LECTURE 32
Filter Inductor Design

A. Detailed Look at Analysis of L-C Filter Inductor:
$I_{ac} \ll I_{dc}$

1. $L \approx \dfrac{N^2}{\Re}$ and $A_L = \dfrac{1}{\Re}$

   Where $\Re$ is magnetic reluctance of the core plus air-gap and A_L is the specific inductance in mH for 1000 turns of wire

2. Copper Windings
   a. Wire sizes, AWG # chosen to meet current specification.
   b. Available Winding Area of the Core: W_A
   c. Mean Length of Wire Turn (MLT) on the particular core

3. Six Inductor Parameters

4. Inductor Design Spec's
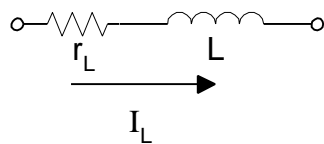
}AWG # Choice for inductor wire windings including skin and proximity effects

5. Inductor Core Constant: $K_g = \dfrac{A_c^2 W_A}{MLT} \ [cm^5]$

6. Trading Cu Wire for Core Iron in the quest for minimum losses

B. Analysis and Design of Inductors

1. Overview

2. Analysis of an Inductor Performance

3. Design Flow for an Inductor

LECTURE 32
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A.   Inductor Design for a L-C Filter: $I_{ac} < I_{DC}$

1.   Air Gap Conditions in Magnetic Cores

The inductance required is specified by the PWM converter circuit needs such as the required ripple allowed in an output filter or the energy storage requirement in a flyback converter. If there is no air gap in the magnetic core, $\Re$(core) is simply $\Re = l_c/(\mu_c A_c)$ for the flux paths in units of H^{-1}. If due to wire or core losses heat builds up and T(core) > $100^{o}C$ μ_c is reduced. **This makes L change with load on a slow time scale and is undesired.** How do we make L less dependent on the core properties?

     With an air gap placed in the core we need to use the series reluctance of core plus air gap , $\Re$(total) = $\Re$(core) + $\Re$(air gap). We cut an air gap in the core both to widen the current i_{range} for which the inductance $L \neq f(i_L)$ and to make the effective permeability less dependent on the core conditions such as core temperature. We will show herein an example of a core with 50% variation of μ yet by employing an air gap in the core we will have only a change in the inductance of 5% as compared to the inductor variation without an air gap which is 50%.

$L \propto N^2/\hat{A}$ (core plus airgap)

     For filter inductor cases $I_{ac} << I_{dc}$ and AC core losses are expected to be small compared to $I^2 R$ losses in the copper wires. If we choose wire size comparable to the skin depth at the frequency of the applied currents, we also expect additional wire losses due to skin effects and proximity effects are low and the dominant loss is simply $I^2_{rms}R_L$. R_L is usually just the ESR term of the inductor. This filter inductor is the easiest inductor case to examine and we will focus on it first. Inductors used with nearly pure AC waveforms will be the most difficult to design and we will defer discussion of these till we cover transformers.

W$_A$ (window area)
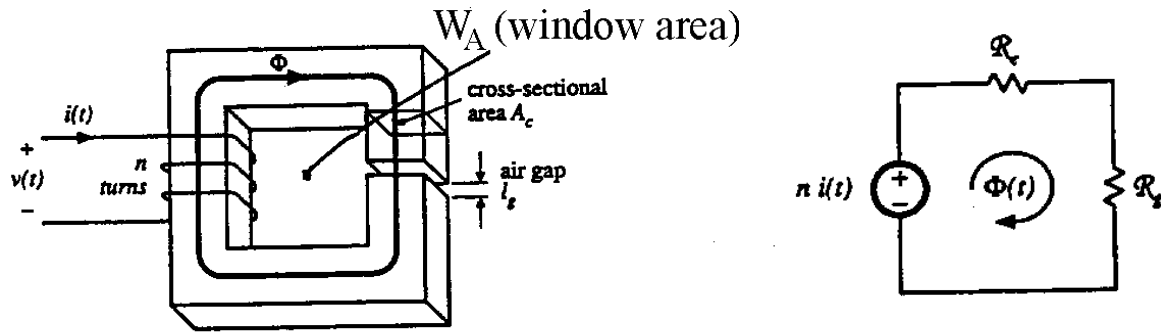
The specification of the core air gap is a crucial part of the core design for an inductor as we will show below.
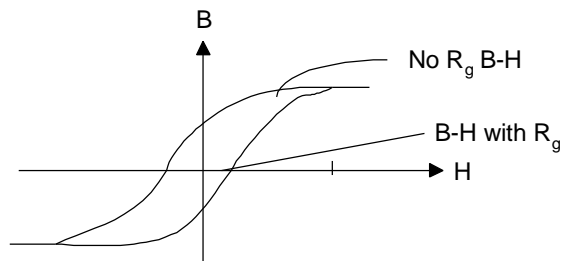
$$\Re_c \;=\; \frac{\ell_c}{\boldsymbol{m}_c \; A_c}$$ for the ferrite core

portion . For the air gap the

portion$\rightarrow \Re_g \;=\; \frac{\ell_g}{\boldsymbol{m}_o \; A_g}$

Often for high permeability cores, $\mu_c = 10^3 \mu_o$, and $\Re_{total} \approx \Re_{gap}$.. **Hence we often asume that the air gap reluctance dominates** $\Re$(air) > $\Re$(core). In this case,

$Ni = \phi \Re_{gap}$ If we solve for N under maximum current in the inductor wires and maximum Magnetic flux density in the core, we find N= Li_{max} / (B_{MAX} A_{core}). This will be a useful relation as will the following one. Now we can use some simple steps to show how the value of L varies in terms of the air gap, the number of turns and core area for flux flow, assuming little flux fringing occurs.

$$Li = N\phi \quad \Rightarrow \quad L \equiv \frac{N^2}{R_{gap}} = \frac{\boldsymbol{m}_o A_c N^2}{\ell_g}$$

$$L = \frac{N^2 \, \boldsymbol{m}_o A_c}{\ell_g} \text{ for an air gap inductor.}$$

We can tune L in terms of the core in two ways :by varying the geometry of core(A_c) or by size of the air gap (l_g). The amount of copper turns also has a big role but it at first seems independent of the core. This is not the

case as N is limited by N= $Li_{max}/(B_{MAX} A_{core})$.

There is both gross and fine tuning of L.

1. N↑↓ L↑↓ Gross tuning via copper!

2. l_g ↑ L↓   We are now looking at the core based

    A_c ↑ L ↑   Fine tuning of L for a design goal

Where A_c is the cross-section of magnetic core to flux flow. By equating the magnetic energy stored in the air gap, ½ B^2/μ_0 , to the electrical energy in the inductor, ½ Li^2, we can find the required air gap as follows.

$$l_g = \frac{\mu_0 L I^2_{max}}{B^2_{max} A_c} 10^4 \qquad (m)$$

with A_c expressed in cm^2. $\mu_0 = 4\pi 10^{-7}$ H/m.

The air gap length is given in meters.

The value expressed above is approximate, and neglects fringing flux and other nonidealities.

