LECTURE 39
CCM to DCM Boundary Conditions

HW #2 DUE next time

A. CCM to DCM Transition Boundary via \( D \) versus \( \frac{I_{av}}{(I_{OB})_{\text{max}}} \) Plots with \( \frac{V_{IN}}{V_{OUT}} \) as a parameter

1. Overview

2. Buck DCM to CCM Boundary Plot

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A. CCM to DCM Transition Boundary
via $D \text{ versus } I_{av}/(I_o)_{max}$ Plots

1. Overview
In CCM we have only two portions of the duty cycle: $D_1$ and $D_2$ which give rise to two circuit topologies during the switch cycle. $D_1$ is actively set by the control circuitry, defaulting the value of $D_2 = 1 - D_1$. In DCM one or more additional circuit conditions are met that result in a DIFFERENT DC TRANSFER FUNCTION:
   
a) Unipolar diode conduction in one of the switches
   b) Low $I_o(DC)$ and high ripple $i_L$ in the current waveform

With both are present then DCM of operation can occur with three circuit topologies present over the switch cycle.

Whenever $I_o(peak) > I_{AV}$ and unidirectional switches present $\Rightarrow$ DCM operation occurs with its mixed desirable and undesirable features both on a DC and AC basis.

Both $I_o(peak)$ and $I_{AV}$ depend on the duty cycle $D$. But each is a unique function of $D$ for each circuit topology. So to set an inequality between them sets up a range of duty cycles.
Hence, when the duty cycle, D, is such that $I_{o}(\text{peak}) = I_{AV}$ we have a transition or border region mapped out between DCM and CCM of operation. This will be unique for each circuit topology. In short when the ratio: $\frac{I_{AV}}{(I_{o})_{\text{peak}}} \approx 1$ the DCM to CCM boundary occurs.

Often in applications we require $V_{o} = \text{constant}$ for a converter while $V_{g}$ (raw DC input) and D (duty cycle) are adjusted accordingly to achieve this, usually by output driven feedback loops. From our prior work on equilibrium conditions, lossless CCM of operation has an ideal output V source characteristic with $I_{o}$ not effecting $V_{o}$. That is $V_{o}$ is constant for all $I_{o}$. $V_{o} = M(D)V_{\text{in}}$ only, with no $I_{o}$ dependence.

We will contrast this with the DCM of operation which has a non-ideal $V_{o}$ source with $I_{o}$ effecting $V_{o}$. Since for DCM, $V_{o} = M(D, R_{L})V_{\text{in}}$. Moreover, as we saw in lecture 38 $V_{o}(\text{DCM})$ usually exceeds $V_{o}(\text{CCM})$ for a fixed duty cycle.

We aim in lecture 39 to reconcile these two equilibrium relations by plotting out the CCM to DCM boundary conditions versus duty cycle. The CCM-DCM boundary transition is best seen by plotting for each circuit topology the following: duty cycle, D, on the ordinate or y-axis versus the ratio $I_{AV}(D)/I_{o}(\text{peak})$ on the abscissa or x-axis. We will get unique plots for the three major converters as shown below in anticipation of the results we will derive herein later.

The above dashed line is CCM-DCM boundary transition
for the Buck topology. For DCM operation various D values are possible to the left of the dashed curve and we will determine them. Note the ideal (load independent) voltage source characteristics of the CCM will not follow into the DCM region of operation.

Next is the boost topology.

\[ \frac{V_d}{V_o} = M^{-1} \]

The above dashed line is CCM-DCM boundary transition for the boost topology in steady-state. To the left of the dashed line we will derive the non-ideal source characteristics, as compared to the ideal CCM voltage source curves.

Next is the buck-boost.

\[ \frac{V_d}{V_o} = M^{-1} \]

The above dashed line is CCM-DCM boundary transition for the buck-boost topology. Depending on the operating Mor DC gain, a unique D is set for the CCM-DCM transition. To the left of the transition boundary we must derive the non-ideal DCM curves. In Lecture 38 we solved for \( V_{\text{out}} \) in DCM of operation by using intuitive linear analysis as well as by solving the quadratic equations resulting from balance conditions in the three circuit topologies. Herein we concentrate on the goal of clearly defining the border between the two regions under all operating conditions.
We will first calculate for each circuit topology the unique relationships for $I_{AV} = f_1(D)$ and $I_o(peak) = f_2(D)$. These are two distinct relations for each topology. Then we will plot $D$ versus $I_{AV}/I_o(max)$ and on this plot delineate with a dashed curve the ratio $I_{AV}(D)/I_{peak}(D) = 1$. This will sketch out the locus of points for the CCM-DCM boundary transition or the dashed boundary curve.

