LECTURE 30

Simple Heat Flow Modeling and Sample Temperature Calculations

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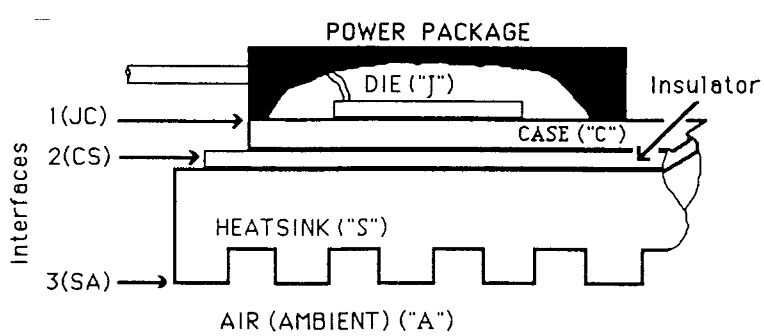
A. General Heat Flow Considerations
1. Steady State Loop Equations

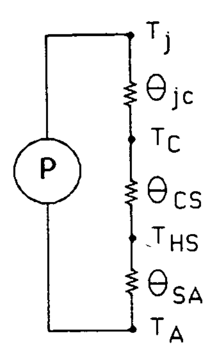
Heat flow in steady state is very easy to model as we saw in lecture 29**.** Steady state thermal analysis is similar to the solution of simple series resistor circuits in the electrical circuits. We make the following equivalencies between thermal and electrical quantities:

ELECTRICAL	**THERMAL**
1. Electrical Current	1. Heat Flow in Watts
2. Resistance	2. Thermal Resistance in $^{o}C/W$
3. Node Voltage	3. Temperature at a Spatial Location ^{o}C
4. Current Loop	4. Thermal Loop
5. Ground Potential	5. Ambient Air Temperature

A steady-state thermal model is shown below. The junction temperature of the solid-state device is given by the thermal loop relation depicted above the figure and the equivalent thermal circuit shown to the right of the figure.

$$T_j = T_A + (\mathbf{q_{SA}} + \mathbf{q_{CS}} + \mathbf{q_{jC}}) \mathbf{x P}$$





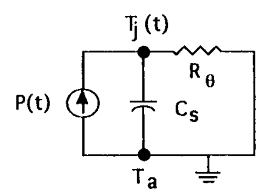
This model is only suitable for steady-state conditions.

1. Transient Heat Flow Models

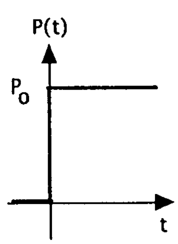
Transient heat flow calculations are much more complicated as we will see below. We need to introduce additional concepts besides thermal resistance. Moreover, the differential equations to model heat flow include both spatial and time variables. These will be diffusive heat flow equations, which are not analogous to simple electrical circuits. But we will do all we can to extend the analogy, since as EE's we know circuits best. Below we show the modified equivalent thermal circuit that includes a heat capacity, which is analogous to the electrical capacitor in that we cannot instantaneously achieve a change in temperature across the thermal capacitor when a sudden pulse of heat energy is applied.

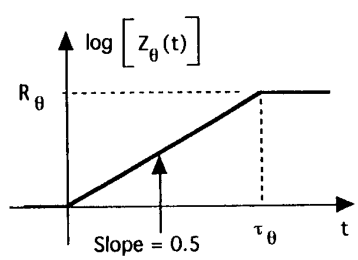
Transient Thermal Impedance

- Heat capacity per unit volume $Cv = dQ/dT$ [Joules /°C] prevents short duration high power dissipation surges from raising component temperature beyond operating limits.
- Transient thermal equivalent circuit. $C_s = C_v V$ where V is the volume of the component.



