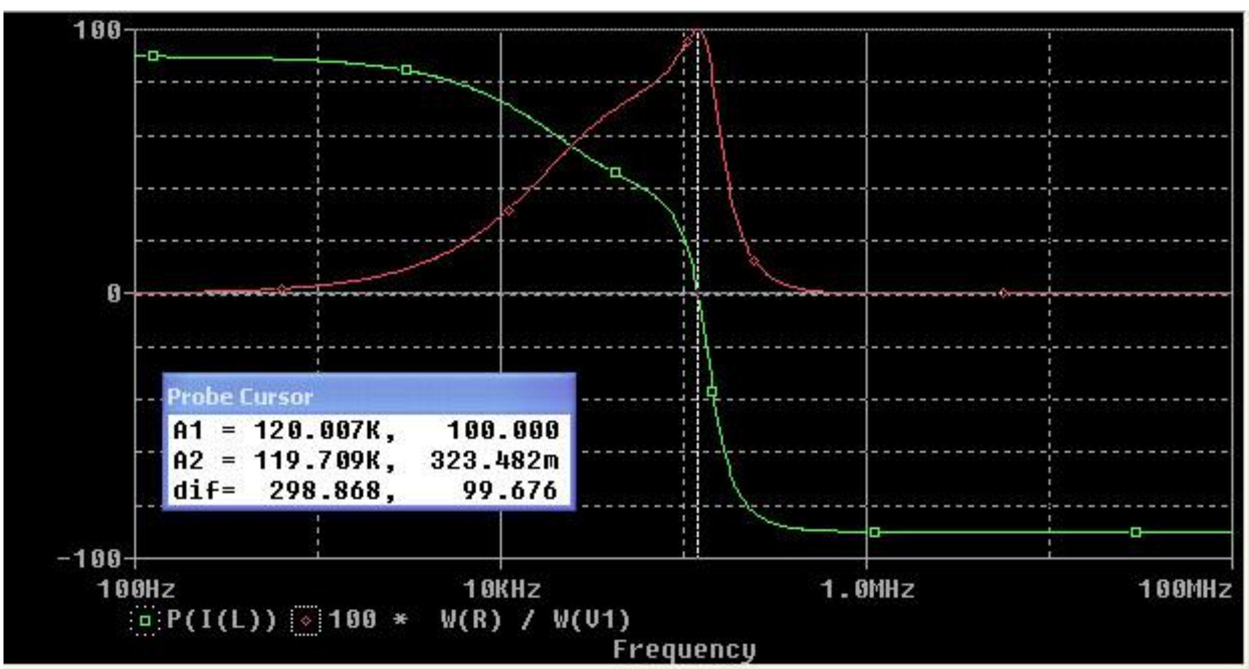


**Figure 23 - Power Factor(%) = Real (output) power / Apparent (input) power(red)  Input Current Phase(green)**

Notice that at resonance(120 kHz), the Power Factor(red) is 100%(markerA1). The input current phase angle (green) is also 0º(marker A2), meaning the input voltage and current are in phase.

**Part 6: CLC Circuit**

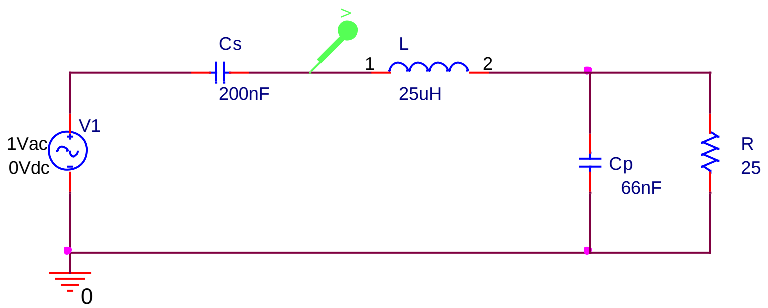


Figure 24 - CLC Tank circuit

Question: How would all of the above simulations be different if L and $C_s$ were connected as in Figure 24, a CLC Tank circuit?

Answer:
There would be no difference because the two devices are still in series with $Z_{par}$. The current through both devices would be the same as before. The only difference would be if the voltage was probed at the node between the two devices, as shown. This voltage would be different than if the same node in the LCC circuit was probed.

**Part 7: ESR Effects on Output Power:**
Simulate the LCC resonant circuit with Capacitor ESR in $C_s$ and $C_p$ and see its effects on the output power. For example choose the ratio of the $C_s$ and $C_p$ ESR to the load resistance to be in the ratio range from 0.01 to 1.

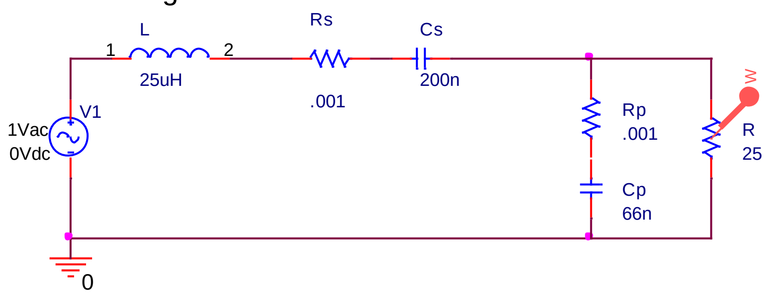


Figure 25 - LCC with Capacitor ESR Configuration

Find the output power for Cp and Cs ESR values of .001Ω, 0.250Ω, 2.5Ω, and 25Ω.

Question: What is the peak output power and resonant frequency for each ESR below:
For $C_p$, $C_s$ ESR = 0.001 Ω peak output power = ??? mW @ ??? kHz

For $C_p$, $C_s$ ESR = 0.250 Ω peak output power = ??? mW @ ??? kHz

For $C_p$, $C_s$ ESR = 2.5 Ω peak output power = ??? mW @ ??? kHz

For $C_p$, $C_s$ ESR = 25 Ω peak output power = ??? mW @ ??? kHz

**Transfer Function Magnitude and Phase**
from: **Budak, Aram. Passive and Active Network Analysis and Synthesis**,
Houghton Miflin Co. Boston, 1974

A system function of a circuit with no independent sources, and zero initial conditions, can be written in the form: $T(s) = \frac{R(s)}{E(s)}$ where E(s) is the Laplace transform of the system excitation signal e(t), and R(s) is the transform of the response signal r(t). Both the excitation and response transforms can be functions of s, with numerator and denominator polynomials in s. Therefore, T(s) can be factored into the form: $T(s) = \frac{N(s)}{D(s)}$ where N(s) and D(s) are the numerator and denominator polynomials, respectively.

T(s) can be a dimensionless transfer function, or have units of impedance or admittance.

| Input | Output | Transfer function | T(s) units |
|---|---|---|---|
| Voltage | Voltage | Voltage amplifier | dimensionless |
| Current | Current | Current amplifier | dimensionless |
| Current | Voltage | Transimpedance amplifier or $Z_{in}$ or $Z_{out}$ | Resistance or Impedance |
| Voltage | Current | Transconductance amplifier or $Y_{in}$ or $Y_{out}$ | Conductance or admittance |

T(s) can then be written: $$T(s) = \frac{a_m s^m + a_{m-1} s^{m-1} + \ldots a_1 s + a_0}{b_n s^n + b_{n-1} s^{n-1} + \ldots b_1 s + b_0} = H \frac{(s-z_1)(s-z_2)\ldots(s-z_m)}{(s-p_1)(s-p_2)\ldots(s-p_n)}$$

Where $H = \frac{a_m}{b_n}$ is the scale factor, factored out so that the coefficient of the highest term of s is unity in both the numerator and denominator. For real system functions, the order of the numerator is less than or equal to the order of the denominator, i.e. $m \leq n$.

The values of s that make the numerator polynomial zero are called the zeros of N(s).
Since N(s) is of order m, it has m zeros, expressed explicitly as $z_1, z_2, \ldots z_m$.
The zeros of N(s) are also the finite zeros of T(s).