1. **Buck or step down topology CCM to DCM Boundary**

We let $V_D = V_{in}$ be the input voltage. Often this is the rectified mains. We will first review CCM conditions and then find $I_{critical}$ at the CCM to DCM boundary.

a. **Review of CCM DC Transfer Function and Inductor Waveforms**

Use $<V_L>_T = 0$  

$\frac{V_o}{V_d} = \frac{t_{on}}{T_s} = D$  

$\frac{I_o}{I_d} = \frac{1}{D}$

During the interval $t_{on}$ $V_L = V_d - V_0$ while for the time $t_{off}$ $V_L = -V_0$. The buck converter operates CCM for the case $I_o$ (average) $> I_{CRITICAL}$. The individual circuit conditions will set both critical and average currents. In short for high DC levels of $I_L$ and low levels of $\Delta I_L$ (ripple), we have only unipolar inductor current. On the other hand when $I_o$(equivalent DC level) goes below a critical level, $I_{CRITICAL}$, we have the possibility of bi-polar inductor current. With uni-polar switches present this may cause the onset of the DCM of operation when one of the switches turns off inadvertently. Below $I_0 = I_{CRITICAL}$ the ideal CCM of operation is no longer valid. Our goal is to quantify the CCM to DCM boundary.
As easily seen in the waveform versus time plots of $I_L$, for $I_o < I_{(critical)}$ DCM occurs and for $I_o > I_{(critical)}$ we get CCM operation. So we know already if $I < I_c \equiv I_{(critical)}$ we are just beginning DCM and at the ultimate limit of CCM. This allows us to quantify the boundary transition as a function of duty cycle, $D$. This boundary is shown schematically in the $I_o - V_o$ plot below of an ideal buck. Solid lines are CCM operation up to the limit of DCM where $I_o - V_o$ plots are no longer vertical lines. This was also shown in our overview plot of the buck circuit, where we plotted $D$ versus the ratio $I_{AV}/I_0(max)$. The boundary plot varies with $D, V_{in}$ and $V_0$ in a complex way we will derive below.
At the Boundary $I = I_{\text{CRITICAL}}$

Below we show the $i_L$ current waveform just as it hits zero and tries to go negative.

The effective DC inductor current is defined by the three equivalent parameters: $I_{LB} = I_{OB} = I_{AV}$. Where the subscript B refers to the boundary of CCM to DCM.

The critical current for the buck converter is $\frac{DT_s}{2L} (V_d - V_o)$.

We can set the inequality:

If $\Rightarrow I_{LB} = I_{av} \leq \frac{DT_s}{2L} (V_d - V_o)$ DCM occurs as discussed above.

We have several ways to satisfy the inequality. For we can choose: $T_{SW}, V_d, V_0, L$ and D to meet the inequality.

There are two major paths for $I_{AV} < I_{\text{critical}}$ and each will give a unique DCM to CCM boundary plot as we will see below for the two separate practical cases: $V_d = \text{constant}$ and $V_0 = \text{constant}$. In either case if $I_{AV}$ decreases $i_L$ will try to go negative but the uni-polar diode switch will not allow it.
A buck converter driving a DC motor often has \( V_o = D V_d \) and \( V_d \) is fixed but \( D \) varies to control \( V_o \). In this case we find at the CCM to DCM edge:

\[
I_{av} = \frac{T_s V_d}{2L} D(1-D) = I_{LB}
\]

**\( V_d \) Constant**

By plotting \( I_{AV} \) versus \( D \):

\[
I_{LB} = I_{oB}
\]

\[
I_{LB, max} = T_s V_d / 8L
\]

\[
0 \quad 0.5 \quad 1.0 \quad D
\]

We find \( I_{LB} \) or \( I_{AV} \) is max. at \( D = 1/2 \)