- Transient thermal impedance $Z_\theta(t) = [T_j(t) - T_a]/P(t)$

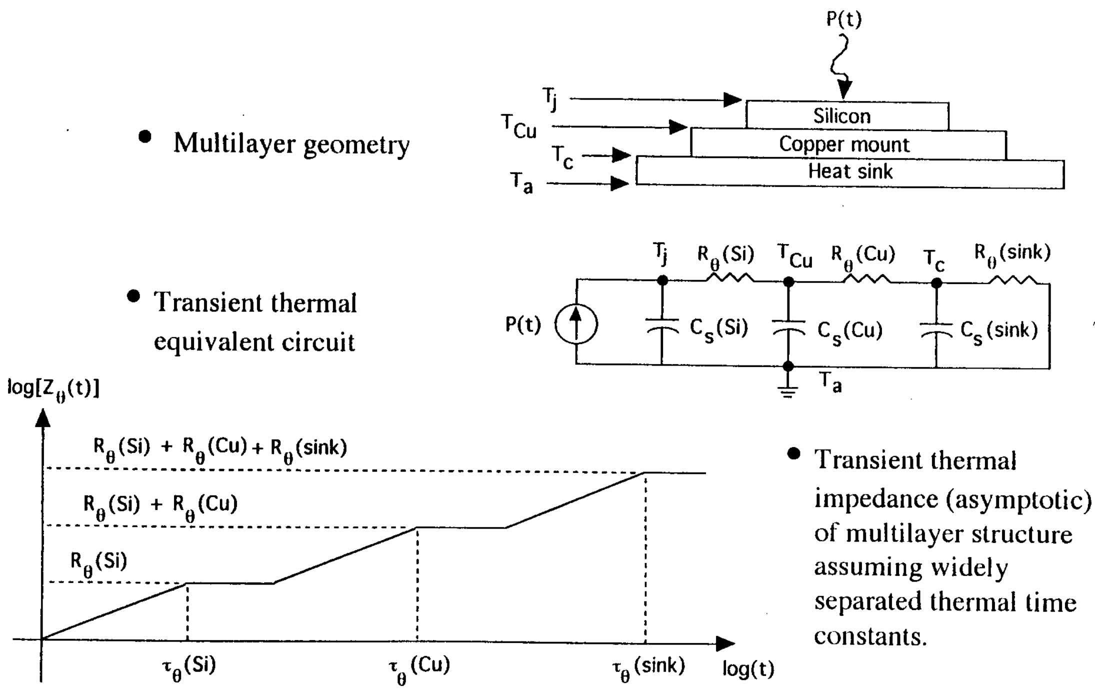




- $\tau_\theta = \pi R_\theta C_s / 4$ = thermal time constant
- $T_j(t = \tau_\theta) = 0.833 P_o R_\theta$

The transient thermal impedance is an artifical construct we make to preserve our desire to employ simple thermal circuits , even to transient heat flow problems. The heat capacity per unit volume of a material requires a finite amount of heat to be absorbed by a material before it's temperature rises. With a step-function of heat applied it results in a thermal time constant describing the temperature rise versus time. Otherwise, with simple thermal resistances only in our thermal loops, we would have instantaneous temperature rises with applied step functions of applied heat. This is not an accurate description of heat flow in real materials, so we introduce **thermal impedance** to replace the simple **thermal resistance** as given above. The thermal impedance is now considered to be a function of time. It starts at zero value of impedance and ends saturating at the steady-state values for long times, for a single layer of material as shown on page 3. For multi-layers we find by analogy with one layer:

Z_θ for Multilayer Structures



We consider transient thermal analysis beyond the goals of this lecture and will consider it no further.

3. Concept of Thermal Resistance in [^{o}C/W] Units

a. Overview of Thermal Circuits

We model conductive heat flow in a solid via a thermal resistance characteristic of the material. Then for a given steady-state heat flow through the material there will be across the material a thermal gradient. The solid could be magnetic cores, solid state switches or even copper wires.

$$R \equiv \frac{\Delta T}{\text{Heat Flow}} \left[\frac{^{\circ}C}{W}\right] \qquad \text{Where } \Delta T \text{ is across the solid}$$

Note that thermal resistance includes both materials properties and geometery: $R \equiv \dfrac{d}{A\, s_T}$ and σ_T = Thermal conductivity W-m^{-1} - ^{o}C

Some typical values of material thermal conductivity's are:

σ_T for <u>pure Al 220W - m^{-1} - ^{o}C^{-1}</u>

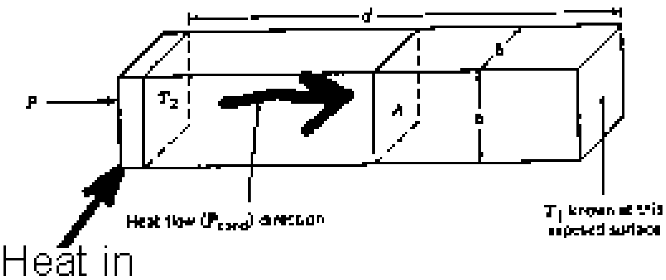
σ_T for ferrites is close to 0.1 W-m^{-1}-C$^{0-1}$

R and σ_T are different and thus have different units.

R depends on both the materials thermal properties, s_T, and chosen dimensions.