Core manufacturers use a parameter A_L, the specific inductance per turns squared, as a way to specify the core gap required. For A_L we assume 1000 turns

$$\frac{L}{n^2} \equiv A_L \equiv \frac{1}{R} = \frac{1}{\frac{m_0 \ell_g}{A_g}} = \frac{A_g}{m_0 \ell_g} \quad \text{Again the units of } A_L \text{ are mH/turn}^2$$

A_L is Usually specified as inductance in mH for a 1000 turn winding.

A_L spec for a core is equivalent to specifying the air gap spacing l_g.

For an l_g feeling consider that 3 thousands of an inch gap corresponds to a sheet of paper. Mylar sheets come in precision thickness down to 1/2 mil.

$20 \leq A_L \leq 500$ mH per $turn^2$ is typical of cores with air gaps. Consider some of the constraints set by B_{SAT} of the chosen core material for an inductor. First the amp-turn limit set by ni< $B_{SAT}A_C$ ℜ, this means

ni $<B_{SAT}/\mu$. This limit on I_{MAX} will in turn limit the maximum energy that can be stored in an ungappped core to B^2(SAT) *l(core)*A(core) $/2\mu$..

For HW #1 show that for a gapped core we get a different value of the maximum energy stored.

Looking ahead, we note that for transformers B_{SAT} will introduce another constraint called the maximum volts per turn. In the case of a sinusoidal voltage, Faraday's law gives V= N dφ / dt =Nω A_C B_{SAT}. Hence V/ωNA_c must be less than $B_{SAT.}$ A_L of 100 is a typical specific inductance core spec by a manufacturer of <u>pregapped</u> cores.

The required A_L is given by:

$$A_L = \frac{10B_{max}^2 A_c^2}{LI_{max}^2} \quad \text{(mH/1000 turns)}$$

Units:
A_c cm^2,
L Henries,
B_{max} Tesla.

$$L = A_L\, n^2\, 10^{-9} \quad \text{(Henries)}$$

That is the core manufacturer will take your A_L spec and the core type you select and then gap the core to meet your requirements. Given the specific inductance , A_L , we multiply by N^2 to get the inductance in Henries. Inductors typically have a range of from 0.1 to 3000 μ Henries and carry from 0.1 to 30Amperes. **A good website for inductor information is http://order.coilcraft.com**

    2.   **Specifing Copper Windings**

There is usually only one Cu wire winding around a magnetic core when making an inductor. The winding is placed in the core air window of area,W_A, which is called the wire winding window. Each wire type(circular, rectangular, tape-foil) we choose to employ has a K(factor) that tells of the total wire area how much is really Cu.

HV	**depends**	**varnished**
wire	**on wire**	**wire**

$$.02 \leq K_{cu} \leq .95$$
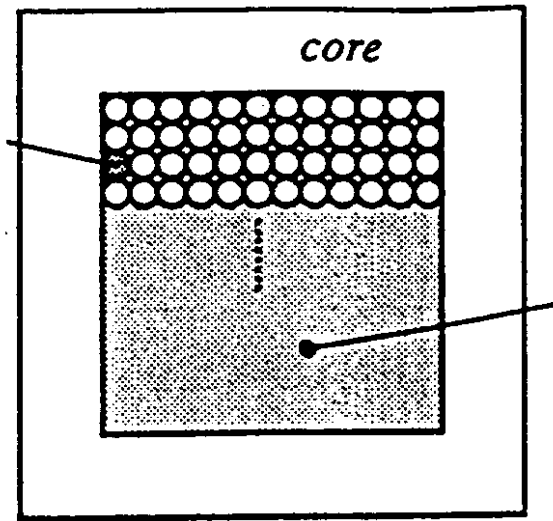
core

For example K(cu foil) ≈ 0.7
From considering

$$\frac{N \ A_{cu}}{A(window)}$$

A_c is area of wire, Wire winding area of the core is W_A.

For core wound filter inductors where $I_{ac} < I_{DC}$ proximity effects, that increase wire resistance, are negligible so we can employ large diameter wires compared to the skin depth at the operation frequency of the curent in the wire. Hence, we could fill entire core window with wire if needed ,with no need for concern with large diameter wire, as no big d/δ effects (skin effect), proximity effects nor harmonic effects occur since $I_{DC}^2 R_{DC} > I_{AC}^2 R_{AC}$. In short for $A_{cuwire\#}$,we can maximize the of wire to meet the DC current level without skin effect (d/δ) fears The wire resistance is simply the DC value as given by:

$$R_L \ = \ r\frac{\ell_{wire}}{A_{cu \ wire \ \#}}$$

ℓ_{wire} is N * mean length of turns around the selected core. That is the core choice does effect the wire resistance for a fixed number of turns. This interdependence of the core size and the copper windings is also illustrated by the core B_{SAT} or B_{MAX} which is a property of core material choice. However it also effects the maximum number of copper turns we can employ at a given coil current via $N = Li_{max}/(B_{MAX} A_{core})$. We will more fully develop the compromises needed in inductor design later.
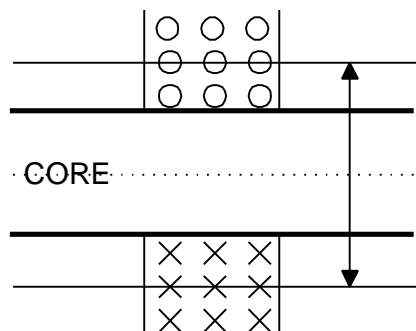
The situation for the wire is also constrained by the area of the wire winding window area, W_A. Given W_A and the choice of wire diameter then we are also constrained in the total number of turns that will fit into the wire winding window

The wire size or wire diameter is then set by knowing the wire type and its associated K value as well as the winding area available on the core, W_A as shown below. Given the required number of turns, n and W_A, wire size is set.

$$A_W \leq \frac{K_u W_A}{n} \quad (cm^2)$$

Select wire with bare copper area A_w less than or equal to this value.

As a check, the winding resistance can be computed:

$$R = \frac{\rho n \, (MLT)}{A_w} \quad (\Omega)$$

3.   Summary of Inductor Parameters

There are seven unknowns(six independent ones) in an inductor specification split between the core and the windings

a. Core Geometry (4 variables)

A_c: cross-section of flux path of chosen core

W_A: wiring window area of chosen core

MLT: Core choice sets <u>mean length</u> of the copper turns

l_g: size of core air gap you specify

b. Wire Geometry: (2 independent variables)

n: number of wire turns required to hit the desired L value

A_{cu}: area of chosen Cu wire #AWG to meet low loss current flow

K_{Cu} the copper fill factor for the wire type chosen which is derived from W_A and the wire area and is not really an independent variable

4. Inductor Design Specs flow of requirements

From the circuit conditions we usually know the maximum expected current in the wires of the inductor. We also know B_{SAT} of the core

$I_{max} \rightarrow H_{max} \rightarrow B_{max} \rightarrow$ we can never exceed $B_{saturation}$

   NI   core

   and   spec

   air gap

That is the circuit will cause a maximum current to flow in the inductor. This current may act to saturate the core unless we design for it. The mmf relation ni=Hx l(core flux path) will be the first step on the path to determining the B inside an ungapped core. It shows that for fixed n and i we can choose core size via l(core flux path) to minimize H and therefore keep B below B $_{SAT}$. The core permeability will allow us to go from H to B for comparison to B_{SAT}. For gapped cores the calculation of H in the core is more complex. We employ $\Re$(total) = $\Re$(core) + $\Re$(air gap) and start from ni= $\Re$(total) ϕ and proceed in a similar fashion to determine B_{MAX} in the core as compared to B_{SAT}. In summary the flow of filter inductor requirements goes as:

♣L(desired) from PWM converter circuit spec's

♣K(cu) from choice of wire # and W_A of the core

♣R_L from choice of N, MLT, and Cu wire #

This assumes we do not violate B_{SAT}, if we do an iterative proceedure discussed below is required to "balance conflicting copper and core requirements". That is we can trade off core and copper.

