# LECTURE 28

## Wire Losses Due to Both Single Wire Skin Effects and Proximity Effects

## A. Single Wire Losses at High Frequencies.

1. Basic Copper Wire Properties
2. Wire Volume, Winding Area and Current Density, J
3. **Magnetic Fields and Wire Current Distributions,J**
4. Cu Wire Sizes, AWG Gauge Number System and Porosity of Wires
5. Wire Fill Factor of a Core : Parameter K
6. Core Wire Window Utilization and Porosity
7. Per Unit Volume Wire Winding Losses

$$\frac{\text{P(ohmic in Cu)}}{\text{unit volume}} = r\, J_{\text{rms}}^2$$

8. Single Wire Conduction and **Skin Effects**

**B**. **PROXIMITY EFFECTS:** Enhanced Wire Loss Due to Collective Effects of Contiguous Layers **of** Overlapped or Interleaved Wire Windings**:**

1. **$R_{AC}$ of a particular wire in a stack of wires will vary with Wire Position in a Stack**
2. Proximity Effect Explained by Surface Mirror Currents **and Associated Current Density Profiles**
3. **The Wire Parameter f and Losses**

## 4. Flux Strength from Non-interleaved versus Interleaved Windings

A. Increased Loss for overlapped wires via the Net MMF "M" Factor

$$P_{loss} \sim \frac{d}{d(f)} M^2$$

d is wire diameter, δ(f) is skin depth and M is the number of overlapped wire layers that contribute to the **net MMF**.

**B. Skin Effect on J distributions in various layered windings and associated mmf vs x. This is the case where D> d(skin effect)**.

C. **Interleaving the primary and secondary winding turns** The Fractional "M" Factor

D. $J^2_{rms}\ R_{AC}$ losses vs the Empirical wire parameter $f \equiv \sqrt{h}\ \frac{d}{d}$ on the x-axis of Loss Nomographs

Goal of φ ≈ 1 for low total wire loss.

η depends on wire insulation's, d is wire diameter and δ is skin effect size at the current drive frequency.

**LECTURE 28**

# Wire Winding Losses due to Both Single Wire Skin Effects and Proximity Effects Due to Stacks of Wire

## A. Cu Wire Conduction Properties at High Frequency

1. Basic Electrical Properties of Copper Wires

.The electrical resistivity of the wire is given by the parameter $\rho = 2.2 * 10^{-6}$ Ω-m @100°C and $\rho = 1.7 * 10^{-6}$ Ω-m @20°C. We try hard not to have the wire temperature exceed 100°C because cores, wires and solid state devices all degrade at higher temperatures

High σ or equivalently low ρ means that in a wire of length l and cross-sectional area A the **total DC wire resistance** is $\mathrm{R}(DC) = \frac{r\,\mathrm{l}(wire)}{\mathrm{A}(wire)}$.

Next we calculate the DC resistance of a length of wire. Let's consider a simple case of a primary wire winding used in a transformer, wound around a magnetic core cross-section of perimeter 10 cm. The core has an open wire-winding window into which the wire is placed. The total winding would usually entail a number of turns to achieve the desired value of the magnetizing inductance for the transformer

For a primary winding of n turns around a 10 cm perimeter core we might estimate the total length of wire for n turns as :

L = n*10 meters. Hence the DC resistance of the wire is:

$R_{primary} = R_L$(AWG Wire #) in Ω/m * L

For example, for #19 wire $R_L = 27$ mΩ/m at 25°C.

So for 60 meters of wire

$R_{primary} = 1.6\ \Omega$

This wire resistance value will hold only for DC currents and is not accurate as the frequency of the current is increased. We find herein that this **simple-minded DC result is as much as a factor**

**of 10-1000 too low in resistance from those measured at high frequency.** This is due to several effects that occur due to interactions between an external magnetic field and current flow in a wire that this magnetic field passes through.

**2. Wire Volume, Winding Area and Current Density,J**

We always seek to use the minimum area wire required for cost reasons when winding any inductor or transformer on a magnetic core. That is with larger diameter wire, and a fixed number of turns required, the core size must be increased to fit the wire coil. Also with smaller diameter wire we employ a smaller amount of associated wire volume on any inductor or a transformer. However, minimum size wire means the current density in the wire will increase, J $\uparrow$, for a given wire current and hence the power loss per unit volume will go as $P \sim \rho J^2$.