The values of s that make the denominator polynomial zero are called the zeros of D(s).
Since D(s) is of order n, it has n zeros, expressed explicitly as $p_1, p_2, \ldots p_n$.
The zeros of D(s) are also the finite poles of T(s).

The zeros of T(s) are the values of s that make $T(s) = 0$,
The poles of T(s) are the values of s that make $T(s) = \infty$.

When dealing with sinusoidal excitation and response, s can be confined to the value $s = j\omega$, therefore:

$$T(j\omega) = H \frac{(j\omega - z_1)(j\omega - z_2) \ldots (j\omega - z_m)}{(j\omega - p_1)(j\omega - p_2) \ldots (j\omega - p_n)}$$

Consider the factor ( $j\omega - s_i$ ) where $s_i$ may be a zero, i.e. $s_i = z_i$, or $s_i$ may be a pole, i.e. $s_i = p_i$. Since $s_i$ is, in general, complex, it can be expressed in terms of its real part α, and its imaginary part β , that is:

$$s_i = \alpha_i + j\,\beta_i$$

The factor ( $j\omega - s_i$ ) then becomes $\left(-\alpha_i + j(\omega - \beta i)\right) = M_i\, e_i^{j\theta_i}$, where

$$M_i = \sqrt{a_i^2 + (\omega - \beta i)^2} \qquad \text{and} \qquad \theta_i = \tan^{-1}\left(\frac{\omega - \beta i}{-\alpha_i}\right)$$

Hence: $$T(j\omega) = H\left(\frac{M_{z1}e^{j\theta_{z1}} M_{z2}e^{j\theta_{z2}} \ldots M_{zm}e^{j\theta_{zm}}}{M_{p1}e^{j\theta_{p1}}\, M_{p2}e^{j\theta_{p2}} \ldots M_{pn}e^{j\theta_{pn}}}\right)$$

$$= H\frac{M_{z1}M_{z2}\ldots M_{zm}}{M_{p1}M_{p2}\ldots M_{pn}}\, e^{j(\theta_{z1} + \theta_{z2} + \ldots \theta_{zm} - \theta_{p1} - \theta_{p2} \ldots - \theta_{pn})} = M(\omega)e^{j\theta(\omega)}$$

where $M(\omega)$ is the magnitude of $T(j\omega)$ and $\theta(\omega)$ is its phase.

In the s-plane the factor ( $j\omega - z_1$ ) represents the vector from $z_1$ to $j\omega$,
the factor ( $j\omega - p_1$ ) represents the vector from $p_1$ to $j\omega$, and so on.

These vectors are shown in Figure 26 below:

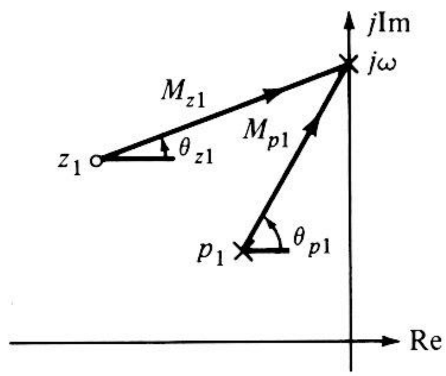


Fig. 2-13 Geometric interpretation

Figure 26 - Geometric Interpretation of T(jω)

The magnitude function M(ω) is H times the product of the zero-to-jω distances divided by the product of the pole-to-jω distances.

The phase function θ(ω), is the sum of the zero-to-jω angles minus the sum of the pole-to-jω angles.

If H is negative, its magnitude may be associated with M(ω), and π radians added to θ.

Since both the magnitude and angle changes arising from the individual terms can be seen readily by looking at the s-plane diagram, the M(ω))-vs.-ω and θ(ω)-vs.-ω <u>may be sketched by inspection.</u>

So what do we need MATLAB for, now that we understand the above?

MATLAB can be a great help in factoring higher order polynomials into the pole-zero format.

Simulation of the LCC Resonant Circuit Using MATLAB

**NOTE**: The simulations that follow are intended to be completed with MATLAB®. It is assumed that the student has a fundamental understanding of the operation of MATLAB®. MATLAB® provides tutorials for users that are not experienced with its functions.

**PROCEDURE**:

This will use the same circuit as previously:

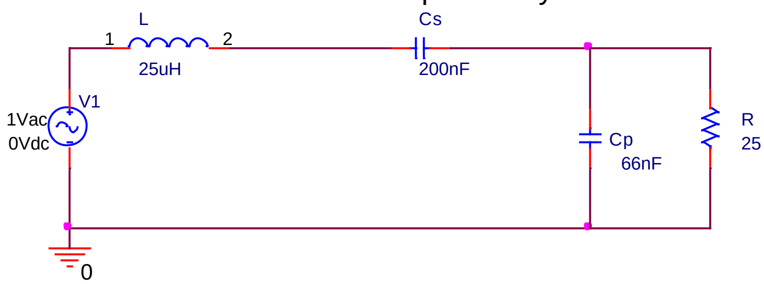


Figure 27 - LCC Tank Circuit

**Part 1**: Input Impedance $Z_{in}$

In the MATLAB Editor, write an m file shown in Figure 25.

**Vm** is a variable voltage, set to 1 volt.

**L** is a variable inductor, set to 25µH.

**R** is a variable ideal resistor, set to 25Ω.

**Cp** is a variable ideal capacitor, set to 66nF.

**Cs** is a variable ideal capacitor set 200nF.

```
Vm = 1;
R = 25;
Cs = 200e-9;
Cp = 66e-9;
L = 25e-6;
Zl = tf([L 0],[0 1]);  % ZL = (sL + 0)/(s*0 + 1) = sL
Zcs = tf([0 1],[Cs 0]);  % Zc = (s*0 + 1)/(sC + 0)
Zcp = tf([0 1],[Cp 0]);
Zpar = 1/(1/R + 1/Zcp);
Zser = Zcs + Zl;
Zin = Zpar + Zser;
bode(Zin)
title('Input Impedance of LCC Tank Circuit');
[z,p,k] = zpkdata(Zin, 'v')  % Leave off the ';' because
                             % it will inhibit the zpk printout
```

Figure 28 - m file for the LCC circuit Input impedance

The tf function is a complex transfer function of the form tf(num,den), where num and den are the numerator and denominator vectors for the coefficients of s, in descending powers of s.

Thus: `Z = tf([L 0],[0 1]);` means $Z = \frac{sL+0}{s*0+1} = sL$ for an inductor.

And `Z = tf([0 1],[C 0]);` means $Z = \frac{s*0+1}{sC+0} = \frac{1}{sC}$ for a capacitor.

Once the above m file is saved, the simulations can be run. First, go to your directory. Find your m file and then run your file. If there is a red message on your MATLAB window, then you need to correct your error. Otherwise, you will see the solution as shown in Figure 26.