For fixed \( V_o \); \((I_{vo})_{max}\) for a buck converter occurs at \( D = 0 \). But \( D = 0 \) means \( V_d = V_{in} = \infty \), via

\[
I_{LB}(max) = I_{oB} / D
\]

\[
I_{LB, max} = T_s V_d / 8L
\]

we can rewrite our expressions

**For \((I_{LB})_{max}\) for \( V_d \) constant**

\[
\frac{T_s V_d}{2L} \downarrow \quad I_{AV} = 4(I_{LB})_{max} D(1-D)
\]

If we divide the period \( 1-D = D_2 \) and \( D_3 \) one can show that \( I_L = 0 \) in DCM at time \( D_1 + D_2 \) as shown on page 10.
\[ D_2 = \Delta_1 = \frac{I_{AV}}{4(I_{AV})_{max}} D \]

From this we solve for \( D \) in terms of the same ratio \( \frac{I_{AV}}{(I_{AV})_{max}} \) or \( \frac{I_{AV}}{(I_{LB})_{max}} \)

\[ D = \frac{V_o}{V_d} \left[ \frac{I_{AV} / I_{max}}{1 - V_o / V_d} \right]^{1/2} \]

We can plot \( \frac{V_o}{V_d} \) (y-axis) versus \( D \) for \( \frac{I_{AV}}{I_{max}} \) as a parameter as on page 10

\( \frac{I_{AV}}{I_{max}} \) (x-axis) for \( V_d = \) constant

\[ \frac{V_d}{V_o} \text{ Constant} \quad \frac{V_o}{V_d} \text{ Constant} \]

\[ \left( I_{V_L} \right)_{max} = \frac{T_s V_d}{8L} \]

\[ I_{AV}(D) = 4(I_{V_L})_{max} D_1 (1 - D_1) \]

\[ \left( I_{LB} \right)_{max} = \frac{T_s V_o}{2L} \]

\[ I_{AV}(D) = \left( I_{V_L} \right)_{max} 1 - D_1 \]

In either case if \( I_{AV} \) decreases \( i_L \) will try to go negative but the diode will not allow it. When the diode stops conducting we go from two known periods of switching, \( D \) and \( 1-D \), to three periods \( (D_1, D_2 \text{ and } D_3) \), only one of which \( D_1 T_s \) is known as shown below from the active switch drive. \( D_2 \) and \( D_3 \) are set by circuit conditions not by switch drive conditions.

c. **Into the DCM Region: Beyond the Boundary**

We will have three independent time periods within the switch cycle as shown on page 10. Note that \( D_2 \) is not equal to \( 1- D_1 \) and the third period \( D_3 \) is another unknown.
We can try to estimate $D_2$ from the two equations that are true for all $V_d$(input) and $V_0$(output) cases.

**EQ1.** $\langle V_L \rangle_{T_s} = 0, \quad (V_d-V_0)D_1T_s + (-V_0)D_2T_s = 0$

$$\frac{V_o}{V_d} = \frac{D_1}{D_1+D_2}, \quad \text{Note EQ 2 } (i_{L\text{peak}}) = V_0/L D_2T_s$$

$$I_{AV} = \frac{1}{2} (i_{L\text{peak}})(D_1 + D_2) = \frac{V_oT_s}{2L} (D_1 + D_2) D_2$$

Equations 1 and 2 are both employed to make plots of $D$ versus $\left(\frac{I_o}{I_{LB}(\text{max})}\right)$, where $I_{LB}(\text{max})$ a scaling factor is either $T_sV_d/(8L)$ for $V_d$ = constant or $T_sV_o/(8L)$ for $V_0$ = constant.

1. **DCM to CCM Boundary for a Buck with $V_d$ (Constant):** where the x-axis scale factor $I_{LB}(\text{max}) = T_sV_d/(8L)$

   • We have an ideal Buck $V_o$ source for CCM only which has $V_o/V_d$ fixed for all possible load currents up to the DCM boundary as shown

   • Dashed curve is the calculated CCM - DCM boundary and it occurs only at low load current.
The DCM to CCM boundary depends on both D and $I_{AV}/I_{max}$. Note how $V_o/V_d$ changes in DCM versus load current and is constant in CCM. We will revisit this in Chapter 10 of Erickson, especially Problem 10.3 which graduate students are to do for homework #2 now, undergraduates will do it later.