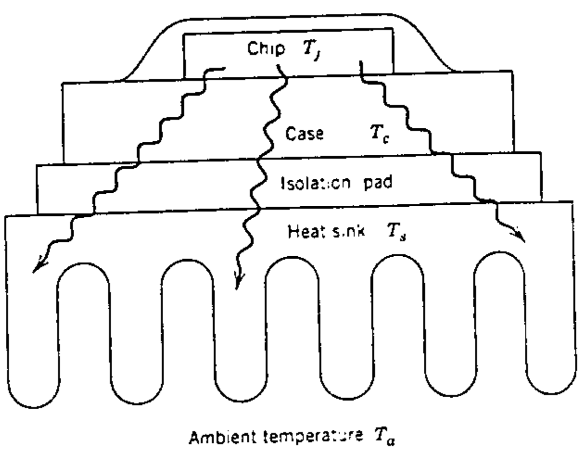
The thermal conductivity of a material and the and thermal resistance of a specific geometric shape of that material will be employed to get heat flow from the interior of the material to a remote surface. Once the heat arrives at the surface it needs to be transferred to the ambient air. Only then do we have a complete thermal circuit with a closed loop. The total series sum of all such thermal impedance's will be the effective thermal impedance to heat flow. Then we can calculate the temperature rise of any particular surface as compare to the ambient temperature ,T_A. We will find that the range of thermal resistance's for **simple air cooled situations** varies over the range:

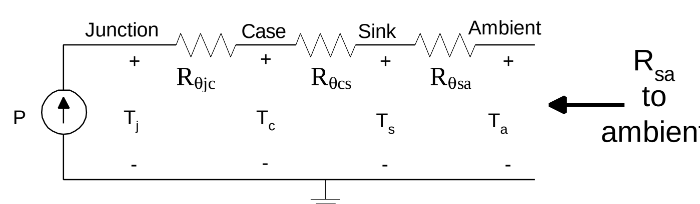
1 °C/Watt ≤ R ≤ 100 °C/Watt



$R_{min}\ \frac{1°C}{Watt}$
will allow 100Watts to flow and cause only a 100 degree temperature rise

We could have a series or tandem stack of different materials that the same heat flows through. An IC Chip on a lead frame is a good example of a tandem stack of material through which heat must flow, from the IC volume where it is created, to the external heat fins where it is transferred to the ambient air. An IC chip on a lead frame has several series R (thermal) occurring sequentially until you reach the surface. At the surface conductive heat transfer ends and radiative and convective heat flow from the surface limits total series heat flow in the air cooled situation. That is the surface to ambient thermal resistance R_{sa} is usually the largest and limits the total heat flow.



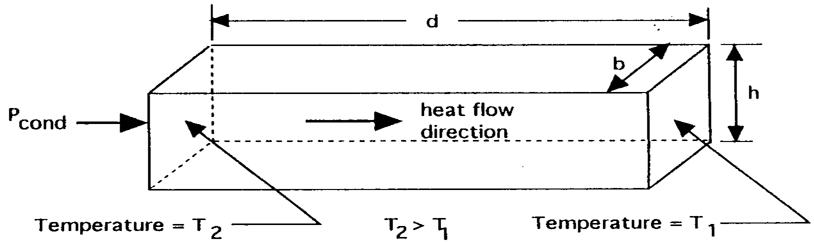


The total thermal resistance is the series stack or thermal loop:

$$\therefore\ R_T(\text{total series R to ambient}) = \underbrace{R(\text{conductive})}_{\text{case}} + \underbrace{R_{sa}}_{\text{surface}}$$

Heat Conduction Thermal Resistance

- Generic geometry of heat flow via conduction



- Heat flow P_{cond} [W/m²] = $\lambda A (T_2 - T_1) / d = (T_2 - T_1) / R_{\theta cond}$
- Thermal resistance $R_{\theta cond} = d / [\lambda A]$
  - Cross-sectional area $A = hb$
  - λ = Thermal conductivity has units of $W\text{-}m^{-1}\text{-}°C^{-1}$ ($\lambda_{Al} = 220\ W\text{-}m^{-1}\text{-}°C^{-1}$).
  - Units of thermal resistance are °C/W

In summary we repeat the simple case of thermal conduction in a solid above in order to nail the concept down.

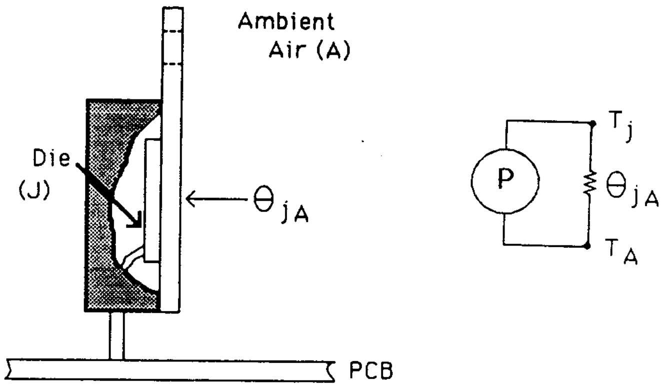
b. Reducing Device Temperature by Employing Heatsinks

Consider the case where we are dissipating1.75 Watts in a free standing diode, shown on page 8, with the only means of cooling being the ambient air cooling,R_{jA}, specified at 65[$^oC/W$]. Assume that the ambient air is at 50^o C without an additional heatsink we will find that the diode junction temperature, T_j, will rise due to heat generation to the following:

T_j =P(lost in the diode)x R_{jA}

T_j =1.75x 65 +50 =164 ^oC

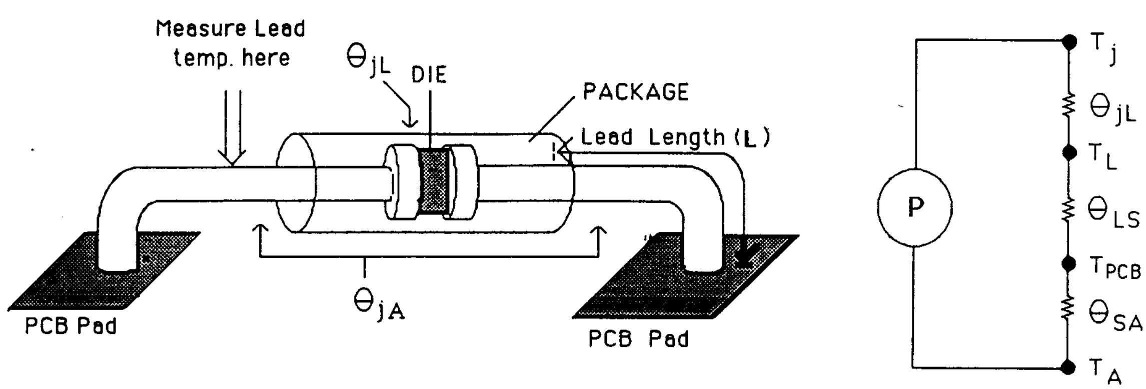
What if this temperature is too high for the diode?? What to do?



We could add a heatsink to the diode so that the thermal circuit has a lower overall impedance. This involves attaching the diode case to a heat sink. The heatsink we choose to employ has a thermal resistance of 14 ^{o}C/W as compared to the value of 65 ^{o}C/W of the non-heat sunk case. Now the device to case thermal resistance is 5 ^{o}C/W and the electrical insulator used to isolate the case from the device has 0.65 ^{o}C/W. So the maximum temperature is given by the series of thermal resistances through which heat flows through:

$$T_j = 1.75x(5+o.65+14)+50\,^{o}C(\text{ambient}) = 84\,^{o}C$$

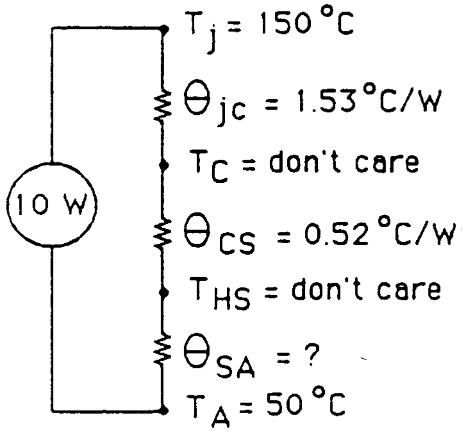
This is nearly half the value of the case with no heatsink. We



show the heatsink case above with the thermal circuit on the right.

c. Minimum Size Heatsink Required

Assume we have a situation where we are dissipating 10W from a device. The device will be attached to a yet to be determined heatsink via the device case using an electrically insulating buffer layer with thermal resistance 0.53 $^oC/W$. The device manufacturer specifies the thermal resistance from device to case as 1.56 $^oC/W$. We now ask what heatsink thermal resistance will keep the diode junction temperature below the 150 ^oC maximum allowed. The situation is summarized below in the thermal loop above. Solving for θ_{SA} we find:



$$\theta_{SA} = (T_j - T_A)/P \; - \theta_{JC} - \theta_{CS} = 5.5\ ^oC/W$$

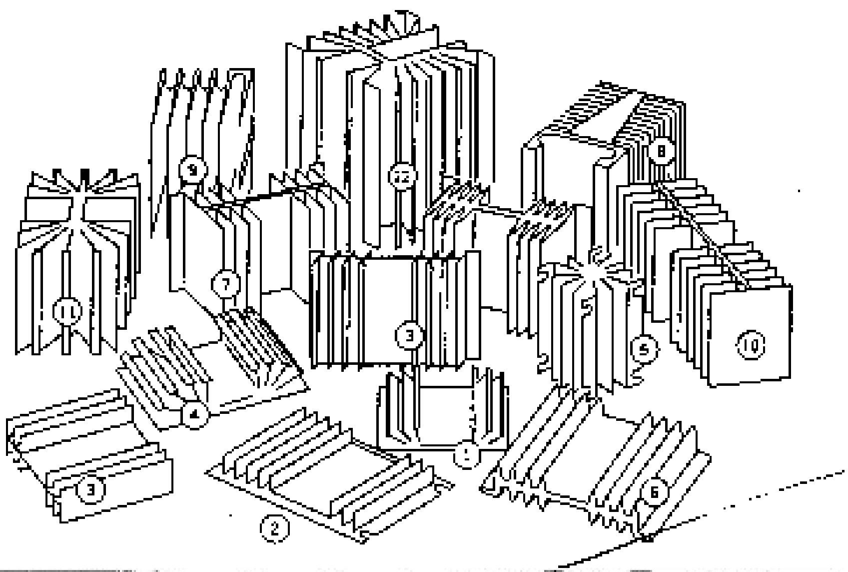
This is the required heatsink thermal impedance to the ambient.