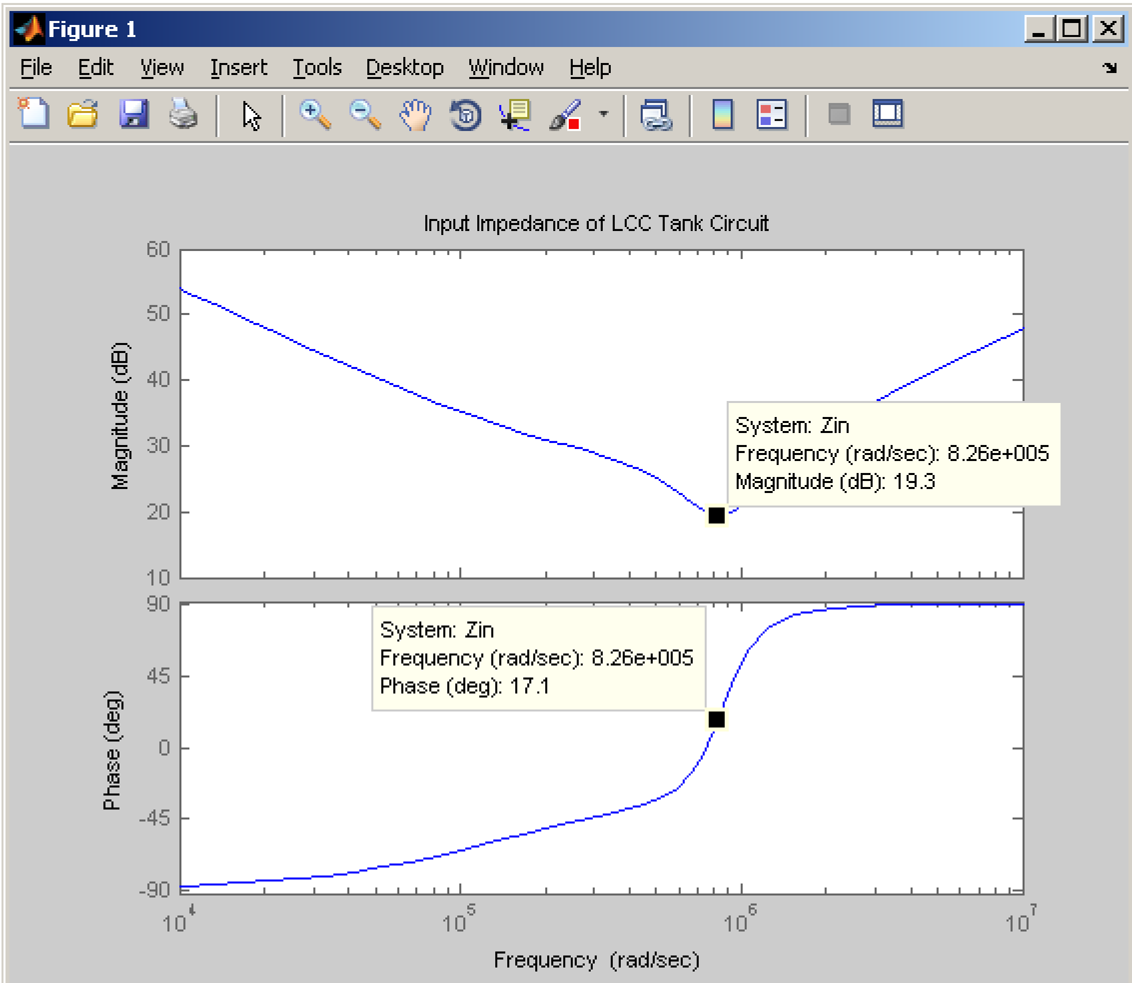


Figure 29 - The output of Zinput_LCC m file.

This shows minimum impedance 19.3 dB(ohms) = 9.2 ohms @ 826 krad/sec = 131 kHz
This matches that found with PSPICE in Figure 3 above.

Remember, from previously:

$$Z_{in} = L\frac{\left[s^3 + s^2\left(\frac{1}{RCp}\right) + s\left(\frac{1}{LCs} + \frac{1}{LCp}\right) + \frac{1}{RLCsCp}\right]}{\left[s\left(s + \frac{1}{RCp}\right)\right]}$$

[z,p,k] = zpkdata(sys) returns the zeros z, poles p, and gain k of the zero-pole-gain model "sys".

The output of the `[z,p,k] = zpkdata(Zin, 'v')` command appears in the command window as follows:

++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++

```
z =  1.0e+005 *           -2.2037 + 8.2743i
                          -2.2037 - 8.2743i
                          -1.6532

p =  1.0e+005 *        0
                      -6.0606

k =  2.5000e-005
```

++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++

zeros:
Two imaginary zeros at s = (-2.2037 +/- 8.2743i)*$10^5$ = 8.56*$10^5$ rad/sec @+/- 75.1º = 136.3 kHz
One real axis zero at s = -1.6532 x $10^5$ rad/sec = 26.3 kHz

Poles: s= 0 and s = -1/$RC_p$ = -6.06 * $10^5$ rad/sec = 96.46 kHz

Gain k = 2.5 x $10^{-5}$ = L = 25 µH

**Part 2: Output Voltage**

Since: $Z_{par} = R\,\frac{\left(\frac{1}{RCp}\right)}{s+\frac{1}{RCp}}$ and $Z_{in} = L\,\frac{\left[s^3+s^2\left(\frac{1}{RCp}\right)+s\left(\frac{1}{LCs}+\frac{1}{LCp}\right)+\frac{1}{RLCsCp}\right]}{\left[s\left(s+\frac{1}{RCp}\right)\right]}$

Then: $\frac{V_{out}}{V_{in}} = \frac{Z_{par}}{Z_{in}}$

So: $$\frac{V_{out}}{V_{in}} = \left(\frac{1}{LC_p}\right)\left(\frac{s}{s^3+s^2\left(\frac{1}{RC_p}\right)+s\left(\frac{1}{LC_s}+\frac{1}{LC_p}\right)+\frac{1}{RLC_sC_p}}\right)$$

Note that there is pole-zero cancellation of the factor: ( $s+\frac{1}{RCp}$)

Note that the order of the numerator is $s^1$ and the order of the denominator is $s^3$, therefore the order of the polynomial ratio is $s^{-2}$. The H term $\frac{1}{LCp}$ is order $s^2$, so this is a dimensionless transfer function, still 3rd order , with the poles of $V_{out}$ equal to the zeros of $Z_{in}$.

Next, plot the output voltage of the LCC circuit by adding the output voltage equation to the LCC m file. Then rerun the LCC m file.

```
Vout = Vm * Zpar/(Zpar + Zser)
figure(2)
bode(Vout)
title('output Voltage of LCC Tank Circuit')
[z,p,k] = zpkdata(Vout, 'v')
wn = sqrt(p(1) * p(2))
```