The primary and secondary wire sizes in a transformer will be set such that J is the same in the primary and secondary wires. Otherwise the heat loss per unit volume will be mismatched in the two windings and optimum heat generation will not occur. In short one winding will overheat before another and be damaged. We will show later tradeoffs between Cu wire volume and magnetic material volume to **minimize total undesired heating** from both wires and magnetic cores.

The wire material of choice is copper. The high ductility of Cu makes tight windings with sharp bends easy to achieve on small size magnetic cores. Tight windings also mean a minimum volume of Cu employed. Never forget that the permeability of copper is the same as that of air so any magnetic field will pass through the wires as easily as through air. Second, due to the relatively small (100-10,000 times $\mu_0$) permeability's of magnetic materials substantial leakage flux occurs outside the core. This means that **leakage flux from magnetic cores will easily leave the core and enter the wires** wound around the cores. This leakage flux in the wires causes high currents to flow at the wire surfaces, which in turn will change the effective wire resistance, especially to high frequency currents as we shall see in parts B and C below.

**3. Magnetic Fields and Wire Current Distributions, J**

We will divide these magnetic field- wire interaction phenomena into two parts.

One is for isolated wires carrying current and is termed the skin effect. Skin-effects in a single isolated wire are due to the magnetic field from the current flow itself penetrating the wire and causing internal eddy currents in the conductive wire. We already covered skin effects in Lecture 26.

The second effect is the interaction between collective magnetic fields created by turns of wires on the current flowing in individual wires and is termed **the proximity effects in stacked layers of wires.** A quick insight into collective magnetic effects from different choices of winding wire coils is as follows. Consider the difference in H fields we achieve when we wind a coil of n turns in a single layer on a large core of magnetic length x. Compare this result versus the result if we wind the same n turns in several stacks on a smaller size core of magnetic length x/10. The latter small size core will require one to employ multiple stacked layers of wires on top of each other to fit N turns, if the core wire winding window did not have sufficient height to fit all required wire turns in one layer. This opens up the subtle details of wire winding configurations on the cores.

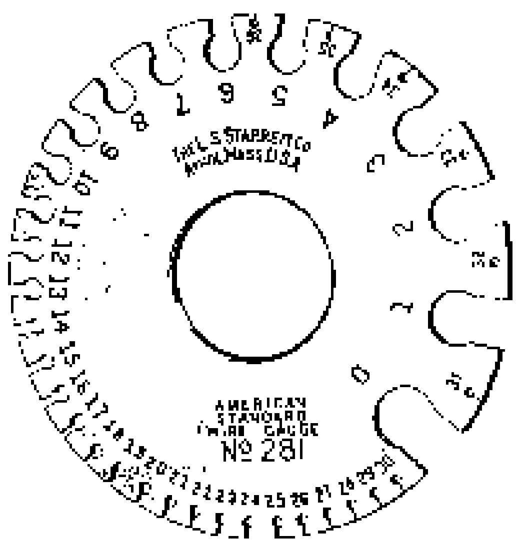
We will see that one long layer of wire turns, on a core of length x, has less magnetic field generated than a stack of wires on a core of length x/10. This occurs due to Ampere's law which says Ni=H x. H differs if all N turns fit along the length x as compared to if the N turns are in multiple layers along a smaller core length x/10. We will revisit this in sections B and C below where the H field that enters the wires will cause large mirror currents to flow that increase wire loss above that from skin effects alone.

## 5. Cu Wire Sizes, AWG Gauge Number System and Porosity of Wires

Different size wires can handle different current levels. In the USA we use the American Wire Gauge (AWG) number system below.

| AWG# | Ampacities in amperes |
|---|---|
| 24 | Familiar laboratory bread board wire |
| 14 | 16 Amperes Conventional House wiring |
| 12 | 20 |
| 10 | 27 |
| 8 | 36 |
| 1 | 102 |
| 1/0 | 121 Ampere- Terminal box wire |

A pictorial view of AWG numbers is given below via an electricians tool to determine the AWG of an unknown wire encountered.