5. Core Constant K_g

We need to specify how the various core and copper quantities are to be considered. We start with a list of parameters and units employed.

Wire resistivity	ρ	(Ω-cm)
Peak winding current	I_{max}	(A)
Inductance	L	(H)
Winding resistance	R	(Ω)
Winding fill factor	K_u	
Core maximum flux density	B_{max}	(T)

The core dimensions are expressed in cm:

Core cross-sectional area	A_c	(cm^2)
Core window area	W_A	(cm^2)
Mean length per turn	MLT	(cm)

The use of centimeters rather than meters requires that appropriate factors be added to the design equations.

Core manufactures give a complete set of K_g values for the various geometries of cores in a core family

$$10^{-7} \leq K_g \leq 10 \qquad 10^8 \text{ range for cores}$$

Little Core   Bigger core
Costs minimum   Costs more for materials

$$K_g \;=\; \frac{A_c^2 W_A}{(MLT)} \;\geq\; \frac{L^2 I_{max}^2\, r}{B_{max}^2 R_L K_{cu}} \text{x } 10^8 \text{ in units of cm}^5$$

core specs   **Design specs for inductor**
you need   **to meet specs for PWM converter circuit.**

Allow the coil $R_L \uparrow K_g \downarrow$ smaller core lower cost
$(LI_{max})^2 \downarrow K_g \downarrow$ smaller core means lower cost
You pay severely for I_{max} spec as $K_g \sim I^2_{max}$. You also pay for energy storage capability. For low I_{ac} circuit conditions you save 10 * on K_g by choosing permalloy or Fe:Si as a core material rather than ferrite, if the frequency of the current is low.

$$\frac{K_g \text{ for Fe:Si cores}}{K_g \text{ for Ferrite cores}} \;\sim\; \left(\frac{1/2\,T}{1.5\,T}\right)^2 \approx 1/10$$

6.   Trading Cu for Core

We can trade off copper wire for ferrite core for a given L value in order to meet conflicting core and copper needs.

$$L^2 \ \sim \ \frac{A_c^2 W_A}{MLT} \ \text{ If } W_A\uparrow \Rightarrow \text{ More Cu windings are possible. If}$$

$A_c \uparrow \Rightarrow$ More Iron or ferrite is required

The point to remember is that for fixed L we can swap Cu wire vs. Iron core to achieve the inductor we seek in a given circuit application. $L = N^2$(wire)A(core)/μ(core)/(core)

. For either transformers or inductors the maximum volt-second ratings go as follows. The inductor voltage waveform integrated over the switch cycle in a PWM converter is $\int v_L$(t)dt. Remarkably the choosen core size (A_c) and number of wire turns required all converge on the B_{sat} specification of the chosen core as follows:

$$\int \frac{v_L dt}{NA_c} \leq B_{sat} \text{ The volt-sec rating for an inductor must not exceed, in}$$

the case of pulsed signals $V_{dc} \Delta t$(pulse duration) $< B_{sat}NA_c$ which is the maximum volt-sec rating. To achieve this condition we can trade N(copper) versus A_c(ferrite size).

We must also consider the peak volt-sec. for an inductor driven by a sinusoidal signal with a peak voltage which is: Vpeak $< B_{sat}wA_{core}N$ for the volt-sec rating to sinusoidal voltages. Again we can trade off the applied frequency ω, core size, A_c, and the number of turns, N , to meet the B_{sat} spec's. If we exceed v-sec spec's L $\rightarrow$ 0 prompting undesired electrical shorts in the circuit and damaging components. We will see later that core saturation also causes a transition from continuous mode operation to discontinuous mode of operation.

B.   Analysis and Design of Inductors

1.   Overview

Analysis of the filter inductor specifications we achieve by various choices of core and wire geometry is considered first. It highlights the **major aspects of inductor analysis where we are given**:

·   All core dimensions (A_c, l_c) and core air gap dimensions (l_g, A_g)

·   All wiring parameters such as AWG# and total length of wire

.   Expected I_L(waveforms) and values such as I(peak), I_{DC}, I_{rms} etc.

In inductor analysis we find:

· electrical properties L, R_L and risks of core saturation as well as estimated core losses

· thermal conditions of the core due to both wire and core losses as well as the effective thermal conductivity of the core to the ambient

In inductor design by contrast, we are given from the PWM converter circuit spec's the electrical and thermal specs of the required inductor.

We must design/choose all $\begin{bmatrix} \text{core specs} \\ \text{wiring specs} \end{bmatrix}$ if we wish the inductor to operate comfortably under those circuit and environmental conditions. Let's get specific with the values involved via several examples.

A ferrite core, shown on the top of page 13 has an Ampere's Law loop length l_{equiv} = 10 cm and cross-sectional area of 2 cm^2. Its saturation flux density is 0.3T, and the core permeability is 1250μ_o. **What inductive energy can be stored in this core? With ten turns of wire, what is the maximum allowed dc current? How many turns would be needed if this core is to be used instead for a transformer with 120 V, 60 Hz input?**

The ungapped core as above has a magnetic reluctance:

$\Re = l/\mu A$ = 0.10 m/[(1250*4π*10^{-7} H/m)(2*10^{-4} m^2)] =3.18*105 H^{-1}

To avoid saturation, keep B < 0.3T. This is equivalent to Ni/$\Re$ < B_{sat}A for the inductor. Thus Ni < (0.3T)(2*10^{-4} m^2)(3.18*10^5 H^{-1}),and we find that Ni < 19.1 turns to achieve our B_{SAT} goal. . If ten turns

of wire are used, the inductor coil should not carry more than 1.91 A if saturation of the core is to be avoided. That is B_{SAT} limits the maximum number of copper turns allowed. The maximum stored inductive energy is proportional to the square of the amp-turn limit divided by twice the reluctance. For this ungapped core, the energy cannot exceed 0.573 mJ

Depth: 2 cm

$\mu = 1250\mu_D$

$B_{sat} = 0.3$ T

2.2 cm

f1 cm
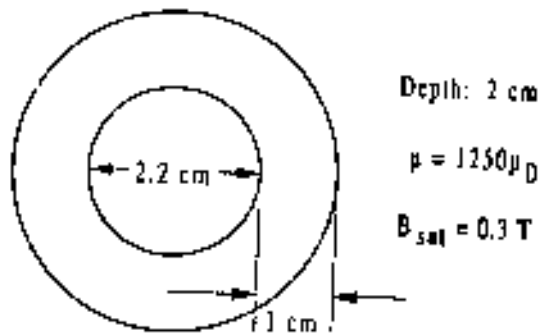
For employing this same core for a low frequency mains transformer, the net dc current is low, but the voltage imposed on the primary induces flux in the core. To avoid saturation, the flux linkage λ must be kept below a limiting value.