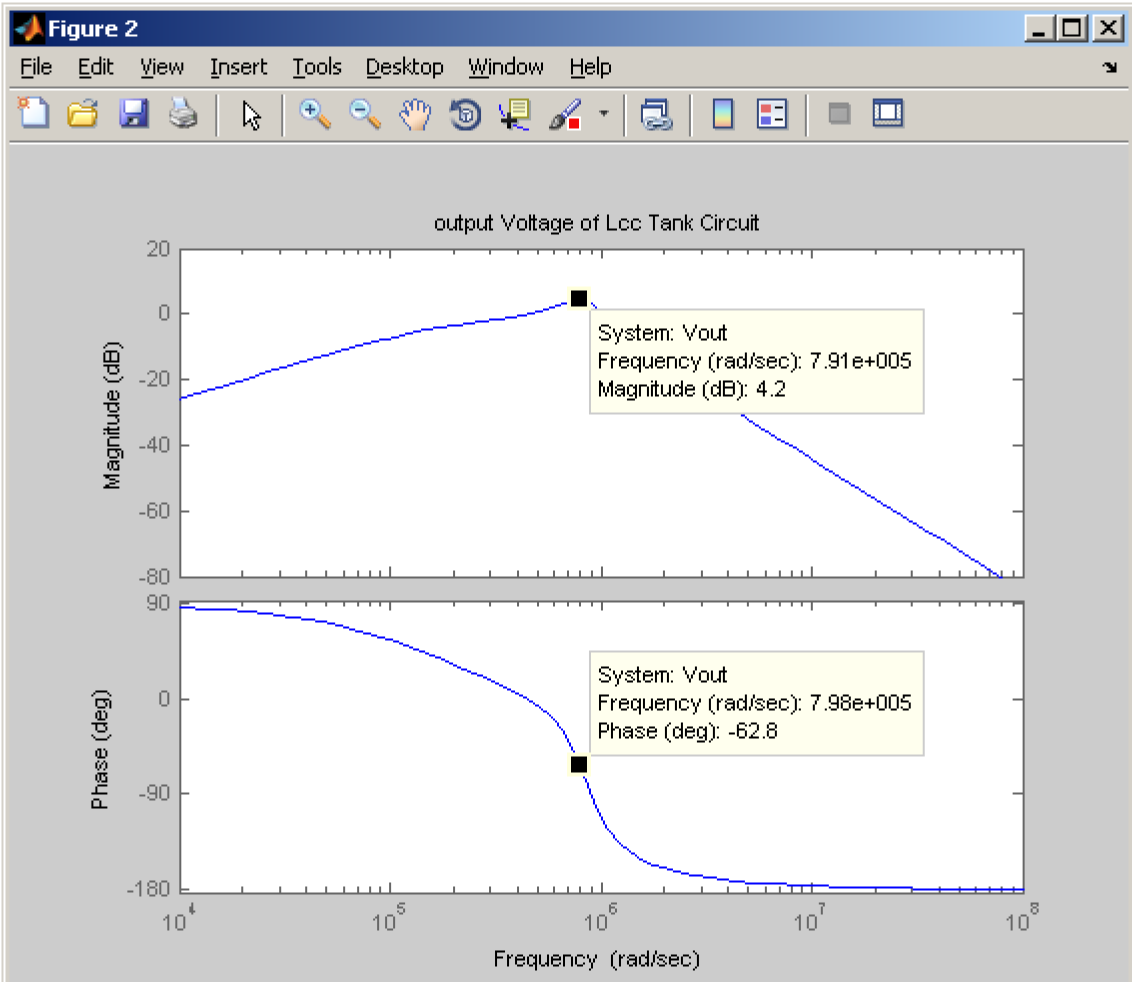


Figure 30 - The Command Window Output of Voutput_LCC m file.

Question: How do the results of Figure 30 compare with the PSPICE results in Figure 5 and 7?
Answer: They are the same. Magnitude = 4.2 dB(V) = 1.6V, phase angle = -65º.

The output of the `[z,p,k] = zpkdata(Vout, 'v')` command appears in the command window as follows:

++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++

Transfer function:

```
                    1.32e-014 s^2 + 8e-009 s
   -----------------------------------------------------------------
   2.178e-026 s^4 + 2.64e-020 s^3 + 2.556e-014 s^2 + 1.328e-008 s + 0.0016

z =  1.0e+005 *      0
                 -6.0606

p =  1.0e+005 *      -2.2037 + 8.2743i
                    -2.2037 - 8.2743i
                    -6.0606
                    -1.6532
k =  6.0606e+011
```

wn =  8.5627e+005
++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++

Question:  Why does MATLAB show the transfer function as 4$^{th}$ order, rather than 3$^{rd}$ order?
Answer:  Because it failed to recognize that there is pole-zero cancellation at:

$$s = \frac{-1}{RCp} = -6.0505 \text{ x } 10^5 \; \frac{\text{rad}}{\text{sec}}$$ , shown in red.

This failure to recognize exact pole-zero cancellation is probably due to round-off error.

If the coefficients of the highest order terms in the numerator and denominator were factored out, this equals the gain factor k:

$$k \; = \frac{1.32 * 10^{-14}}{2.178 * 10^{-26}} = \; 6.0606 * 10^{11}$$

Question:  How does “k” above compare to the “H” in the symbolic transfer function?
Answer:  They are the same, i.e.

$$H = \left(\frac{1}{LC_p}\right) = \frac{1}{25\mu H * 66nF} = \; 6.0606 * 10^{11} \; \left(\frac{rad}{\text{sec}}\right)^2$$

Question:  Why is there a -270º phase shift from low to high frequencies?
Remember back at Figure 7, we couldn’t answer why the phase angle went from +90º at   low frequencies, to -180º at high frequencies.

Now, knowing the 3$^{rd}$ order transfer function:

$$\frac{V_{out}}{V_{in}} = \left(\frac{1}{LC_p}\right)\left(\frac{s}{s^3 + s^2\left(\frac{1}{RC_p}\right) + \; s\left(\frac{1}{LC_s} + \frac{1}{LC_p}\right) + \frac{1}{RLC_{sC_p}}}\right)$$

And knowing the pole and zero values from MATLAB, we can use the graphical method of Figure 26, to see that the zero at s=0 contributes +90º at all frequencies.  The two complex, and one real pole contribute 0º at s=jω=0(due to symmetry about the horizontal axis), but contribute -90º apiece as s=jω -->∞. Therefore the phase angle goes from +90º at low frequencies to -180º at high frequencies.

**Part 3A:  Input, or Inductor Current**

Symbolically:

$$\left(\frac{I_{in}}{V_{in}} = \frac{1}{Z_{in}} \quad = \frac{1}{L}\frac{\left[s\left(s+\frac{1}{RCp}\right)\right]}{\left[s^3 + s^2\left(\frac{1}{RCp}\right) + s\left(\frac{1}{LCs}+\frac{1}{LCp}\right) + \frac{1}{RLCsCp}\right]}\right)$$

So the poles of $Z_{in}$ become the zeros of $I_{in}$, and the zeros of $Z_{in}$ become the poles of $I_{in}$. The numerator is order $s^2$, and the denominator is $s^3$, so the ratio is order $s^{-1}$1, giving the transfer function units of $\frac{1}{sL} = \ admittance$, as expected.

Now plot the inductor current of the LCC circuit by adding the inductor current equations to the LCC m file.

```
Iind = (Vm - Vout) / Zser
figure(3)
bode(Iind)
title('Inductor Current of LCC Tank Circuit')
[z,p,k] = zpkdata(Iind, 'v')
wn = sqrt(p(1) * p(2))
[A,B] = damp(Iind)
```

Then rerun the LCC m file.

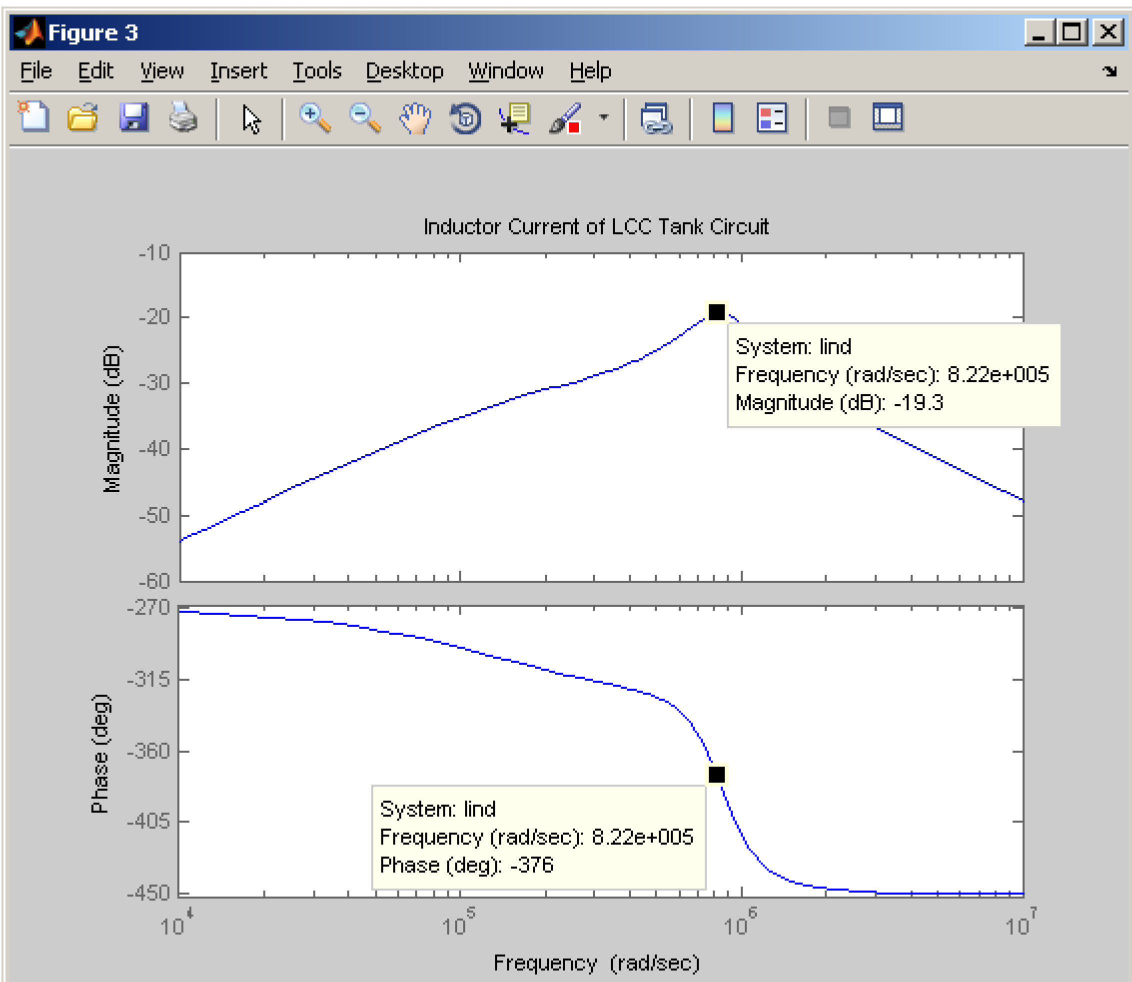

Figure 31 - The output of Input or Inductor Current_LCC m file

Question: How does the peak current magnitude and phase angle compare to what was simulated with PSPICE?
Answer: The results are the same. See Figure 10. $I_{in\ peak}$ = -19.2 dB(A) @ -21º. The phase angle in Figure 31 could have been shifted +360º, to show the phase angle = -16º at the peak current.