American Wire Gauge AWG 4/0 is "the biggest wire size available" #50 is "the smallest wire size available" We will show below that AWG# follows a "db scale"

Note also that a cable containing three #10 conductors is termed a 10-3 cable. There are also some special wires such as Litz wire which has 345 strands of #35 wire in parallel to form one wire with special high frequency properties as we will explain below.

AWG #'s are such that the cross-sectional area of wire doubles every change of 3 AWG sizes. That is in term of wire diameters , AWG wire changes are like base 10 decibels in that if one increases or decrease AWG # by 20, the wire diameter decreases or increases by a factor of ten respectively. Thus for example, #10 wire has ten times the diameter of #30 wire.

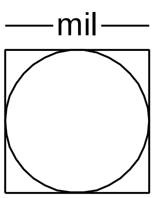
Some other factoids about the AWG system

Increase AWG # by 6 and we find the diameter varies by D/2

Increase AWG # by 10 and we find the wire cross-section area varies as Area/10

Increase AWG # by 3 and the wire area changes by $A_w/2$

Wire area seems a simple concept but there is a very troublesome unit of wire area for metric challenged U.S. engineers that causes lots of headaches. This is the concept of circular mils.



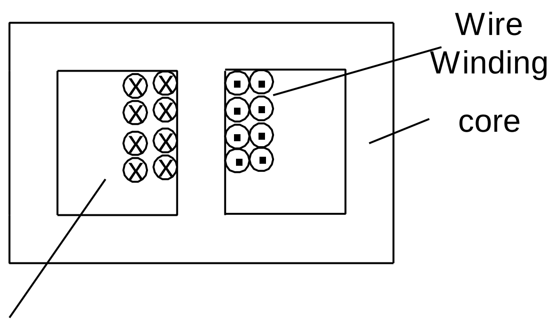

Area of square 1 mil on a side is $(\text{mil})^2$

In the USA we define a Circle inside the square as 1 circular mil. This unfortunately is the standard. Thus a1 $cm^2$ area is then 200,000 cir mils in USA units

Next we consider conductor spacing factor or porosity for different shape wires. For example to achieve the wire turns required, we could use round shape wire, square shape wire and even tape like wire. Clearly, each shape wire would have a different **porosity** or density of turns. We use a symbol η for this wire porosity parameter from which we get an effective wire diameter: $d_{eff}$ = d(actual wire diameter)$\sqrt{h}$. **Again h is termed the wire porosity** and is specified by the wire manufacturer for each type of wire geometry. Roughly speaking, voids between wires waste 21% of the winding cross-section area. Wire insulation further reduces the useful area, especially with the smaller diameter wires used to minimize high frequency losses, because insulation is a larger percentage of small wire diameters.

## 5. Magnetic Core Cu Wire Fill Factor

The wire size and required number of turns to a great extend dictate the magnetic core size needed for either an inductor or a transformer. The core size needs to have a open wire winding area into which **all of the wire turns fit**. The wire windings will partially or totally fill the two air windows in the core as shown below. We quantify this partial fill by an empirical parameter:




$A_w$ is the area of core window available for wiring as given by core manufacturer. $A_c$ is the area of Cu wire chosen for the winding. This will differ for the primary and secondary windings in transformers. For an inductor we have only one winding on the core but for a transformer we will have two or more windings. . for each turn. If N is the number of turns of Cu wire then we define:

$$K(\text{Cu fill factor}) \equiv \frac{N\,A_{cu}}{A_W}$$

K is an empirical factor unique to each wire geometry and insulation type as well as core cross-section. The wire area includes both copper and any electrical insulation placed on the outside of the wire. We list on the next page some typical values found in practice for commonly employed wires of various shapes and cross-sections.

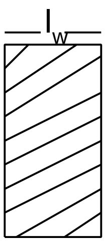
| Wire Type | Typical K |
|---|---|
| Litz wire bundle | 0.3 |
| Round wire with plastic insulation | 0.5 |
| Bare round wire with varnish | 0.8-0.9 |
| Foil, tape or sheet of wire | 1.0 |

## 6. Core Wire Window Utilization and Wire Porosity

We ask -How large a core window is necessary to contain the ampere-turns of all the required windings on a transformer?. Ultimately, this is determined by maximum allowable power dissipation in both the core and in the wire windings. This power must be dissipated as heat flow from the core/wire winding mass to the ambient. The balance between heat generated and heat flow from the core/winding mass to the environment causes an equilibrium associated temperature rise. In practice this must now exceed 100 degrees C where wire insulation degrades as do the core magnetic properties.