Given a sinusoidal 120 Vrms input at 60 Hz = $2\pi f$ =

$$\int 170\cos(120\pi t)dt = \lambda = NBA < NB_{sat}A$$

$$170 \text{ V}/(120\pi NA) < 0.3 \text{ T, or } \mathbf{N>7516}$$

This huge number of turns is unwieldy for a core of this size. What if the frequency was raised to 60 kHz? Very tiny wire would be required for mains frequencies, and the wire resistance would be high. Notice in the above example that if N = 7516 turns, the coil will be able to handle no more that 2.54 mA of DC current because of the limit on amp-turns. That is ni< B_{SAT}/μ , the amp-turn limit. This suggests that the above core will be extremely sensitive to any unwanted dc component.

Using the above-described core from the top of page 13, we now gap it to achieve a goal of more precise L values given large variations in core permeability. This will also increase energy storage capability.

A core for an inductor application has a loop length of 10 cm and cross-sectional area of 2 cm^2. The permeability is $1250\mu_o \pm 50\%$, given $B_{sat} = 0.3$T. It is proposed to alter the core by adding a 1 mm air gap, as shown below. Clearly, a gapped core will have a more reliable range of effective permeability given the wide range in the material properties. For the gapped core, find the maximum inductive <u>energy</u> storage. Define an *effective permeability* from the total reluctance, such that $\mu_{eff} = \mathfrak{R}_{tot}A/l$. What is the variation of this μ_{eff}? If an inductor is formed by wrapping ten turns around the core, what are the expected values and tolerances of *L?*

From the prior torroidal core geometry of page 11, the ungapped core has a reluctance of $3.18*10^5$ H^{-1} and a practical limit $Ni < 19.1$ A - turns. The energy storage capability was a fraction one millijoule. With the air gap added, the total reluctance is now changed to:

$$\mathfrak{R}\text{total} = \mathfrak{R}\text{core} + \mathfrak{R}\text{gap} = 4.29*10^6 \text{ H}^{-1}$$

The extra reluctance of the air gap now gives an amp-turn limit of 258 A – turns and the maximum energy storage becomes 7.73 mJ. The increase in energy storage capability is proportional to the reluctance increase, and is more than an order of magnitude. The gapped core is shown below as compared to the ungapped core of page 13.

We will show that the airgap allows core μ to vary by 50% while the effective permeability with the air gap varies only by 5%. This will make the variation in the target inductance value to be much less. This reduced sensitivity to μ variations is a very desired property in the design of inductors.

Depth: 2 cm
$\mu = 1250\mu_0 \pm 50\%$

2.2 cm

1 mm

$B_{sat} = 0.3$ T

1 cm
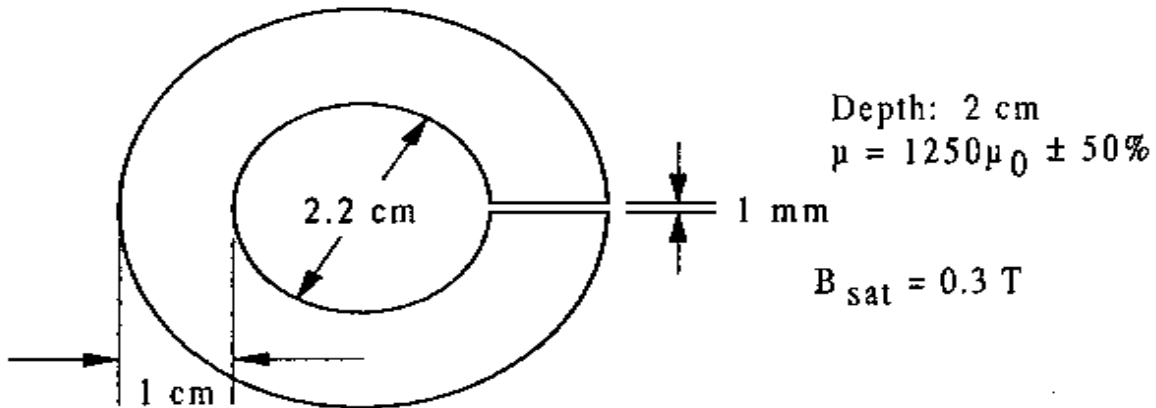
Thus the total permeability is now $92.7\mu_o$ with the air gap, and the core is functionally equivalent to a core material having, a relative permeability of about 93. The variation of the inductance occurs because the permeability of the ferrite could be as low as $625\mu_o$ or as high as $1875\mu_o$. With the airgap, the total reluctance will be somewhere between $4.19*10^6$ H^{-1} and $4.61*10^6$ H^{-1}. These values correspond to effective permeabilities of $95.0\mu_o$ and $86.3\mu_o$, respectively , which is a much reduced variation from the core variations, less than 10%-a factor of ten lower variation than without the air gap. In practice, the value might be given as $90 \pm$ 5%. The value $90\mu_o$ gives $\Re_{total} = 4.4 *10^6$ H^{-1}. A ten-turn coil therefore produces $L = 23$ $\mu H \pm 5\%$. This is usually considered to be an excellent tolerance level for an inductor.

2.   Analysis of Inductor Operation Given Core and Wire Specifications

a. Electrical and Magnetic Conditions

As a second example, we are aiming for an inductor that can handle $4A_{rms}$ at 100 kHz. We specify all wire and core materials as shown below.


4A @ 100 kHz
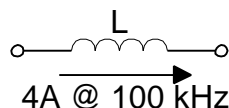
To achieve our goal we will
Use Litz wire:
$N = 66$ turns

A_c of Litz = .64 mm^2

The core has an air gap for linearizing "L" over a wider current range. Specicically the air gap length L_g = 3 mm, W_A(wire winding window)= 144 mm^2, V_c(core) = 13.5 cm^3, Ac(core) = 1.5 * 10^{-4} m^2, B(core) < B$_{max}$ = 180 mT which is specified by the chosen core material.

Now $\phi_{max} = B_{max}A_c$; ϕ_{core}(max) = 180 mT, ϕ_{max} = = 2.6*10^5 Wb for A_c = 1.5*10^{-4} of this particular core.
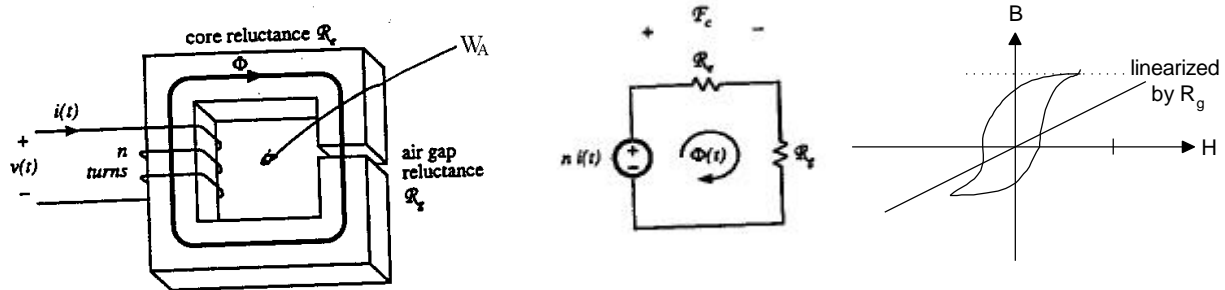
$L \equiv \dfrac{Nf}{I}$; Where I is peak sinusoidal current. NI = H * Σ l (air gaps); assume $H_{core} \to 0$ for $\mu \to \infty$. The peak field intensity is:

$H = \dfrac{NI_p}{L_g}$; with N = 66, $I_p = \sqrt{2}$ 4, and L_g = .003

H(peak) = 1.25 * 10^5 A/m. This corresponds in the air gap region with μ_o to B_{gap}(peak) = 4*π*10^{-7} H(peak) = 160 mT. B(core) > B(gap) = 180 mT as shown below due to fringing.