Consider second case. Given a 26W power loss device (diode) @ 125^{o}C and the fact that the case has an equivalent thermal $R_{case} \approx 1.3\ ^oC/W$ for ambient conditions @ 55^{o}C. What is the required R_{sa}(heat fin) to keep the device below 125°C?

$$R_{sa}\,(\text{required}) \;=\; \frac{125-55}{26} \;-\; 1.3\ \frac{^\circ C}{W}$$

$R_{sa} = 1.4\ ^oC/W$

To achieve this R_{sa} value we choose a corresponding convective / radiative heat sink from manufacturers parts and their corresponding R_{sa} spec's. Clearly we seek the lowest cost
one.



Good choices for this application:
1.3 | 1.7
conservative | aggressive
more costly | cheaper

Heat sink no.	1	2	3	4	5	6	7	8	9	10
$R_{\theta sa}$ (°C/W)	3.2	2.3	2.2	0	2.1	1.7	1.3	1.3	1.25	1.2
Vol. (cm^3)	76	99	181	0	198	298	435	675	608	634

Some common power packages are shown below with their thermal resistances.

Thermal Resistances of Common Power Packages

Package	J-A min.	J-A max.	J-C min.	J-C max.
TO-3	—	30.0	0.7	1.56
TO-3P	—	30.0	0.67	1.00
TO-218	—	30.0	0.7	1.00
TO-218FP	—	30.0	2.0	3.20
TO-220	—	62.5	1.25	4.10
TO-220FP	—	62.5	2.78	4.40
TO-225	—	62.5	3.12	10.0
TO-247	—	30.0	0.67	1.00
DPACK	71.0	100.0	6.25	8.33
D^{2}PACK	50.0	62.5	1.00	2.00

The final heat flow solution combines the common power package with or without a heatsink attached to the power package to achieve the lowest possible thermal resistance required by the application.

d. Maximum Ambient Operating Temperature, T_A, for Operation Without a Heatsink

Consider the situation where we know that a device junction temperature T_j cannot exceed 150°C and the maximum ambient temperature is to be estimated for a case of a power loss of 1 W in that device. We estimate the thermal resistance without a heat sink at 65°C/W. Hence the maximum allowable ambient temperature will be given by:

$$T_A(max) = T_j(max) - P \times \theta_{jA} = 150°C - 1 \times 65°C = 85°C$$

We cannot tolerate an ambient temperature in the power supply environment above 85°C if T_j is to meet spec's

e. Estimating an Unknown R_{sa} from Known Convective / Radiative Conditions

We have seen that the surface to ambient thermal resistance is usually the largest and thereby limits heat flow and also sets maximum temperatures for a fixed heat flow. The surface to ambient thermal resistance is composed of two parts.

$$R_{sa} = R\left(\frac{\text{radiative}}{\text{cooling}}\right) + R_c\left(\frac{\text{convective}}{\text{cooling}}\right)$$

1. Radiative Thermal Resistance

We will outline below on page 12 the components to the radiative heat loss so that we are able to estimate values of radiative resistance from the known geometry and surface emissivity values. This is handy for situations where the thermal resistance is unknown. We will then further simplify the expression so it depends only on the temperature difference of the surface and

Radiative Thermal Resistance

- Stefan-Boltzmann law describes radiative heat transfer.
  - $P_{rad} = 5.7x10^{-8} EA [(T_s)^4 - (T_a)^4]$; $[P_{rad}] = [watts]$
  - E = emissivity; black anodized aluminum E = 0.9 ; polished aluminum E = 0.05
  - A = surface area $[m^2]$through which heat radiation emerges.
  - T_s = surface temperature [°K] of component. T_a = ambient temperature [°K].
- $(T_s - T_a)/Prad = R_{\theta,rad} = [T_s - T_a][5.7EA \{(T_s/100)^4 - (T_a/100)^4\}]^{-1}$
- Example - black anodized cube of aluminum 10 cm on a side. T_s = 120 °C and T_a = 20 °C
  - $R_{\theta,rad} = [393 - 293][(5.7)(0.9)(6x10-2)\{(393/100)^4 - (293/100)^4\}]^{-1}$
  - $R_{\theta,rad}$ = 2.2 °C/W

that of the ambient to the fourth power,ΔT^4. We will also need to know the full area ,A, of the heat sink as well as it's emissivity,E. Where the range of emissivities is:.05 $\leq$ E(rad emissivity) $\leq$ 1.0.