Assuming we do not violate equilibrium temperatures for either the wire or the core we now ask the question in a simpler way. How much of the window area is actually useful copper area where current flows? A wire winding bobbin, if used, significantly reduces the available window area, again impacting smaller, low power applications. In a small transformer with a bobbin and with high voltage isolation requirements, **perhaps only 25% or 30% of the window area is actually current carrying copper**. Electrical codes impose voltage isolation safety requirements on transformers such as the need for 3 layers of insulation between primary and secondary windings. We also must insure minimum leakage or creepage distances of 6 to 8mm from primary windings to secondary windings where leakage current may flow around the ends of the insulation layers at both ends of the winding. Nearly 1 cm of window breadth is lost to this safety requirement, severely impacting window utilization, especially with smaller cores in low power transformers. The increased separation between primary and secondary coils also results in higher leakage inductance in the transformer, as we will see later.

Below we consider the window winding area of a given magnetic core is h high by $l_W$ wide in area. We revisit the **empirical parameter called the wire porosity.**




Consider that in a stack of turns h high by $l_w$ wide we place a n turns employing wire of diameter d. We can say that the wire porosity is:

$$h \equiv \frac{\text{wiring}}{\text{porosity}} \equiv \frac{p}{4}\, d^2 \,\frac{n}{l_w h}$$

$.7 \leq \eta \leq .8$ is typical for a range of insulated wires.
Note that η reduces the effective conductivity of the wire, or increases the effective resistivity of a wire, because the wire insulation takes up area that is not employed for current flow.
In short η becomes the Effective diameter of wire:
$d_{eff} = d\sqrt{h}$ - Typically η introduces a factor of 0.9

$$R_{eff} = \frac{r\, l(wire)}{A_w} = \frac{r\, l(wire)}{\frac{p\, d_{eff}^{\,2}}{4}} \quad \text{But } d_{eff} = d\sqrt{h} \Rightarrow \frac{r l(wire)}{\frac{pd^2}{4} h} = R/\eta$$

Higher R in the wire means Higher $I^2R$ losses for fixed current flow! The load not the wires determines current flow.

In summary, $R_{eff} \sim \frac{1}{h} * R_{geometry}$ (without porosity corrections)

### 7. Power Dissipated Per Volume in the Wire Windings

We will consider equilibrium heat flow in lecture 29 including wire and core losses together. Because heat equations and core losses use input heat in P/cm$^3$ we need to find wire losses in the same units of W/cm$^3$.

**power**
**P(per unit volume) =** $r_{cu}(\Omega\text{-cm})\, J_{rms}^2\ (A/cm^2)^2$ **in W/cm$^3$ units of copper**

A traditional rule-of-thumb for mains 50-6OHz transformers is to operate copper windings at a current density of 450 A/cm$^2$(2900 A/in$^2$). However, smaller high frequency transformers can operate at higher current densities because there is much more heat dissipating surface area in proportion to the heat generating volume.

The effective wire volume used for heat flow is actually smaller by the factor K as we saw above. That is the winding volume is:

$V_{cu} = k\left(\frac{Cu\ \text{fill}}{\text{factor}}\right) * V_w$ (core window volume)

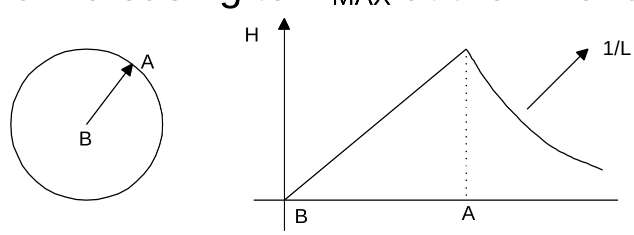
For $\rho = 2.2 * 10^{-8}\ \Omega$-m and J in A/mm$^2$ units which is more useful than A/cm$^2$ we find:

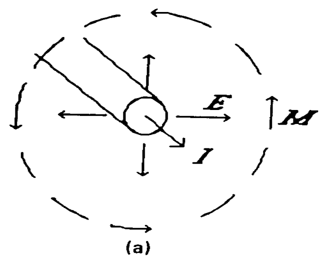
$$P\left(\frac{\text{power per}}{\underline{\text{winding}}\ \text{volume}}\right) = 22\ K(\mathit{fillfactor})\ J_{rms}^2 \quad \text{units mW/cm}^3$$

## 8. Single Wire Conduction and Skin Effects

### a. H fields Surrounding Wires

One wire with DC **current** flowing causes a magnetic field that is both enclosing and penetrating the wire. The H field intensity outside the wire versus current flow within is given by by Amperes Law: $\int H \cdot dl = H2\pi r = I$. That is H(outside the wire) $= I/2\pi R$. The field decays with the distance form the outer radius of the wire. Inside the wire, for a constant DC current distribution versus wire radius, H can be shown to increase linearly with radius, starting at zero at r=o and increasing to $H_{MAX}$ at the wire radius, r=a.



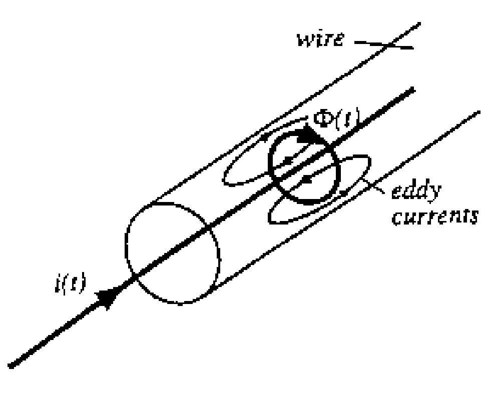



The fields around a conductor in free air: (a) field around a free wire; (

The above figure is a DC model of a single wire carrying current with constant current density and the associated H profiles. Position A represents the surface of the wire when r=A and B is the center of the wire when r=0. Because we have a DC current flow, the penetrating DC H field entering the wire will not cause any eddy currents in the wire. Hence, the current density distribution remains constant across the radius of the wire. This will not be true for AC currents in wires as shown below. The AC H fields, which penetrate the wire, will indeed induce additional currents in the wire. This will alter the current density distributions in the wire dramatically.

**b. Eddy Currents in the Wire Created by AC H Field Penetration into the Wire**

The induced eddy currents in the Cu wire, due to AC H fields penetrating the wire, act to enhance current flow at the edge of the wire and decrease current flow at the center of the wire as shown below. The induced eddy currents oppose the applied H.



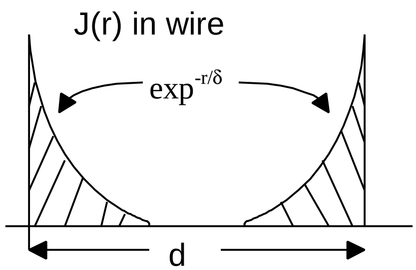

Flow of i(t) causes $H_\phi(t)$ fields. $H_\phi(t)$ in turn causes elliptical voltages to appear in the wire which drive longitudinal elliptical eddy currents to <u>oppose</u> $H_\phi(t)$. The eddy currents act to cancel out the applied current in the center of the wire.

Looking at a wire cross-section above better explains the net effect on the J(r) profile to reduce current flow in the middle of the wire.

Current flows in the wire path(s) that result in the lowest expenditure of energy. At low frequency, this is accomplished by minimizing $I^2R$ losses and constant current density profiles of DC currents result. At high frequency, current flow in the path(s) that minimizes inductive energy dominates. That is, energy transfer to and from the magnetic field generated by the current flow. Energy conservation causes high frequency current to flow near the surface of a thick conductor even though this may result in much higher $I^2R$ losses. If there are several available paths, HF current will take the path(s) that minimize inductive energy.