Lets say that the NI(mmf) pushes 26 µWebers through the total reluctance: $\mathfrak{R}_T = \mathfrak{R}$(core) + $\mathfrak{R}$(air gap) using Ni = $\mathfrak{R}_T\phi$. However for fixed ϕ we find $f_{gap} = f_{core}$ but due to the fringing we fully expect $B_{gap} \ll B_{core}$

Fringing flux
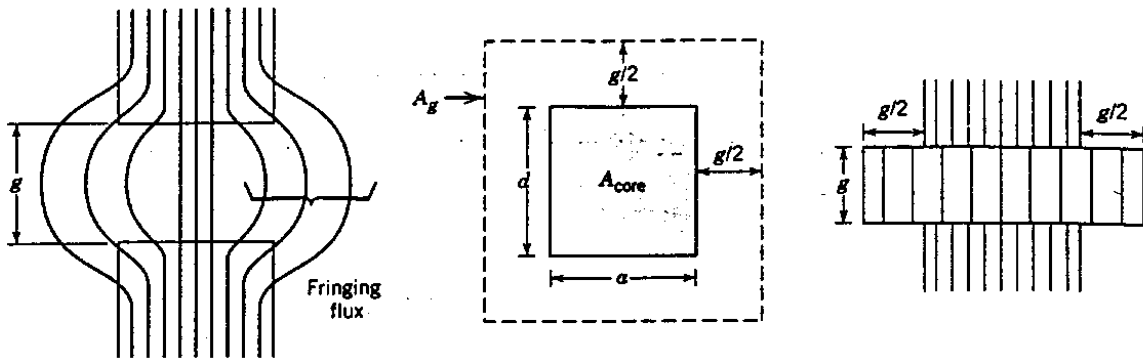
B(gap) < B(core) for same flux ϕ.

$$B_{core} = \frac{A_g}{A_c} B_g; \quad \frac{A_g}{A_c} = \frac{1.7}{1.5}$$

Typically $\frac{A_g}{A_c} = 1.13$

Given that B(air gap) = 160 mT

$$B_{core}(max) = \frac{1.7}{1.5} * 160 \approx 180\,mT$$

$$L \equiv \frac{N f}{i} = 310 \text{ mHenrys}; \quad N = 66, \phi = 2.6 * 10^{-5}, i(peak) = \sqrt{2}\ 4$$

A typical E type magnetic core with air gap is shown below. For an inductor the air gap sets the desired stored energy value. There will be a biggest air gap allowable, which does not introduce excessive flux leakage. Note that the center leg of the core is around which the inductor wire is wound. Its center leg flux splits equally between the two outer flux return legs whose area need only be half as big if core material cost are an issue. Winding the wire on the center leg, where the air gap lies, will expose the wire to leakage flux.
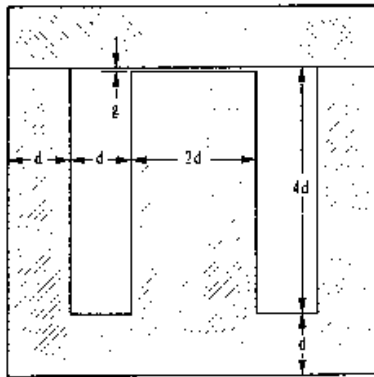
DEPTH d depth d of the magnetic core will set convective cooling limits as we will show in later parts of this section.

Let's consider another numerical example. An E-E core with air gap is to be chosen for inductor for a dc-dc converter. The air gap is created by grinding down the center post of width 2d to a shorter length than the two side legs. The air gap g will be kept less than about d/10 to minimize the fringe flux. **The inductor is to carry 5 A on average, and will be used in a 10 V to 5 V buck converter running at 100 kH**z. Current ripple should not exceed 1% peak-to-peak. Given the high frequency, a ferrite core with $B_{sat} = 0.3T$ is chosen. Furthermore, a value of $\mu=2000\mu_o$ might be typical in this frequency range.

First, let us determine the necessary energy storage and air-gap volume. When the inductor carries current, the inductor voltage is +5 V, and its current should change by no more than 50 mA. The duty ratio of the switch mode converter is 5/10, and the on time is 5 μs. Thus 5 V = Ldi/dt, 5 V = L(0.05A)/5μs, L = 500 μH.

The inductor should store $1/2Li^2 = 6.25$ mJ. If all this energy is stored in the air gap, suggests that the gap volume, at a minimum, should be given by

6.25 mJ = $(0.3T)2Vgap/2\mu_o$, Vgap = $1.75*10^{-7}$ m^3

For the maximum gap of d/10 the gap volume is 0.2d3. To provide the required volume we need to insure that, d > 9.56 mm. Let us choose d = 10 mm, and examine the results. With d = 10 mm and g = 1 mm, the reluctance can be determined with the simple magnetic circuit. The

length of a large loop around both outer legs will be 20d, the length through the center post is 4.9d, the outside legs have magnetic area of d^2, and the core and gap have area $2d^2$. The leg reluctances will be determined by a loop half the total length, so $l = 10d$ and $\Re_{leg} = 10d/(\mu d^2) = 10/(2000\mu_o 2d^2) = 9.75*10^4$ H^{-1}, while the center post reluctance will be $\Re_{post} = 4.9d/(2000\mu_o 2d^2) = 9.75*10^4$ H^{-1}.

The air gap reluctance of $3.98*10^6$ H^{-1} is a factor of ten higher than the others in the core legs and the total reluctance is the parallel combination of the outer legs in series with the gap and center post reluctance, so $\Re total = 4.28*10^6$ H^{-1}. To meet the requirement that $L > 500$ μH the number of wire turns we must provide is set by $N^2/\Re_{total} > 500$ μH. We cannot forget however, to avoid saturation, the ampere-turns must not exceed the limit so that $Ni < 0.3(4.28*10^6)2d^2$. The conflicting inductance requirement means $N > 43$ turns. In the extreme, if saturation is to be avoided at 5 A of current, the number of turns should not exceed 51. Thus it appears that **43 turns wire on this core will avoid saturation and provide adequate energy storage with a 1 mm gap.**

What about wire size? Given the 5 A current value, copper wire on the order of #16 AWG or #14 AWG will be needed. The window area $4d^2$ is 400 mm^2. With #14 wire, 43 turns will require 90 mm^2. A fill factor of (90 mm^2)/(400 mm^2) = 0.225 is therefore involved. Notice that if d is made much smaller, there might not be enough room for the required number of wire turns of the given diameter. If in the linear B-H range of the core $B \leq 180$ mT; $\phi \sim i$ $\Rightarrow L \neq f(i)$.

b. Heat Flow and Core Surface Temperature

Next we calculate losses from Cu wires carrying J and core loss with B_{peak} levels. Both wire winding loss and core loss will be calculated and the total power loss will be inputted into heat flow calculations to estimate the core surface temperature under operating conditions.