Next we do D vs. $I_o/I_{LB,\text{max}}$ plots for $V_o$ fixed conditions.

2. **DCM to CCM Boundary for ($V_o$ Constant) the x-axis scale factor $I_{LB,\text{max}} = T_s V_o/(8L)$**
   - We have an ideal buck $V_o$ source for CCM only. We have fixed $V_d/V_o$ for all load currents up to the DCM boundary as shown on page 12.
   - Dashed curve is the derived boundary from CCM to DCM. At the boundary we find: $I_{AV} = I_{max} (1 - D)$

Note the special case on the x-axis $D = 0$ $\Rightarrow$ $\frac{I_{AV}}{I_{max}} = 1.0$
Similar plots of the DCM to CCM boundary can be made for the other two basic converter topologies as we will do below.

2. **Boost or Step-up Topology**
   
   a. Review of CCM $V_o/V_d$ Transfer Function and Inductor Waveforms

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**Diagram:**

- Graph showing $V_o = constant$.
- Boundary lines for $Vd/Vo = 1.25$, $2.0$, $5.0$.
- Axis labels for $D$, $(I_o/IL_{B,max})$, $(I_o/IL_{B,max}) = Ts V_o/2L$.

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**Text:**

- Similar plots of the DCM to CCM boundary can be made for the other two basic converter topologies as we will do below.

**2. Boost or Step-up Topology**

a. Review of CCM $V_o/V_d$ Transfer Function and Inductor Waveforms
The above switch on and switch off circuit topologies are **WRONG** for the boost circuit. For HW#2 please draw the correct circuit topologies, that the equations below will satisfy by using the boost circuit below in two switch states.

\[
\begin{align*}
<V_L>_{Ts} &= 0; \quad V_dD + (V_d - V_o)(1-D) = 0; \\
V_o &= \frac{1}{1-D} = \frac{1}{D'}
\end{align*}
\]

Moreover for the lossless converter \( I_{out} = I_{in} (1-D) \)

The DC transformer model for the CCM boost topology would be:

\[
\begin{align*}
\frac{V_d}{1:D'} \quad \text{or} \quad \frac{V_d}{D':1}
\end{align*}
\]

b. **At the DCM to CCM Boundary**

At the CCM-DCM boundary the DC current reduces until the ac current tries to go negative and the uni-directional switch cannot follow. \( I_{LB}(\text{average}) = I_{AV} = \frac{1}{2} i_L(\text{peak}) = \frac{V_d}{2L} D_s T_s \)

This equation is valid just at the boundary.
The average output current, \( I_{LB}(\text{average}) \), also occurs at CCM-DCM boundary. We choose to examine the case \( V_o (\text{constant}) \) and \( V_d (\text{input}) \) or \( V_{in} \) varies which corresponds to a crude rectified ac mains for \( V_{in} \) and a feedback circuit where \( V_0 \) is kept constant. We will find \( I_{LB}(\text{boundary}) \), \( I_{OB}(\text{output at the boundary}) \) versus the \( D_1 \) duty cycle set by the timing

\[
I_{LB}(\text{average at the boundary}) = I_d = I_{AV} = \frac{T_s V_o}{2L} D_1 (1 - D_1)
\]

The maximum occurs at \( D_1 = \frac{1}{2} \) and is called \( I_{LB}(\text{max}) \). Now we learned before that the inductor current equals \( I_{in} \) for a boost converter. We also know at the boundary that \( I_{out} = I_{in} (1 - D_1) \) using the simple CCM relation in steady-state.

\[
I_{OB} = I_{out} = I_0 = i_L (1 - D_1) = \frac{T_s V_o}{2L} D_1 (1 - D_1)^2
\]

EQU. #1 at the border

\[
(I_{AV})_{\text{max}} = \frac{T_s V_o}{8L} \quad \text{occurs at } D_1 = \frac{1}{2}
\]

Equ #2 at the border

\[
(I_{OB})_{\text{max}} = \frac{2}{27} \frac{T_s V_o}{L} \quad \text{at } D_1 = \frac{1}{3}
\]

Where \( 2/27 = 0.74 \). Plotting these two equations versus \( D \) on page 15 we find \( (I_{AV})_{\text{max}} \) occurs at \( D = 0.5 \), while \( (I_o)_{\text{max}} \) occurs...
at \( D = 0.33 \); as shown.