The radiative thermal resistance does not act alone as there is also convective cooling operating that acts in parallel. We will only crudely estimate this heat path below.

2. **Convective Thermal Resistance**

$$R_C = \frac{\Delta T}{P_{conv}}, \quad R_C = \frac{K_C}{A(m^2)}$$

K_C = 0.13 for 10cm vertical height for a black or blue oxidized Al plate. Consider an area of 10x30cm and T_S = 120 & T_A = 20. Then we find, R_c = 2.2 $^oC/W$

For fixed ΔT conditions of 100°C: $R_{rad} = \frac{K_c}{A(m^2)}$ then K_C =0.13

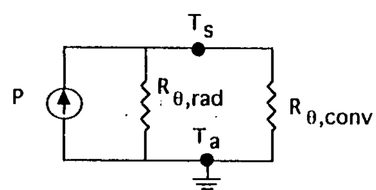
Next we take into account that in practice both radiative and convective heat flow paths are operating in parallel.

3. $R_{total} = R_{rad}\ in.parallel.with\ \frac{\Delta T}{P_{radiative}}$

Both radiative and convective thermal resistances act in parallel to achieve a total thermal resistance $R_{sa} \approx \frac{1^\circ C}{W}$. This situation of parallel radiative and convective heat flow paths is summarized below.

Combined Effects of Convection and Radiation

- Heat loss via convection and radiation occur in parallel.
  - Steady-state thermal equivalent circuit



  - $R_{\theta,sink} = R_{\theta,rad} R_{\theta,conv} / [R_{\theta,rad} + R_{\theta,conv}]$

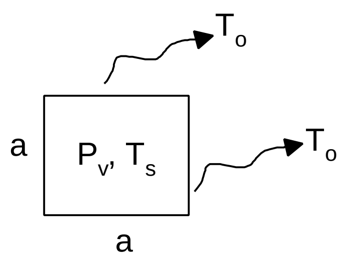
- Example - black anodized aluminum cube 10 cm per side
  - $R_{\theta,rad} = 2.2\ ^\circ C/W$ and $R_{\theta,conv} = 2.2\ ^\circ C/W$
  - $R_{\theta,sink} = (2.2)(2.2)/(2.2 + 2.2) = 1.1\ ^\circ C/W$

4. Heat Flow Issues/Trends Versus Characteristic Size of the Device

To develop a rule of thumb for minimum size volume that will still dissipate high power and remain below 100°C we do the following general solution for heat flow in and out of a cube of dimension a on a side.

a. Overview: Passive Cooling

Useful trends of heat loss and the characteristic dimension "a" can be derived.



$P_{sp} = \frac{P}{V}$: Heat generated per unit volume due to both wire and core losses.
T_s -Temp of heated object @ its surface in constant with the ambient.
a^2 - area of object

we consider that R_{sa} is only passive cooling. With <u>no</u> water cooling we have only two mechanisms as detailed above:

1. R(rad): Radiative heat transfer from a hot black body
2. R(convective): Convective air cooling to the ambient

In summary the temperature difference form the surface to the ambient is

$\Delta T = T_s - T_a \equiv R_{total}[P_v]$

Where, for example, P_v is the power of the heat flow into the cube from both windings and core in the case of magnetic devices.

$P_v = P_{sp}(core)*V_c + P_{sp}(wire)*V_w$

With T_s fixed, we estimate that $R_{total} = K_1/a^2$, and find $P_w = K_2a^2$. Assume V_c(core) and V_w(wire) are both ~ a^3. We find $P_{sp} = K_3/a$. Where K_2 depends on materials choice of the cube of volume a^3 and "a" is the characteristic dimension of the cube.

Case #1:

Now if we consider that the heat is generated solely by the copper winding loss per unit volume = $K_{cu}\,\rho\,(\Omega\text{-cm})J^2$