**FOR HW# 6 derive and plot the same current density and H versus radius profiles for high frequency currents. Be as quantitative as possible**. Employ the handout of Bessel function current profiles from Professor Collins to derive current density and H profiles versus radius as shown below**.**

J decays exponentially from the outer surface of the wire towards the center with a spatial decay constant δ, $J(r) \sim e^{-r/\delta}$ as shown.




$$d \equiv \sqrt{\frac{r}{pm\,\mathrm{f}}}$$

Using constants for Cu media we find:

$$\delta(\mathrm{cu}\ @100^{o}\mathrm{C}) = \frac{7.5\ \mathrm{cm}}{\sqrt{\mathrm{f}}}$$

| | | | | |
|---|---|---|---|---|
| Frequency | 50 Hz | +5 KHz | 20 KHz | 500 KHz |
| δ(skin depth) | 10.6 mm | 1.06 mm | 1/2 mm | 0.106 mm |

## B. PROXIMITY EFFECTS: **Collective Effects of Layers of wiring**

### 1. $R_{AC}$ of a Particular Wire in a Stack of Wires will vary with Wire Position in a Stack

For single conductors of size d(wire diameter) > δ(Skin depth) we expect simple resistance changes at high frequency as compared to DC. Consider for a single wire various values of $\frac{d}{d}$ for #20 AWG wire and the effects of $R_{AC}$ for a single wire as shown below.

$$R_{ac} = R_{dc}\left(\frac{d}{d}\right) \qquad d = 1000\mu$$

| f | d/δ(μ) | $R_{Ac}/R_{DC}$ Ratio |
|---|---|---|
| $10^4$ | 300 | 3.3 |
| $10^6$ | 80 | 12.5 |

Surprisingly for two wires next to each other $R_{ac}/R_{dc}$ exceeds this single wire skin effect value due to proximity effects of several wires causing a collective magnetic field. That is when we wrap several layers of wire turns next to each other we find that $R_{ac} > R_{dc}$ by more than expected from simple single wire skin effects as discussed in part A due to collective H fields being created by the wire stack. Depending on the location of the wire we penetrate different wires with different H levels. The net currents in any single wire do not change but the current consists of mirror currents of opposite direction flowing on the outer surfaces. These are called proximity effects, which further increase $R_{ac}$ above the values for single isolated wires, as we will show below. That is $R_{eff}$ increases above simple skin effect values due to the cumulative proximity effect of N turns of wire. Recall, NI = H x (magnetic path length). Moreover, the resistivity increase due to proximity effects in any one wire now depends on the wire position in the cumulative magnetic field pattern. Different cumulative H arises after each wiring layer is added for fixed current, I, and magnetic path length,x, as described below.

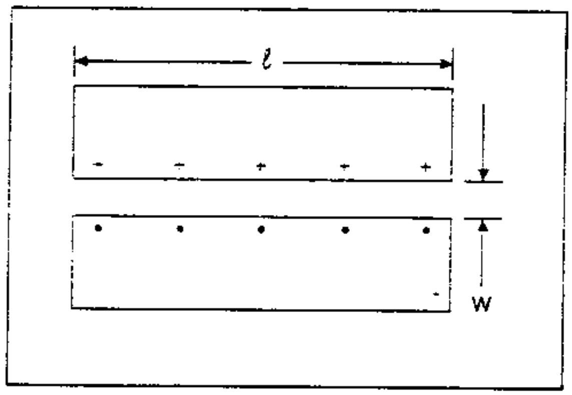
Looking ahead, the additional resistance for a second layer of wiring in a multi-turn wiring stack will be 5 times that of the first layer of wiring. This occurs because 2xI flows on one wire surface and I flows on an opposing surface in the opposite direction. The net current in the wire is unchanged at I. But the $I^2R$ losses are five times as great. The resistance of the third layer will be 13 times as large expected as shown below on page 21 due to 3xI flowing on one surface and 2xI flowing on the opposite surface.

We next set down the basic details of the proximity effect to explain better why we **have surface current flows in opposite directions on the same wire** prior to quantifying resistivity increases.

## 2. Proximity Effect Explained by Surface Mirror Currents **and Associated Current Density Profiles**

### a. Overview of Conditions that Alter Current Profiles

When two conductors are in near proximity and carry opposing high frequency currents, the current density distributions in each wire spread across the entire opposing wire surfaces facing each other in order to minimize the total magnetic field energy transfer (minimizing inductance). That is current flow in the wire cross-section, *l* x w shown below, fills the entire length *l of the wire* shown below BUT DOES NOT fill the entire width w of the wire. These non-uniform spatial current density profiles will cause unique $I^2R$ losses. Recall that NI=H*l.* As *l* increases H will decrease. The above fully spatially extended , high frequency current density profile, across $l$ but spatially confined across w current pattern is shown below. Note also how the currents in the two wires will concentrate at the opposing surfaces of the two wires. Current flow toward you is represented by a dot and away from current flow is a cross.