Rewriting both currents in terms of the maximum values versus \( D \) we find:

\[
I_{AV}(D) = 4D_1(1-D_1) (I_{AV})_{\text{max}}
\]

\[
I_o(D_1) = 27/4 D_1 (1-D_1)^2 (I_o)_{\text{max}}
\]

When \( i_L \) tries to go negative we go from the left curve below to the right:

c. **Into the DCM Region: Beyond the Boundary**

Assume that as the output current decreases that both \( V_d \) and \( D \) are unchanged up to \( I_{\text{CRITICAL}} \).

The volt-sec balance on the inductor \( \langle V_L \rangle_{Ts} = 0 \) in DCM case to the right on the above figure gives:

\[
D_2 \neq 1 - D_1, \quad D_2 = 1 - D_1 - D_3 \]

\[
V_d D_1 T_s + (V_d - V_o) D_2 T_s = 0
\]
This leads to equation #3
Eq #3 \[ \frac{V_d}{V_d} = \frac{D_2 + D_1}{D_2} \] \[ \Rightarrow \] In the lossless case \[ P_o = P_d \]
\[ \frac{I_o}{I_d} = \frac{D_2}{D_1 + D_2} = \frac{I_{out}}{I_{in}} \]

Next we turn to the average or DC conditions for \( I_{in} \) or \( I_d \) at the boundary via the simple triangle rule Average value= \( \frac{1}{2} I_{peak} \Delta t \)
\[ (I_d)_{AV} = (I_{L})_{AV} = \frac{V_d}{2L} D_1 T_s [D_1 + D_2] \]
\[ \downarrow \quad \downarrow \]
\[ 1/2 \text{ peak} \quad \text{total time duration}(D_1 + D_2) \text{ for } I>0 \]

The average output current is related to the average input current:
\[ (I_o)_{AV} = (I_o)_{AV} \ast \frac{D_2}{D_2 + D_1} \]

Since \( I_0 \) (average at the boundary) in terms of \( V_d \), \( D_1 \) and \( D_2 \) is:
Eq#4 \[ (I_o)_{AV} = \frac{T_s V_d}{2L} D_1 D_2 \]

Using Equations 1,3 and 4 above we find
\[ D = f \left( \frac{V_o}{V_d} \right), \frac{I_o}{I_{max}} \]

\[ D(\text{Boost}) = \left[ \frac{4}{27} \frac{V_o}{V_d} (\frac{V_o}{V_d} - 1) \frac{I_o}{I_{max}} \right]^{1/2} \]

If we plot \( D \) (y-axis) versus the now familiar ratio \( I_0/I_{OB}(max) \) on the x-axis where \( I_{OB}(max) = 2/27 T_{sw} V_0/L \):

• We achieve an ideal boost \( V_o \) source for CCM only, where \( V_0 \) is flat versus \( I_0/I_{OB} \).
• The dashed curve is the derived DCM-CCM Boundary in steady-state. Note that the D required to achieve DCM of operation changes with \( V_0 \) and \( I_0/I_{OB}(max) \) conditions Below we consider additional salient points
$D \to 0$, $I_o/I_o^{\text{max}} \to 0$. For all $V_0/V_d$ curves

- For $I_0/I_{OB}^{\text{(max)}} > 0.9$ only CCM occurs with ideal $V$ source curves as shown below

Warning: We have assumed that we have an operating feedback loop so that in DCM, $V_o$ is kept constant during each $T_s$ by varying $D$, $V_o = V_d/1-D$. If there is no feedback however then at light load $V_o \to$ dangerously high.