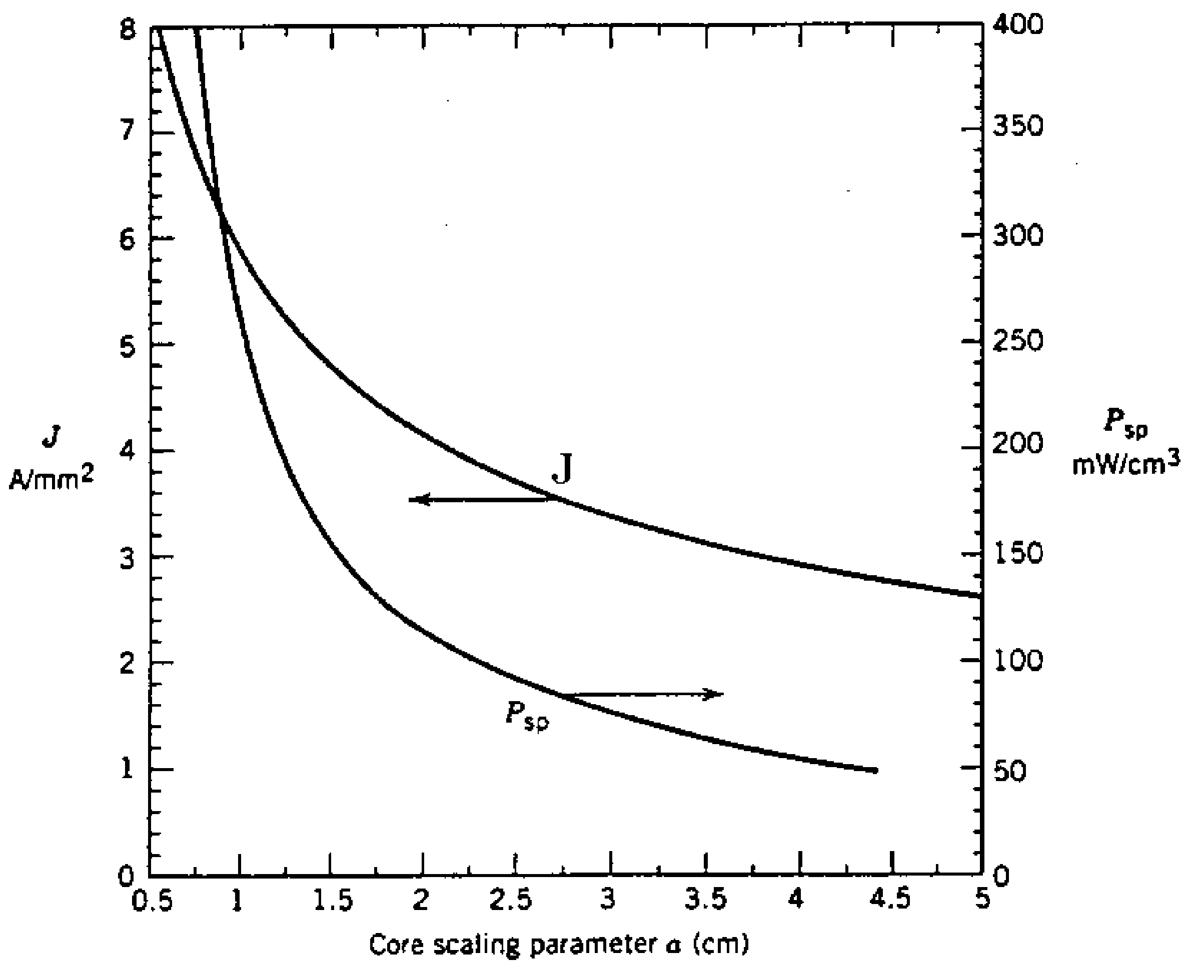
$$\frac{P}{V}(\text{loss}) = 22\,K_{cu}\,\frac{R_{ac}}{R_{dc}}\,J_{rms}^2$$

Solving for J_{RMS} @ P_{sp} (mW/cm^3) of the cube material then the maximum allowed current density goes as:

$$J_{rms} \approx \frac{K_3}{\sqrt{K_{cu}}\;a}$$

The rule of thumb is to use “a” as a characteristic dimension of scale.

As a ↑ the J_{max} value will <u>decrease</u> and if a ↓ then J_{max} will increase as shown below.



Case #2:

Heat is generated solely by the core loss.

If $P_{cu} \sim f^{1.3}B^{2.5}$, then B_{ac}(max allowed by thermal considerations alone) $\sim K_4/(f^{1/2}a^{0.4})$

5. Specific Example Where Thermal Spec's are Known and we Derive Values for Comparision

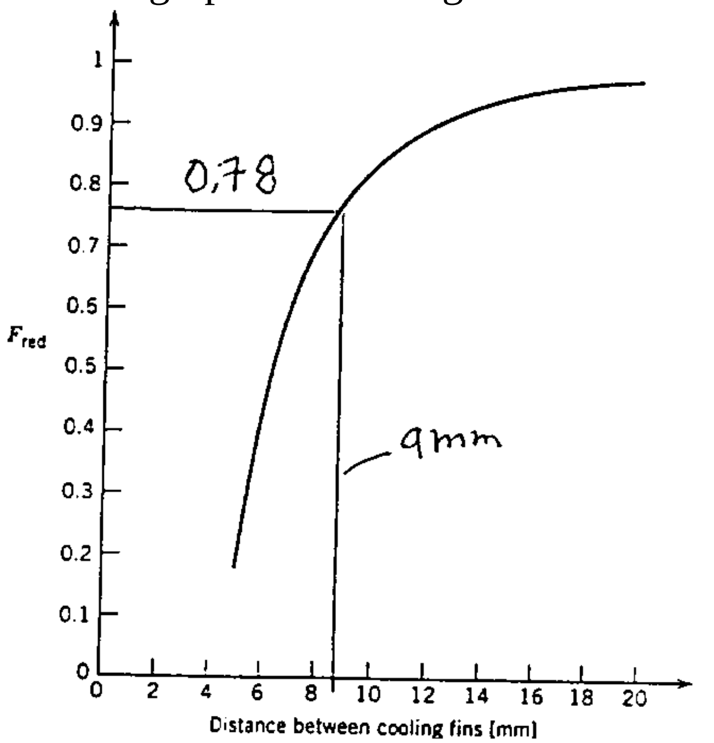
Let's cheat and use heat sink #7 from page 10 where we know manufactures spec, yet we try below to calculate R_{sa} solely from dimensions and compare to the original spec. to see if our calculations are realistic.