+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++

Transfer function:

```
4.356e-033 s^5 + 5.28e-027 s^4 + 2.471e-021 s^3 + 1.056e-015 s^2   + 3.2e-010 s
--------------------------------------------------------------------------------------------------------
1.089e-037 s^6 + 1.32e-031 s^5 + 1.496e-025 s^4 + 9.28e-020 s^3 + 3.356e-014 s^2 + 1.328e-008 s + 0.0016
```

```
z =  1.0e+005 *          0
                   -0.0000 + 4.4721i
                   -0.0000 - 4.4721i
                   -6.0606 + 0.0000i
                   -6.0606 - 0.0000i


p =  1.0e+005 *    -2.2037 + 8.2743i
                   -2.2037 - 8.2743i
                   -6.0606
                   0.0000 + 4.4721i
                   0.0000 - 4.4721i
                   -1.6532


k =      40000


wn =   8.5627e+005


A =  1.0e+005 *
                   1.6532
                   4.4721
                   4.4721
                   6.0606
                   8.5627
                   8.5627


B =       1.0000
         -0.0000
         -0.0000
          1.0000
          0.2574
```

0.2574
++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
Question: Why does MATLAB show the transfer function as 6th order, rather than 3rd order?
Answer: Because it failed to recognize that there is pole-zero cancellation at:

s = -6.0606
0.0000 + 4.4721i
0.0000 - 4.4721i

Question: What is the meaning of k=40000?
Answer: k is the same as H in the symbolic equation:

$$\frac{I_{in}}{V_{in}} = \frac{1}{Z_{in}} \quad = \frac{1}{L}(\dots) = \frac{1}{25}\mu H = 40{,}000\ H^{-1}$$

Question: Why does the input current go through -180º of phase shift from low to high frequencies, when the output voltage went through -270º?

Answer: Remember, the transfer function (input admittance) is:

$$\left(\frac{I_{in}}{V_{in}} = \frac{1}{Z_{in}} \quad = \frac{1}{L}\frac{\left[s\left(s+\frac{1}{RCp}\right)\right]}{\left[s^3 + s^2\left(\frac{1}{RCp}\right) + s\left(\frac{1}{LCs}+\frac{1}{LCp}\right) + \frac{1}{RLCsCp}\right]}\right)$$

After pole-zero cancellation, MATLAB calculated the two real axis zeros at:

z = 1.0e+005 * 0
-6.0606

And the two complex poles and one real axis pole at:

p = 1.0e+005 * -2.2037 + 8.2743i
-2.2037 - 8.2743i
-1.6532

We can use the graphical method of Figure 26, to see that:

The real axis zero at s=0 contributes +90º at all frequencies.

The real axis zero at $s = -\frac{1}{RC_P} = -6.06 * 10^5 \left(\frac{rad}{\sec}\right)$ contributes 0º at s=0 and +90º at high frequencies.

The two complex poles contribute 0º at low frequencies(due to symmetry about the real axis), and -180º at high frequencies.
The real axis pole contributes 0º at low frequencies, and -90º at high frequencies.

Therefore, at low frequencies, the phase angle is +90º, contributed entirely by the zero at s=0.

At high frequencies, the two zeros contribute +90º apiece, and the three poles contribute -90º apiece for a phase angle total of -90º.

---

**Part 3B: Using MATLAB Loops to Compute Input Current**

A MATLAB loop can be used to find the inductor current phase zero crossing. First define the input frequency range as a vector. Write a loop function to compute the various impedances, voltages, and currents at each frequency. Then find zero crossing of the inductor current phase. Create a new m file with the following code:

```
Vm = 1;
R = 25;
Cs = 200e-9;
Cp = 66e-9;
L = 25e-6;

% define frequency as a vector, from 10^5 rad/sec
% to 10^7 rad/sec in steps of 1krad
w = 1e5:100:1e6;
an = size(w); % a vector [1 9901]
jmax = an(2); % 9901 = # of w data points.

%define impedance equations as f(w) and calculate Vout and IL
for i=1:jmax
    % Impedance of each element
    Zind(i) = 0+j*w(i)*L;
    ZCs(i) = 0-j*(1/(w(i)*Cs));
    ZCp(i) = 0-j*(1/(w(i)*Cp));
    ZR(i) = R;

    %Impedance of Inductor in series with Cs
    Zs(i) = ZCs(i) + Zind(i);

    %Impedance of Parallel R and Cp
    Zp(i) = 1/(1/ZR(i) + 1/ZCp(i));

    %Output Voltage divider
    Vout(i)= Vm * Zp(i)/(Zp(i) + Zs(i));

    %input or inductor current
    IL(i) = (Vm - Vout(i))/Zs(i);
end

%find the resonant frequency by finding the phase zero crossing
for i= 1:jmax
    if phase(IL(i))<0, x=i;
        break;
    end
end
wo = w(x-1) + (w(x) - w(x-1))/2;

%plot the inductor current in degrees
figure(3)
semilogx(w,phase(IL)*180/pi)
```

```
title('Inductor Current Phase in LCC Tank Circuit')
grid
sprintf('The resonant frequency is %d.', wo)
```

Run the file, with the following results:

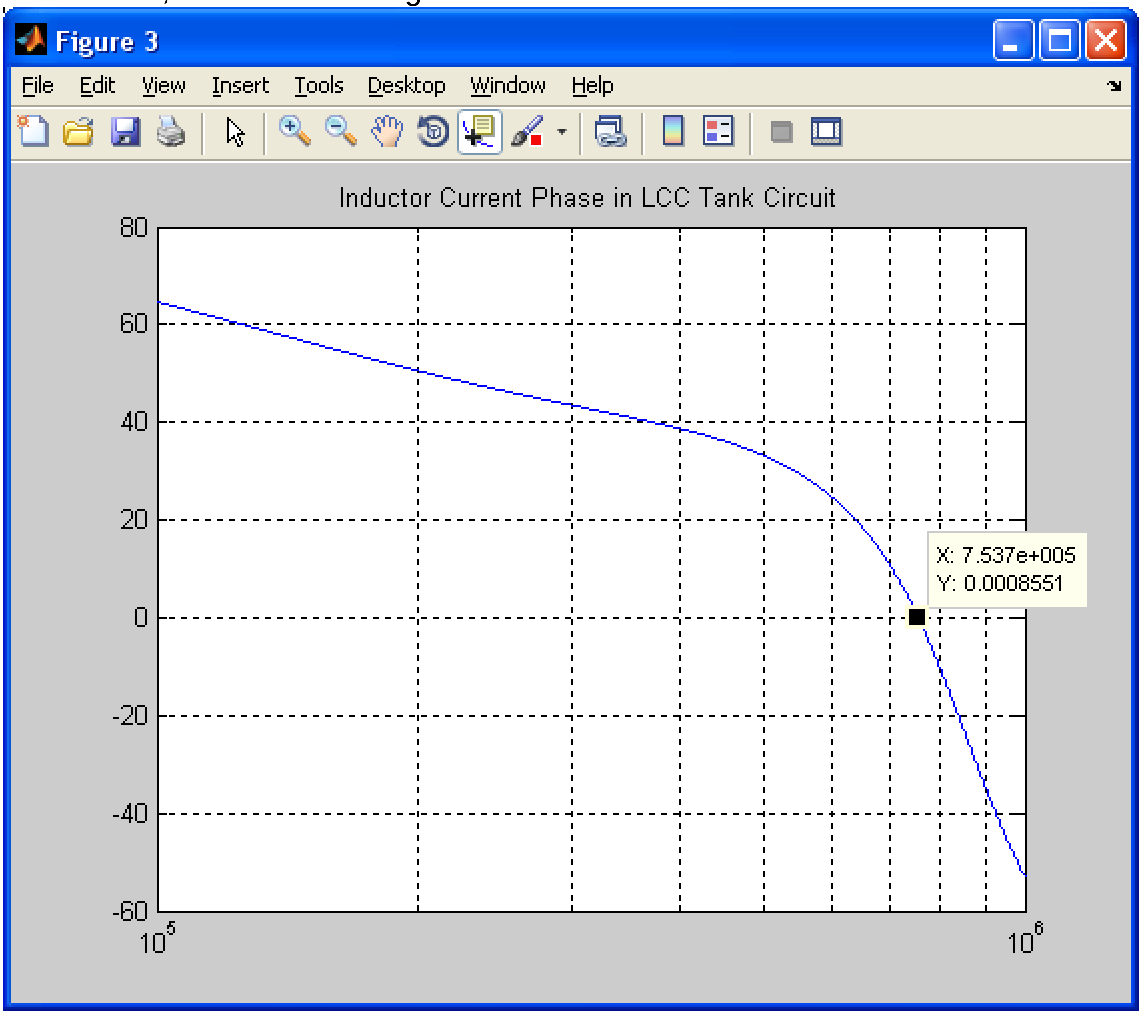


**Figure 32 - Input Current Zero Crossing**

Use Tools->Data Cursor, and drag the cursor to the zero crossing.

And in the command window:
ans = The resonant frequency is 753750.

The zero crossing is 753750 rad/sec = 120 kHz.

Question: How does this compare to what PSPICE computed?
Answer:  This is an almost exact match with Figure 23.

**Part 4: Current Division in the Output**
Symbolically:

$$I_{C_p} = I_{in} * \frac{R}{R + \frac{1}{sC_p}} = I_{in} \frac{s}{s + \frac{1}{RC_p}}$$

and

$$\left( \frac{I_{in}}{V_{in}} = \frac{1}{Z_{in}} \quad = \frac{1}{L} \frac{\left[ s \left( s + \frac{1}{RCp} \right) \right]}{\left[ s^3 + s^2 \left( \frac{1}{RCp} \right) + s \left( \frac{1}{LCs} + \frac{1}{LCp} \right) + \frac{1}{RLCsCp} \right]} \right)$$

Therefore:

$$\left( \frac{I_{C_p}}{V_{in}} = \frac{1}{L} \frac{s^2}{\left[ s^3 + s^2 \left( \frac{1}{RCp} \right) + s \left( \frac{1}{LCs} + \frac{1}{LCp} \right) + \frac{1}{RLCsCp} \right]} \right)$$

Notice there is pole-zero cancellation at the factor: $\left( s + \frac{1}{RCp} \right)$

By inspection(visualizing Figure 26 in your mind), state:

| Question: | Answer |
|---|---|
| Mag $\lvert I_{Cp} \rvert$ at s=jω=0 | Zero, due to two zeros at s=0 |
| Mag $\lvert I_{Cp} \rvert$ at s=jω→∞ | Zero, due to 2nd order numerator, 3rd order denominator |
| $\theta_{Cp}$ at s=jω=0 | 180º due to two zeros at s=0 |
| $\theta_{Cp}$ at s=jω→∞ | -90º, due to two zeros and three poles |

Now calculate and plot the $C_p$ capacitor current by adding the capacitor current equations to your original LCC m file.

```
ICp = Iind * R /(Zcp + R)
figure(4)
bode(ICp)
title('Output Capacitor Current of LCC Tank Circuit')
[z,p,k] = zpkdata(ICp, 'v')
wn = sqrt(p(1) * p(2))
```

Then rerun the LCC m file.

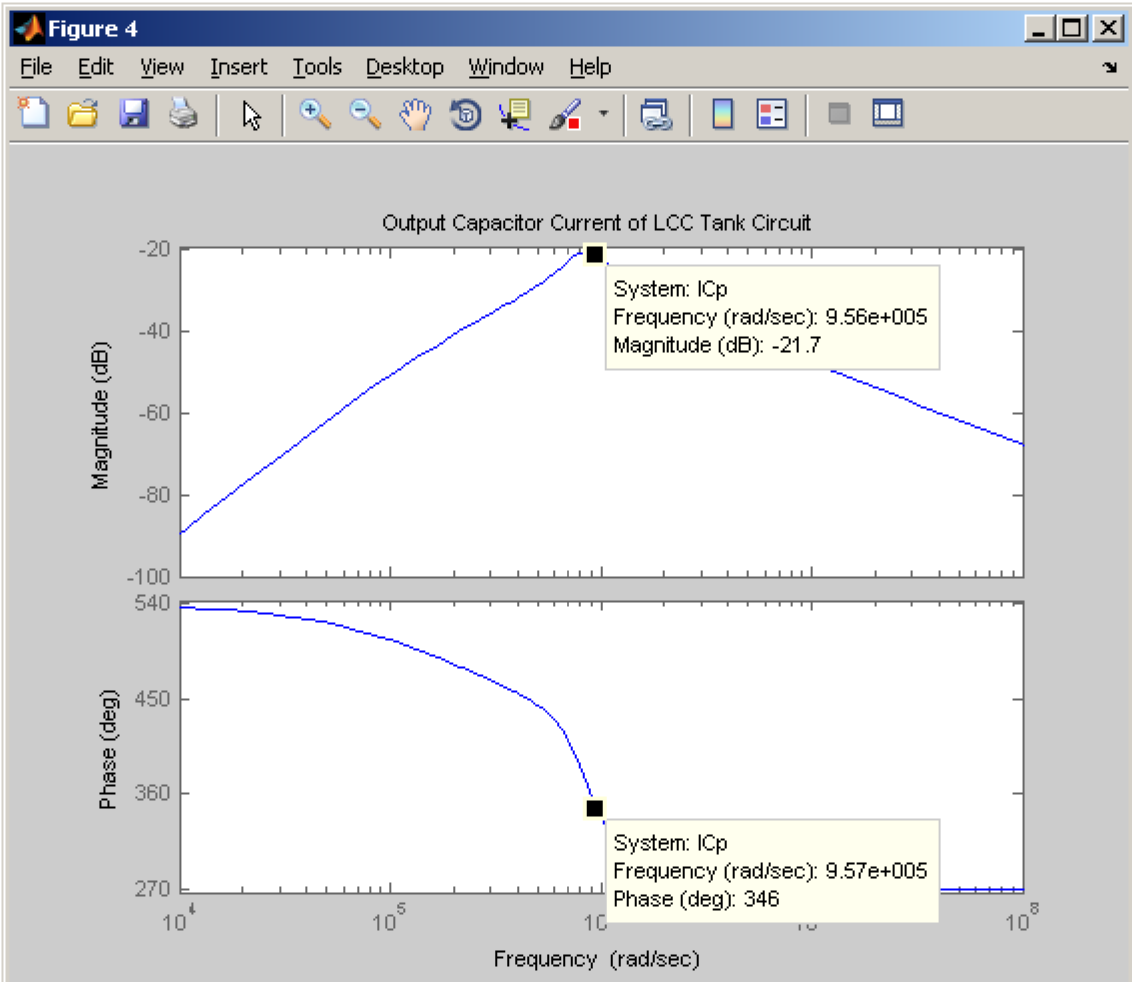

Figure 33 - The Output Capacitor $C_p$ Current _LCC m file.