**Example:** For the 120 W Boost Converter below we choose $C$ very large so we always have DC output.

$V_d$(input) is crude DC and varies from 12 to 36 V but $D$ changes, via an undisclosed feedback loop, to keep the output fixed at 48 V.
In steady state the output current in steady state is:
\[ I_o = \frac{120W}{48} = 2.5A \]

\[ f_s = 50 \text{ KHz}, T_s = 20 \text{ msec} \]

We are asked to find \( L(\text{max}) \) which \textbf{insures DCM operation} only. We want to avoid entirely CCM of operation. That is the choice of \( L \) must \textbf{not} exceed \( L(\text{max}) \). \( L < L_{\text{MAX}} \) to insure DCM of operation. Using CCM equations, which are valid only at the border, we find the range of \( D \) required.

\[ \frac{V_o}{V_d} = \frac{1}{1-D} \quad \text{for } V_o \text{ (fixed)} \text{ and } 12 \leq V_d \leq 36. \]

Then we find \( \frac{1}{4} \leq D \leq \frac{3}{4} \) in order to keep \( V_o \) fixed at 48 V.

\[ I_o = \frac{T_s V_o}{2L} D(1-D)^2 \]

We solve for \( L_{\text{max}} \) and note the smallest \( L_{\text{max}} \) would occur at \( D=0.75 \)

\[ L \leq \frac{T_s V_o}{I_o(\text{min})} \cdot 0.75(0.25)^2. \]

Hence for a given \( T_s = 20 \text{ msec}, V_o = 48 \text{ V}, \text{ and } I_o = 2.5 \text{ A}, \)

\( L \leq 9 \text{ mH guarantees} \) we always operate INSIDE the edge of the CCM to DCM boundary \textbf{on the DCM side}.  

2. Buck-Boost Topology

a. Review of CCM $V_0/V_d$ Transfer Functions and Inductor Waveforms

The basic buck-boost circuit with switches is shown below:

In the CCM of operation the $V_L$ and $I_L$ waveforms for buck-boost are shown below.

The corresponding topologies for the two CCM switch conditions are:
In steady-state the volt-sec on the inductor yields:
\[<V_L>_T = 0; V_d DT_s + (-V_0)(1-D) T_s;\]

Hence we find \(\frac{V_o}{V_d} = \frac{D}{1-D}\) and for a loss free converter we also know \(\frac{I_o}{I_D} = \frac{1-D}{D}\)

The DC Transformer Model for the Buck-boost in Steady State is:

b.  **At the DCM Border**
At the CCM-DCM Boundary the inductor current, \(i_L\), just reaches zero and tries to go negative but the circuit diodes will not allow it. We now write expressions for the inductor current average, \(I_{AV}(boundary)\), versus \(V_d\) and \(D\) as well as versus \(V_0\) and \(D\).
\[ I(\text{average at boundary}) = I_{AV} = \frac{1}{2} i_L(\text{peak}) = I_d = \]

\[ I_{LB}(\text{average}) = I_{av}(V_d, D) = \frac{T_s D_1 V_d}{2L} \]

Max at \( D = 0 \)

Using at the DCM to CCM boundary

\[ V_d = V_o \left( \frac{1 - D_1}{D_1} \right) \Rightarrow T_s V_o (1 - D_1) = I_{AV} (\text{boundary}) = I_{AV}(V_o, D). \]

In the buck-boost if \( I_C(\text{capacitor}) = 0 \), the output current is: \( I_0 = I_d (1 - D) / D \) and at the border we find:

\[ I_0 = I_L - I_d = \frac{T_s V_o}{2L} (1 - D_1)^2, \text{ which is Max at } D_1 = 0 \]

\[ I_{LB}(\text{max}) = I_{OB}(\text{max}) = T_{SW} V_o / 2L \]

\[ (I_{AV})_{\text{max} @ D = 0} = \frac{T_s V_o}{2L} \text{ and } I_{AV} = (I_{LB})_{\text{max}} (1 - D_1) = I_{LB}(D) \]

\[ (I_{OB})_{\text{max} @ D = 0} = \frac{T_s V_o}{2L} \text{ and } I_{OB} = (I_0)_{\text{max}} (1 - D_1)^2 = I_{OB}(D) \]

Assuming \( V_o \) is constant with respect to \( D \) (via feedback), if we plot \( \frac{I_{AV}}{(I_{AV})_{\text{max}}} = 1 - D \) we find the linear solid line connecting the two axii as shown on page 22.
If we plot \( \frac{I_{OB}}{(I_{OB})_{max}} = (1-D)^2 \) we find the dashed line shown above.