a. $$R_c \equiv \frac{1}{1.3\ A_c\ F_{red}} \left(\frac{d_{vertical}}{\Delta T}\right)$$

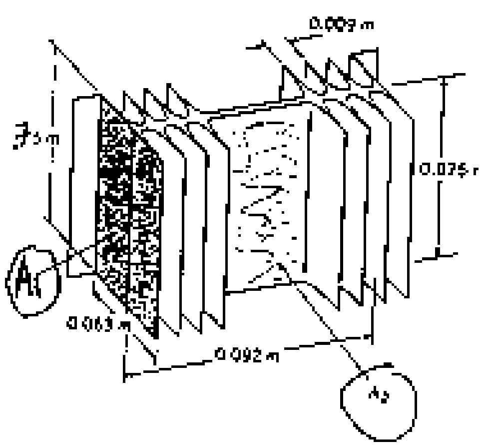
For d_{vert} about 10 cm and ΔT about 100°C we get R_C=0.13/A in °C/W units. Actual heatsinks have a series of fins, not smooth surfaces. F_{red} is a multiplicative reduction factor which depends on vertical fin spacing on the heat sink. It accounts for the reduction of natural convective cooling For heat sink #7 d = 9 mm the distance between cooling fins and this results in F_{red} below unity as shown on the next page. A_c is only for the vertical fins where convective cooling dominates. This involves an effective area that can be gleaned from heatsink manufacturers data sheets as shown on the next page.

b. F_{red} from manufacturing Data

We get F_{red} from the graph below using d = 9mm to find F_{red} = 0.78.



b. A_c from Geometry



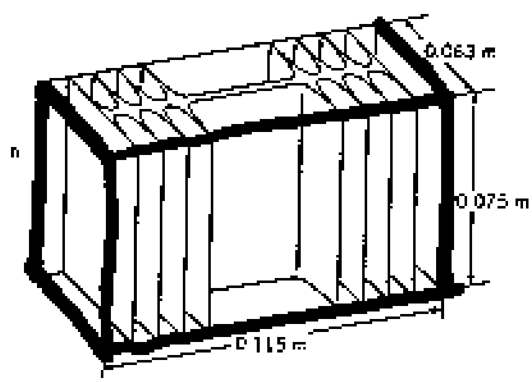
$A_c = 2A_2 + 16\ A_1 = .089\ m^2$
With $A_2 = .075*.092$
and $A_1 = .075*.063$.
We find $A_c = 0.089 m^2$.

d. Calculate Convective and Radiative Thermal Resistances R_C and R_R

$$R_C = \frac{1\left(\frac{.075}{100}\right)^{1/4}}{1.34(.089)(.78)} = \frac{1.8°C}{W}$$

Calculate the Outer Area Cooling Effects Due to Radiative Effects using our formula for radiative thermal resistance at surface temperatures of 120°C and ambient temperatures of 20°C for black anodized Al of surface area A: $R_r = \frac{0.12}{A_r}$

A_r is the outer area of a box which encloses the fins. We do not count the open top and open bottom.



$A_r = 2(.115)(.075) + 2(.063)(.075) = .027\ m^2$

Recall that we derived that the radiative thermal resistance is given by the relation below when the surface temperature is 120°C and the abmient temperature is 20°C for black oxidized aluminum of surface area A.

$$R_r = \frac{0.12}{0.027} = 4.5°C/W$$

The total thermal resistance is the parallel combination.

$$R_{sa} = \frac{R_c R_r}{R_c + R_r} = 1.3°C/W$$

☺ Agrees with the manufacturers data.

In summary we consider heat flow and temperatures in the various devices as being part of the proper design of a power electronics system. One because operation of any devices above 100°C for extended periods of timemay result in catastrophic failure. Second, for silicon solid-state devices it is well known that for every 10°C rise in operating temperature the estimated operating life of the device is halved. It pays in reliability of operation to have as low an operating temperature as possible