Question: How does this $I_{Cp}$ result compare to the PSICE simulation in Figure 14 and 16?
Answer: They should match very closely.

Question:Were you able to see this bode plot magnitude by inspection, before the simulation was run?
Answer: Yes, the magnitude is bandpass, due to two zeros at s=0, and another at s→∞.

Question:Were you able to see this bode plot phase by inspection, before the simulation was run?
Answer: Yes. This simulation should have subtracted 360º to show 180º at low frequencies, and -90º at high frequencies.

The output of the `[z,p,k] = zpkdata(ICp, 'v')` command appears in the command window as follows:

++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++

Transfer function:

```
7.187e-039 s^6 + 8.712e-033 s^5 + 4.077e-027 s^4 + 1.742e-021 s^3   + 5.28e-016 s^2
----------------------------------------------------------------------------------------------
1.797e-043 s^7 + 3.267e-037 s^6 + 3.788e-031 s^5 + 3.027e-025 s^4  + 1.482e-019 s^3 + 5.547e-014 s^2 + 1.592e-008 s + 0.0016
```

z = 1.0e+005 * 0
0
0.0000 + 4.4721i
0.0000 - 4.4721i
-6.0606
-6.0606

p = 1.0e+005 * -2.2037 + 8.2743i
-2.2037 - 8.2743i
-0.0000 + 4.4721i
-0.0000 - 4.4721i
-6.0606
-6.0606
-1.6532

k = 4.0000e+004

wn = 8.5627e+005

++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++

Again, MATLAB failed to do pole-zero cancellation on the poles marked in red and blue.

The k = 4.0000e+004 is equal to $\frac{1}{L} = \frac{1}{25\mu H} = 4 * 10^4 H^{-1}$.

**Part 5A: Output Power Calculation**

Symbolically:

$$I_R = I_{in} * \frac{\frac{1}{sC_p}}{R + \frac{1}{sC_p}} = I_{in} \frac{\frac{1}{RC_p}}{s + \frac{1}{RC_p}}$$

Therefore:

$$I_{in} = \frac{V_{in}}{LRC_p} \frac{s}{\left[s^3 + s^2\left(\frac{1}{RCp}\right) + s\left(\frac{1}{LCs} + \frac{1}{LCp}\right) + \frac{1}{RLCsCp}\right]}$$

$P_{out}$ can be computed three ways:

$$P_{out} = \frac{V_{out}^2}{R} = I_R^2 * R = V_{out} * I_R$$

Any of the three formulas give the identical transfer function:

$$P_{out} = \frac{V_{in}^2}{L^2 C_p^2 R} \frac{s^2}{\left[ s^3 + s^2\left(\frac{1}{RCp}\right) + s\left(\frac{1}{LCs} + \frac{1}{LCp}\right) + \frac{1}{RLCsCp}\right]^2}$$

Note that this is a $6^{th}$ order transfer function, even though there are three energy storage elements. The $P_{out}$ has two zeros at s=0. The 6 poles are double copies of the 3 poles of the cubic equation.

By inspection(visualizing Figure 26 in your mind), state:

| Question: | Answer |
|---|---|
| Mag $P_{out}$ at s=jω=0 | Zero, due to two zeros at s=0 |
| Mag $P_{out}$ at s=jω→∞ | Zero, due to $2^{nd}$ order numerator, 6th order denominator |
| $\theta_{Pout}$ at s=jω=0 | 180º due to two zeros at s=0 |
| $\theta_{Pout}$ at s=jω→∞ | -360º, due to two zeros(+180º) and six poles(-540º) |

Now compute the output power of the LCC circuit by adding the following equations to your original LCC m file.

```
IR = Iind * Zcp/Zpar
%Pout = Vout * IR
Pout = Vout * Vout /R
%Pout = IR * IR *R
figure(5)
bode(Pout)
title('Output Power of LCC Tank Circuit')
[z,p,k] = zpkdata(Pout, 'v')
```

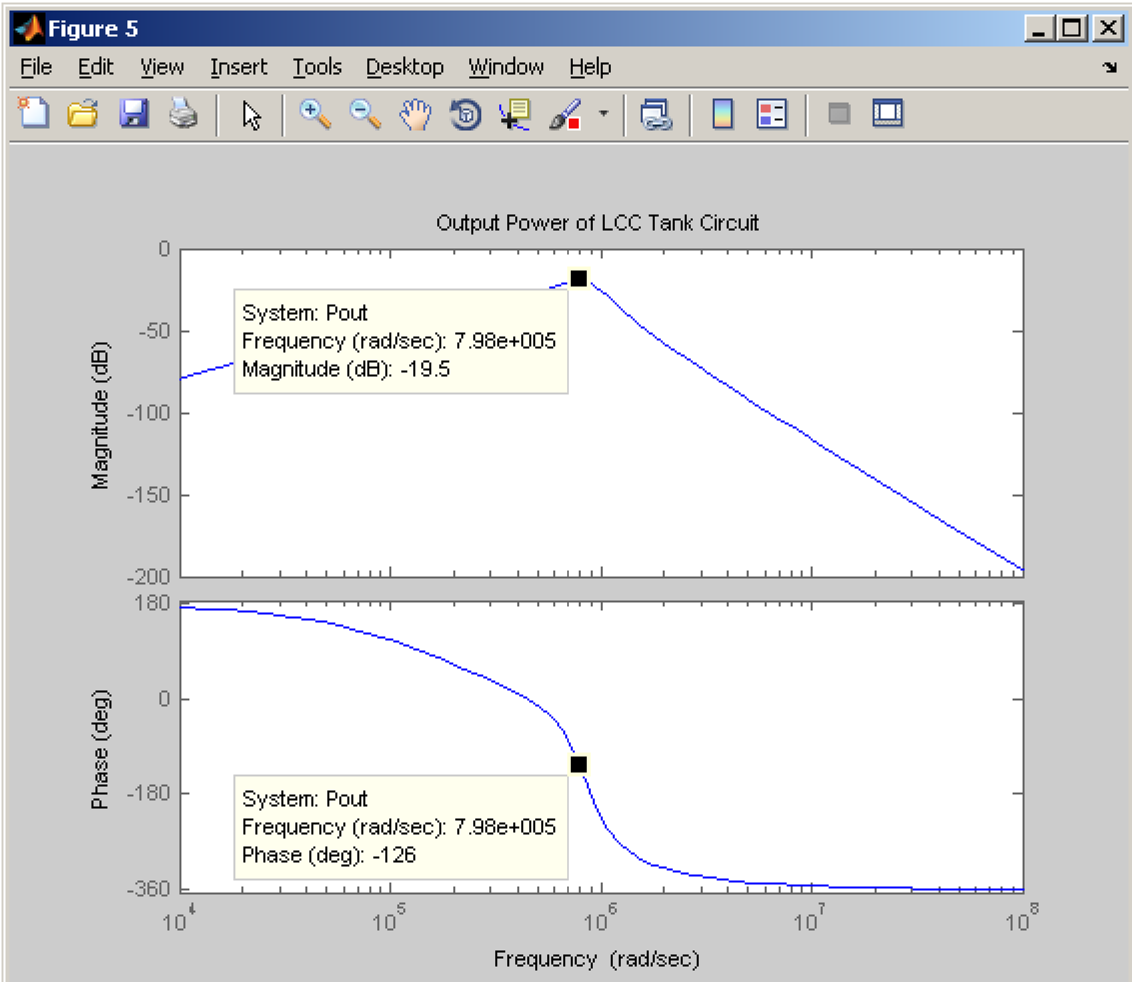

Figure 34 - Output Power

Question: How does this power output compare to the PSPICE simulation.
Answer: -19.5 dB(W) = 105.9 mW at 798 krad/sec = 127 kHz. This compares almost exactly with Figures 17 and 22.

Question:Were you able to see this bode plot magnitude by inspection, before the simulation was run?
Answer: Yes, the magnitude is bandpass, due to two zeros at s=0, and four more at at s→∞.

Question:Were you able to see this bode plot phase by inspection, before the simulation was run?
Answer: Yes. 180º at low frequencies, and -360º at high frequencies.