Copper Fill Factor for Wire Winding Losses-

We wind the Cu wire for the inductor in N turns around the above chosen magnetic core using the chosen wire.

$$K_{cu} \equiv \frac{N\ A_{cu\ wire\ \#}}{A(window)} = 0.3 \text{ for Litz wire}$$

Notice the extremes possible for K(Cu) which is unitless. $K_{cu} = 0.95$ varnished tape-like wire or 0.02 for HV wire with thick insulation. In the present L design we chose the number of wire turns as:
N = 66, $A_{cuwire\#}$ = 0.64mm^2, therefore A(wire winding window) = 140 mm^2 for the chosen core

For calculating wire winding Loss we use the expression:
$P_{total} = P_v$(wire/volume)*wire volume. And $P_v = \rho J^2$. For Cu with $\rho = 2.2*10-5\ \Omega$-m we get $P_v = 22J_{rms}^2$ if J_{rms} is in A/mm^2 units. The effective volume of Cu wire if we fill the entire open window $V_{Cu} = K_{Cu}*V_w$(core window volume used). For our case $V_{Cu} = 12.3$ mm^3.

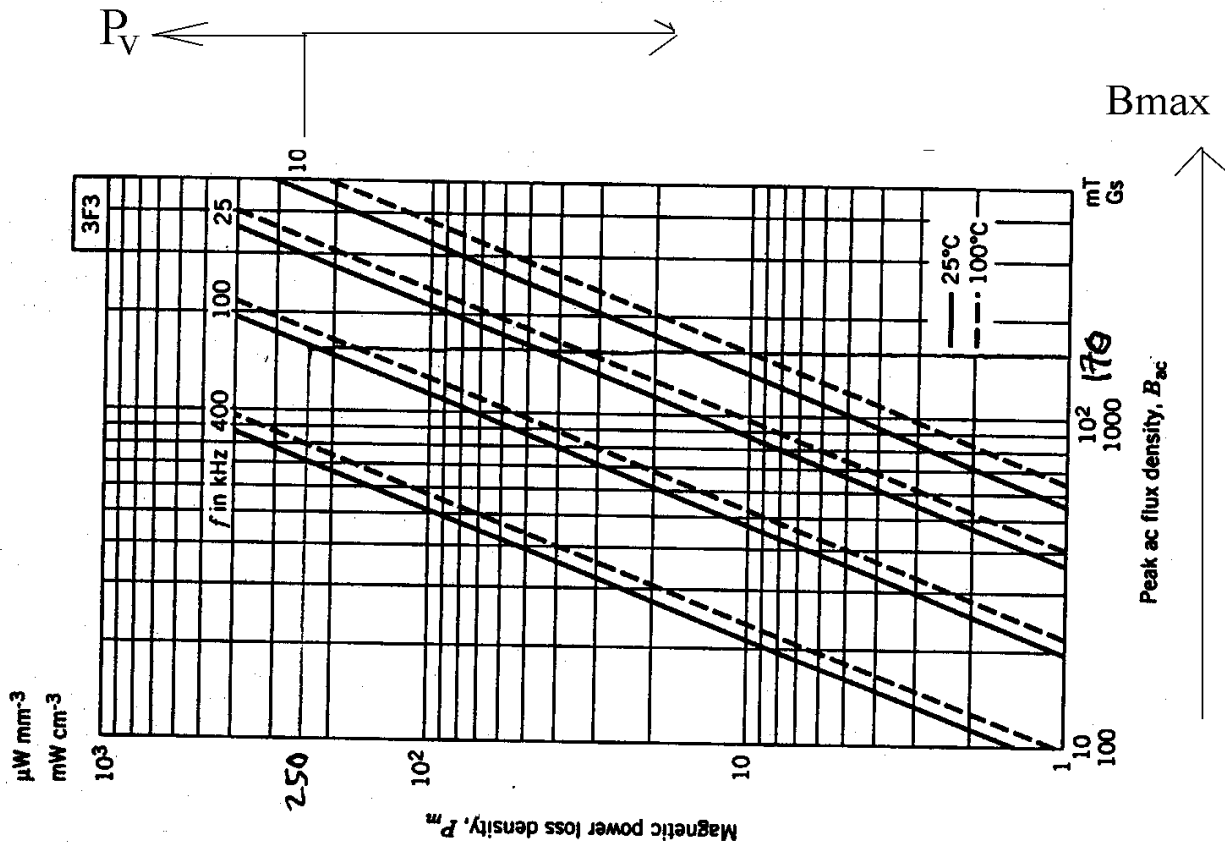$$P = 22\ K_{cu} \quad J_{rms}^2 \qquad V(wire) = 3.2 \text{ W}$$
$$\downarrow \qquad \downarrow \qquad \qquad \downarrow$$
$$.3 \quad (\frac{4\,A}{0.64\,mm^2})^2 \qquad 12.3 \ \ mm^3$$

This is only half of the total power loss in the inductor that has to flow to the ambient air via convection and radiation. Next we estimate core losses which depends on B(peak) not B(rms) and the chosen core material as well as the chosen operating frequency.

Magnetic Core Loss: $P_v = K_{fe}\ B_{max}^b$

We saw B(max) = 180 mT so that for the chosen 3F3 core material we use the graph to find quickly P(loss core) from B(max) as shown below on the nomograph for 3F3 material.

P_V

Bmax

3F3

10

25

100

400

f in kHz

25°C
100°C

mT
Gs

10^2
1000

Peak ac flux density, B_{ac}

μW mm⁻³
mW cm⁻³

10^3

250

10^2

10

1
10
100

Magnetic power loss density, P_m

$$P_v \begin{pmatrix} 3F3@ \\ 100\,KHz \end{pmatrix} = 250\ \frac{mW}{cm^3}$$

$$P_T = P_v * V(core) = \frac{250\,mW}{cm^3} * 13.5\ cm^3 = 3.3\ W$$

## Total Loss = P(winding) + P(Core) = 3.2 + 3.3 = 6.5 W

c. Heat Flow Out and Equilibrium Surface Temperature

Knowing the total heat flow, ambient temperature and core thermal properties we can estimate T(core) under operating conditions. Heat loss from the core surface has two major terms radiative and convective as given below. Together they form R_{th}. P(heat)R_{th} = T(core) -(Tambient)

We choose a Black Core Material with an emissivity for thermal radiation $P_{rad} = \sigma \varepsilon A \Delta T^4$. We will specify E = 0.9 and T(ambient) ≈ 40°C

Core Heat Flow: Passive Cooling. We will do this in 8 steps below.