**c. In the DCM Region: Beyond the Border**

If \( i_L \) tries to go negative in the buck-boost circuit the uni-polar diode prevents it. So DCM occurs with three time periods \( D_1, D_2, \) and \( D_3 \) shown below. Doing volt-sec balance on the inductor in the three periods we find:

\[
<V_L> = 0; \quad V_0 D_1 T_s + (-V_0) D_2 T_s = 0; \quad \text{Where } D_2 \neq 1 - D_1; \quad \text{Rather } D_2 = 1 - D_1 - D_3
\]
\[
\frac{V_a}{V_d} = \frac{D_1}{D_2} \quad \Rightarrow \quad \text{In a loss less converter operating in DCM}
\]
\[
P_\text{o}(\text{out}) = P_d(\text{input})
\]
\[
\frac{I_o}{I_d} = \frac{D_2}{D_1} \quad \Rightarrow \text{We can calculate the value at the border}
\]
\[
(I_L)_{AV} = \frac{1}{2} (I_{L,\text{peak}})(D_1 + D_2)
\]
\[
\downarrow \quad \downarrow
\]
\[
\text{Height} \quad \text{time}
\]
\[
= \frac{V_d}{2L} DT_s(D_1 + D_2)
\]
One can plot the duty cycle at the DCM to CCM border, D, on the y-axis for \(V_0 = \text{constant}\) as a function of the output current, \(I_0\), on the x-axis. We scale the x-axis as \(I_0 / I_{OB}(\text{max})\). Where \(I_{OB} = T_{SW} V_0 / 2L\). In short,
\[
D = f\left(\frac{I_0}{(I_0)_{\text{max}}}, \frac{V_0}{V_d}\right)
\]
\[
D = \frac{V_o}{V_d} \left[\frac{I_o}{(I_{OB})_{\text{max}}}\right]^{1/2} = D(\text{DCM to CCM Border})
\]

Summary of Buck-Boost CCM to DCM Border
1. We have an ideal \(V_o\) source for CCM operation which is flat with \(I_o/I_{OB}(\text{max})\) throughout the CCM region.
2. The dashed curve is the CCM-DCM boundary which varies position with both the ratio \(V_d / V_0\) and the ratio \(I_0 / I_{OB}(\text{max})\).
3. Note that \(V_d / V_0\) versus D is no longer flat in the DCM region of operation. \(D \rightarrow 0\) pins all curves to one operating point. \(I_0 / I_{OB}(\text{max}) = 0\) for all \(V_d / V_0\) curves as shown on page 24.
5. Simple Numerical Example

Consider the 10 Watt buck-boost converter below with \( f_{SW} = 20 \text{ kHz} \) and with a steady-state output voltage at 10 V. We have a load such that the current drawn is 1.0 A at the output. C is assumed very large so \( V_o = \text{constant} \) and \( L = \frac{1}{20} \text{ mH} \). **CAN you tell if this is a DCM or CCM equilibrium condition??**

\[ \frac{V_o}{V_g} = \frac{10}{15} = \frac{D}{1-D} \]

which implies that for given steady-
state circuit conditions above \( D = 0.4 \), but only if we really operating CCM, but we may not be. To determine at the outset the mode of operation we first find \( I_{\text{CRITICAL}} \) for the buck-boost circuit as follows:

\[
I(\text{critical}) = \frac{T_s V_o}{2L} = \frac{0.05\text{ m sec}*10}{2(0.05\text{ mH})}
\]

\( I(\text{critical}) \) = 5 A for this buck-boost

\( I_o \) (at DCM-CCM boundary) is given by the well known and simple CCM equation.

\[
I_o = (I_o)_{\text{max}}(1-D)^2 = 5(0.6)^2 = 1.8 \text{ A which is < 5 A so we cannot be operating CCM as we first assumed.}
\]

Surprise! \( \Rightarrow \) DCM not CCM operation is occurring in steady-state.

But if we have DCM operation the \( D \) value is then given by a different steady-state relationship:

\[
D(DCM) = \frac{V_o}{V_d} \sqrt{\frac{I_o}{(I_o)_{\text{max}}}} = \frac{10}{15} \sqrt{\frac{1.0}{5}}
\]

\( D(DCM) \) = 0.3.

**Finally, For HW#2 Due next time:**

1. Answer any questions asked throughout lectures 37-39.
2. Erickson Chapter 5 and 10
   a. Graduate Students: Problems 5.4, 5.14 and 10.3.