The transfer function for $I_R$ appears in the command window as follows:
+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++

Transfer function:

2.875e-040 s^6 + 5.227e-034 s^5 + 3.743e-028 s^4 + 1.685e-022 s^3

+ 6.336e-017 s^2 + 1.28e-011 s

-----------------------------------------------------------------
7.187e-045 s^7 + 8.712e-039 s^6 + 9.871e-033 s^5 + 6.125e-027 s^4

+ 2.215e-021 s^3 + 8.765e-016 s^2 + 1.056e-010 s
++++++++++++++++++++++++++++++++++++++++++++++++++++++++++

The output of the `[z,p,k] = zpkdata(Pout, 'v')` command appears in the command window as follows:
++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
Transfer function:

6.97e-030 s^4 + 8.448e-024 s^3 + 2.56e-018 s^2
-----------------------------------------------------------------
4.744e-052 s^8 + 1.15e-045 s^7 + 1.81e-039 s^6 + 1.928e-033 s^5

+ 1.424e-027 s^4 + 7.632e-022 s^3 + 2.581e-016 s^2

+ 4.25e-011 s + 2.56e-006

z = 1.0e+005 *
0
0
-6.0606
-6.0606

p = 1.0e+005 *
-2.2037 + 8.2743i
-2.2037 - 8.2743i
-2.2037 + 8.2743i
-2.2037 - 8.2743i
-6.0606 + 0.0000i
-6.0606 - 0.0000i
-1.6532 + 0.0000i
-1.6532 - 0.0000i

k = 1.4692e+022
++++++++++++++++++++++++++++++++++++++++++++++++++++++++++

When the pole-zero cancellation in red is performed, the remaining poles and zeros match the $P_{out}$ transfer function.

Question: How does the gain k above compare to the H of the transfer function?
Answer: They are the same, i.e.

$$H = \frac{(V_{in})^2}{L^2 C_p^2 R} = \frac{1^2}{(25\mu H)^2 (66nF)^2 (25\Omega)} = 1.4692 * 10^{22}\, W \left(\frac{rad}{\sec}\right)^4$$

Note: $P_{out}$ can be computed in three ways, i.e.

```
%Pout = Vout * IR
Pout = Vout * Vout /R
%Pout = IR * IR *R
```

But the two % commented out lines do not compute correctly!
Question: Do you know why? Try it and see!
Answer: ??????????????

**Part 5B: Output Power Calculation using MATLAB Loops**

Start a new LCC m file with the following code. This will define a range for the frequency (w) in rad/sec, from $10^5$ to $10^7$ rad/sec, in steps of 1 krad/sec. In a loop, for each of these 9901 data points, the program will compute the impedance of each element, then calculate the output voltage and power. The peak of this power will be printed out in the command window, and a Figure 5 will display the output power sweep..

```
% define frequency as a vector, from 10^5 krad/sec
% to 10^7 krad/sec in steps of 1krad
w = 1e5:1000:1e7;
an = size(w); % a vector [1 9901]
jmax = an(2); % 9901 = # of w data points.

%define impedance equations as f(w) and calculate Vout and P
for i=1:jmax
    % Impedance of each element
    Zind(i)  = 0+j*w(i)*L;
    ZCs(i)  = 0-j*(1/(w(i)*Cs));
    ZCp(i)  = 0-j*(1/(w(i)*Cp));
    ZR(i)  = R;

    %Impedance of Inductor in series with Cs
    Zs(i)  = ZCs(i)  + Zind(i);

    %Impedance of Parallel R and Cp
    Zp(i)  = 1/(1/ZR(i)  + 1/ZCp(i));

    %Output Voltage divider
    Vout(i)= Vm * Zp(i)/(Zp(i)  + Zs(i));

    %output power
    P(i)  = (Vout(i))^2/ZR(i);
end

%pick the largest value of P and print it out
max(abs(P))

%plot P(i) on a semilog scale grid
figure(5)
semilogx(w,abs(P))
title('Output Power of LCC Tank Circuit')
grid
```

Running this m file should yield the results below:

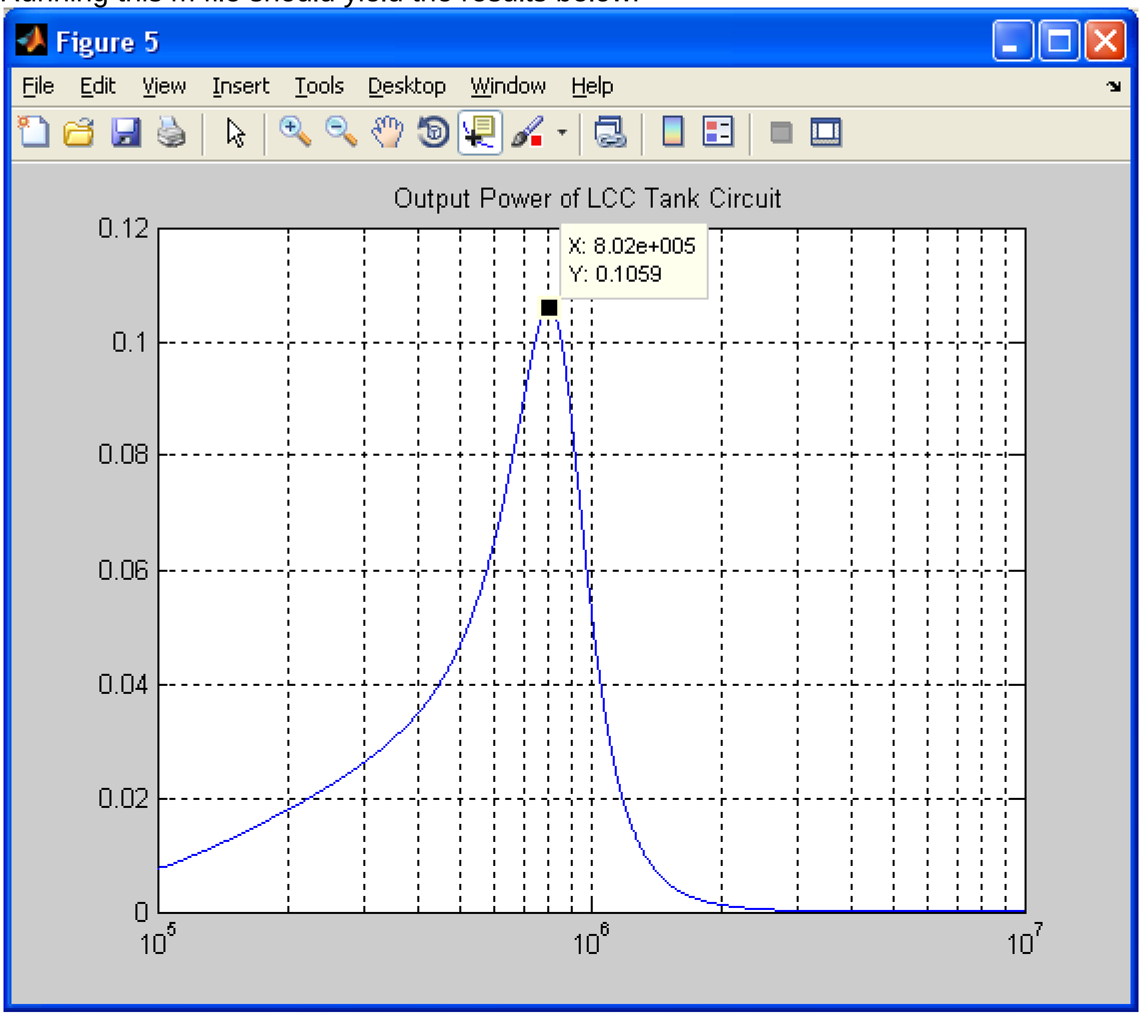

Figure 35 - Output Power Calculation

In the command window:
ans =

0.1059

>>

This should be the peak output power = 105.9 mWatts.

Question: How does this peak power calculation compare to PSPICE?

Answer: This compares exactly with Figures 17 and 22.