1. Radiative Heat Flow
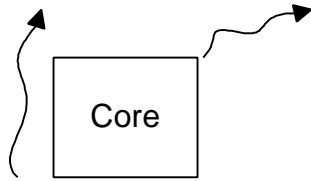
$$P_{rad} \equiv 5.7 * 10^{-8} \, E \, A[T_s^4 - T_o^4]$$

   2. Convective Heat Flow

$$P_c = \frac{1.3A[T_s - T_o]^{5/4}}{\text{core vertical height}} = \frac{L_3 A \Delta T^{5/4}}{d_a(core)}$$

$T_s \equiv 100$, $T_o \equiv 40 \Rightarrow \Delta T \equiv 60°C$. The value of d_a depends on the chosen core geometry.

   3. Both core surface heat loss mechanisms act in parallel so $R_{th} = R_{rad}|\,|R_{convective}$. We will use R_{th} to calculate T_s(core) from the heat flow equation:

$$[P_{heat\,flow}] \, R_{th} = \Delta T;$$

Now lets get values for R_r, R_c and R_{th}.

   4. $\underline{R_{radiative}}$

$$R_r = \frac{60}{5.1(.006)[(\frac{373}{100})^4 - (\frac{313}{100})^4]} = 20°\,C/\,W$$

   ↓

total surface area of inductor

   5. $\underline{R_{convective}}$

$$R_c = \frac{1}{1.3(.006)} \left[\frac{.035}{60}\right]^{1/4} \quad \begin{array}{l} \rightarrow \; d(vertical) \\ \rightarrow \; \Delta T = 60°C \end{array}$$

   ↓

total surface area

6. $R_c = 20^oC/W$

7. Now we can estimate how hot the core gets with 6.5 W of losses and a R_{th} of 10°C/W with an ambient temperature of 40°C.

$R_{total} \equiv R_c|\ |R_T \approx 10^oC/W$

$[T_{core} - 40] = R_{total} \qquad [P_{total}]$
$$\downarrow \qquad\qquad \downarrow$$
$$10^oC/W \quad 6.5W$$

$T_{core} \approx 105^oC$ Hey this is close to the practical limit

8. <u>Effect of Overcurrent on $T_s(core)$</u>

$I(4A) \rightarrow I(5A)$ for heavy loads
If the core is still in the linear range and we do not exceed B_{sat}
We will maintain the same inductance due the linear effect of the air gap.

$$L = \frac{N f}{i}\ L \neq f(i)\ \equiv\ constant$$

However we will see much bigger inductor power dissipation.

<u>Winding Loss</u>	<u>Core Loss</u>
$P_w \sim J^2$	$P_c \sim K_{fc}\ B^{\beta}_{max}$
Up by $(\frac{5}{4})^2 = 56\%$	B_{max} up $(\frac{5}{4})$
if $R \neq f(i)$	Follow core loss nomogram to find
The 4Arms wire power loss	$P_c \uparrow$ by 88%
was $\frac{P}{W}$ (4) $= 3.2$	The $4A_{rms}$ core loss $P_c(4)$ was 3.3

$P_T(15) = 1.56\ [3.2] + 1.8[3.3] = 11$ W
Assume 10°C/W unchanged for cooling the surface of the core.

$[T_{core} - 40] = R_{total}[11W]$; Assume R_{total} unchanged $= 10°C/W$

$T_{core} \approx 150°C$ (Much hotter than we would want for any inductor)

<u>Summary</u>

Pure air-core inductors have extremely low permeability's and will take up more room than inductors with cores. When a ferromagnetic core is used with an air gap, effective permeability's on the order of $100\mu_o$ are possible, and practical values of inductance can be obtained. Even though core materials exhibit wide variations in their μ values, the addition of an air gap allows inductor designs with tolerances of $\pm$ 5% or better cores with air gaps are usually required for inductor designs that meet design specifications such as L+/- ΔL. Large ΔL variations occur in ungapped cores as compared to gapped cores. The gap also serves as the primary energy storage element and extends the range of currents in the inductor coils before core saturation occurs..

Copper loss is an $I_{RMS}{}^2R$ value, representing the RMS current squared times the total wire resistance. If the wire volume is known instead, the loss is (ρJ^2) per wire volume, where J is the RMS current density. Core manufacturers often assist in the computation of copper loss by reporting an average wire length per turn for the specific core where wire is wound around. Without such a number, it can be difficult to estimate wire length. Given the average length per turn, resistance becomes straightforward:

R_{wire} = (resistance per meter) x (length per turn) x N

Wire size is an important aspect of the inductor design a given wire can handle only a limited current density to avoid excessive power loss. The window of a given core must hold enough copper to avoid excessive heating of the wire. The issue of wire loss influences key details in magnetic core geometry.

For an inductor design, saturation limits the amp-turn value. The current density limit in the wire also represents an amp-turn limit. A rule of thumb results from this equivalence:

An inductor core has a dominant Ni limit imposed by core saturation and a second amp-turn limit imposed by chosen wire size. The wire winding window area of the core is chosen so that these two limits are equal.

Consider what happens if the rule is violated. If the core window area is too small, the required wire could overheat before the saturation amp-turn limit is reached. If the window is too large then saturation will limit the core's capability, and the copper windings might be underutilized. Losses and saturation limits make magnetic design a significant challenge. The need for smaller, more efficient cores continues to drive the development of new materials and geometry's. We will end the lecture with a last example of an inductor made employing a pot core shown
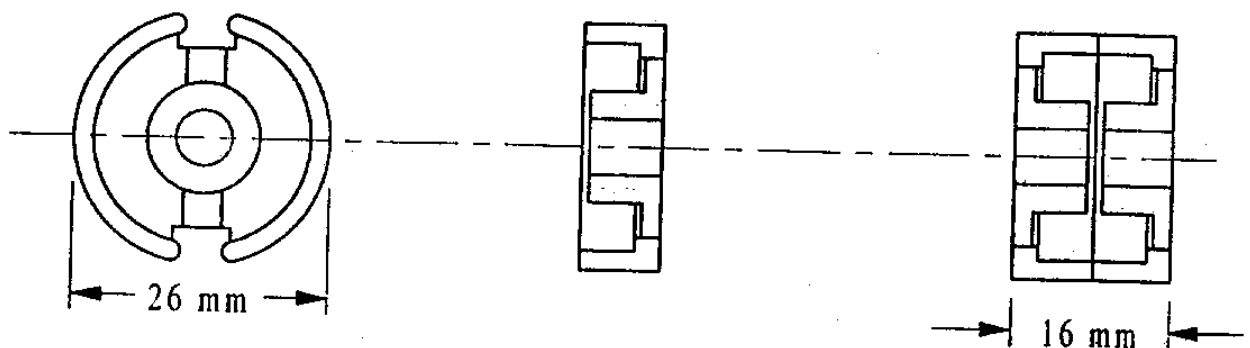
a) Top view, one   b) Cross-section, one   c) Assembled core
    core half   core half with air gap   halves, with air gap

Pot core, standard size 2616 (26 mm diameter, 16 mm thickness).

The specific Pot core for this final example has been prepared with an air gap of 0.23 mm. The manufacturer rates the core to have inductance

of 400 mH with 1000 turns. The ferrite material used to form the core has $A = 2000\mu_o$, and saturates somewhat above 0.3 T. Additional geometric and magnetic information is provided in the table on page 24. The core is wound with 20 turns of wire. Determine the inductance value and the dc current rating of the inductor. Find the equivalent permeability. What is the inductance tolerance if the ferrite permeability is $2000\mu_o + 100\%$ - 20%? Recommend a wire size for this inductor.

The SPECIFIC INDUCTANCE value, A_L, on this core will correspond to 400 mH/1000^2 = 400 nH per $turn^2$ as given in the table on page 24. Twenty turns would have 400 times this inductance, so $L = 160$ μH for this case.

Let us confirm this result. If the core has length 37.6 mm and area 94.8 mm^2, its reluctance should be $l_{core}/(2000\mu_o A_{core}) = 1.58*10^5$ H^{-1}. The gap reluctance should be $l_{gap}/(\mu_o A_{gap})$. With a gap length of 0.23 mm, this gives $2.39*10^6$ H^{-1}. The total reluctance is $2.55*10^6$ H^{-1}. The inverse of this values is specific inductance, $A_L = 392$ nH/$turn^2$.

Information for Pot Core

Characteristic	Value
Magnetic path length l_{core}	37.6 mm
Magnetic core area A_{core}	94.8 mm^2
Effective air gap area A_{gap}	76.5 mm^2
Core volume	3.53 cm^3
Specific inductance, A	400 nH/$turn^2$
Curie temperature, T_{curie}	200°C minimum
Core loss, 100 kHz	100 mW/cm^3 at RMS flux of 0.1 T
Core loss, mathematical fit	$3.2 \times 10^{-3}B^{2.6}f^{1.8}$ W/cm^3 for a given frequency and RMS flux
Window area	57.4 mm^3, or 40.6 mm^3 with bobbin in place
Mean length of wire turn	53 mm

This is only 2% different from the 400 nH/$turn^2$ value reported by the manufacturer. The equivalent permeability is the value that produces $\Re = 2.555*10^6$ H^{-1} when the core geometry values are used. Thus $l_{core}/(\mu_e A_{core}) = 2.55*10^6$ H^{-1}, and the value should be $\mu_e = 1.56*10^{-4}$ H/m. This is $124\mu_o$ (the manufacturer's value is $\mu_e = 125\mu_e$).

The inductance tolerance can be evaluated based on the core reluctance. If the core permeability can vary between $1600\mu_o$ and $4000\mu_o$, the core reluctance will fall between $7.89*10^4$ and $1.97*10^5$. The total reluctance will fall between $2.47*10^6$ and $2.59*10^6$ H^{-1}. The specific inductance falls between 386 and 405 nH/turn^2. With twenty turns, the inductance is between 154 μH and 162 μH. The designer will probably label the part as 160 μH $\pm$ 5%, especially since error in the air gap spacing has not been included in the analysis.
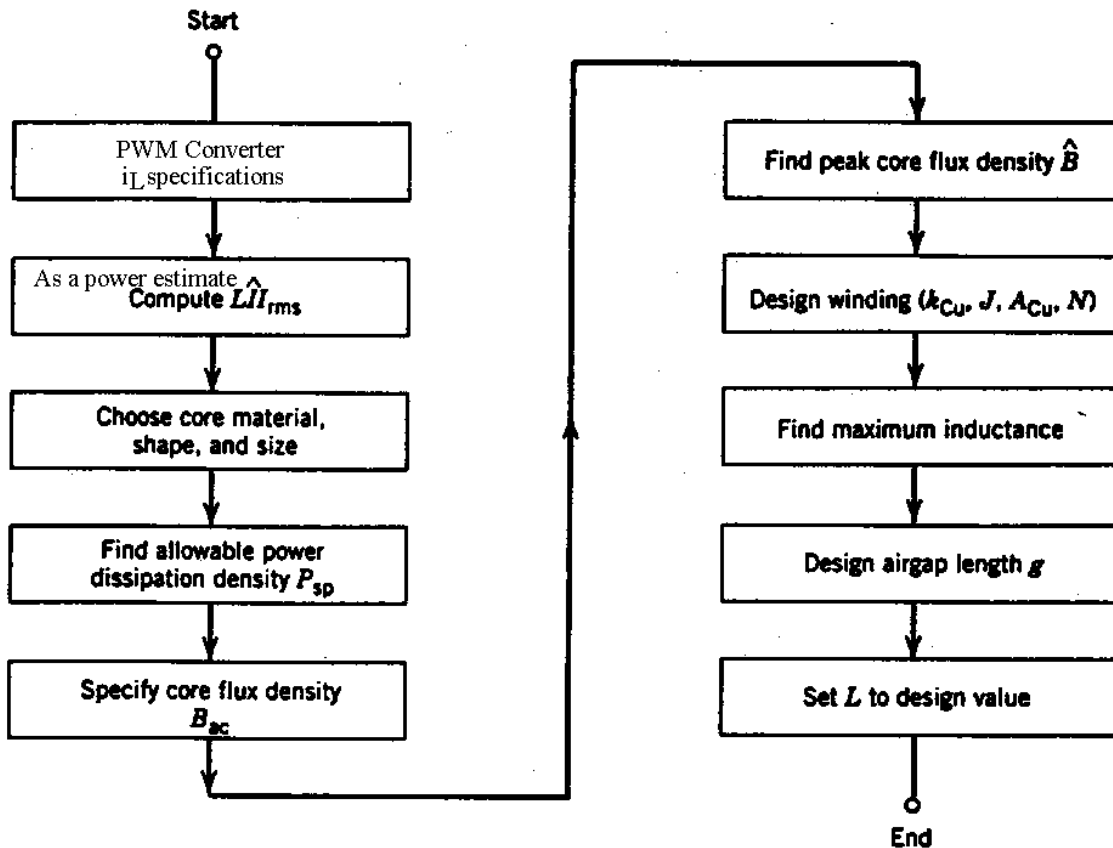
To avoid saturation, the flux density should be kept below 0.3 T. This requires Ni < (0.3 T)$A_{core}\mathcal{R}_{total}$, or

Ni < (0.3 T)(94.8*10^{-6} m^2)(2.55*10^6 H^{-1}); Ni < 72.5 A-turns.

With 20 turns, the dc current should not exceed 3.63 A. From Wire Tables, this current requires #16 AWG. There is little reason not to fill the window with as much wire as possible to keep losses low. A plastic bobbin will be used in most cases to make winding easy. This will leave 40.6 mm^2 for wire. A fill factor of 0.7 allows just over 28 mm^2 for copper (the fill factor is higher than 0.5 because the bobbin area was already accounted for). Twenty turns of #16 AWG wire requires 26 mm^2, and should fit. The maximum current density will be about 275 A/cm^2. Looking ahead to design we will follow an opposite flow path compared to analysis.

3.   Inductor Design Flow

In inductor design we specify l_g(gap) based on core and copper requirements that arise out of circuit requirements.

Start

PWM Converter i_L specifications

As a power estimate Compute $L\hat{I}I_{rms}$

Choose core material, shape, and size

Find allowable power dissipation density P_{sp}

Specify core flux density B_{ac}

Find peak core flux density $\hat{B}$

Design winding (k_{Cu}, J, A_{Cu}, N)

Find maximum inductance

Design airgap length g

Set L to design value

End

Flowchart of a single-pass inductor design procedure. The procedure assumes the existence of an extensive database of characteristics of all cores available to the inductor designer. The characteristics include the allowable power dissipation density for the design temperature range.

Next time in lecture 33 we will go through each step of inductor design.