*Proximity Effect*

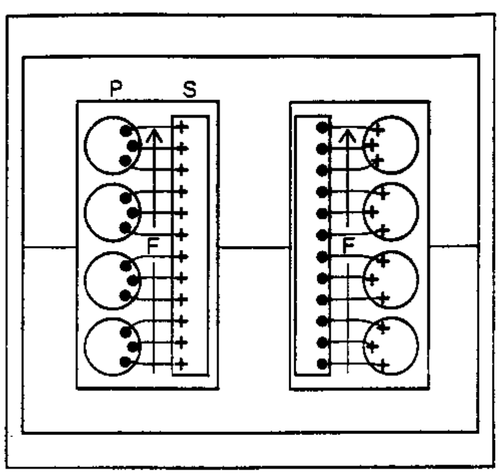
Current does not flow on any other wire surfaces because that would increase the geometrical volume of the magnetic field and require greater energy storage and transfer**. Inductance is thus minimized by this concentrated current profile in each wire, but ac resistance in the wires is made higher**. In the preferred configuration, the current distributes itself fairly uniformly across the two opposing surfaces. An important principle is demonstrated here: If the H field (and the current that produces it) is given the opportunity to spread out along a longer magnetic path $l$ , it will do so in order to minimize energy storage and transfer. The stored energy (and inductance) between the conductors varies inversely with the length of the magnetic field path,$l$.. Transformers are a good case to examine since layers of primary and secondary turns are so close to each other that new more complex current distributions arise on the surfaces of wires.

## b. One Layer of Turns and Wire Current Distributions in a Transformer

A given transformer shown below has two windings, which fit in one layer of turns. Two cases will be explored. Each dot or cross in the wires represents one Ampere of current flow. The primary carries 3A and the secondary 12A in the **one case where the four turn primary wires are connected in series.** This could be the case for a 4-turn primary winding carrying 3A, opposed by a single turn secondary carrying 12A.

In a second transformer winding case, the four primary wires could be connected in parallel, giving a effective 1-turn primary carrying 12A with 12 A flowing in a single turn secondary.

When the conductors are thicker than the skin effect size, $\delta$, the high frequency currents flow near the surfaces of the wires in closest proximity, thus terminating the magnetic field with minimum energy. **In either case, the H field pattern spreads itself across the entire wire-winding window and the minimized energy is stored between the windings if the current flow is altered as shown below**. The currents are driven to the surfaces of the wires by the minimization of magnetic energy.

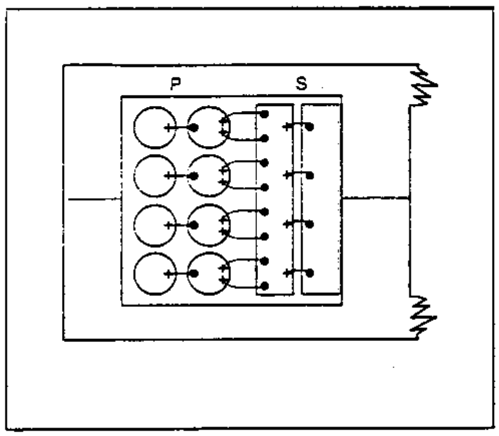



*Single Layer Windings*

The above case is for single stacks of primary and secondary wire windings. The case of multiple stacks of wire winding is next.

c. Case of Two Stacks of Wire Windings: **Suprisingly High $I^2R$ Losses:**

We next examine the case for two layers of stacked wires in a transformer winding containing both primary and secondary coils. Wire winding stacks occur due to finite core winding window size which doesn't allow all N required turns for a wire of a given diameter to lie in one simple stack as first discussed. Below is the case for two primary and two secondary stacks of wire windings wound in the following manner. The secondary windings are wound first around the core cross-section and then the primary windings are wound around the primary windings.




*Two Layer Windings -- Series*

In the two layer, series wire connection, illustrated above, an 8-turn primary carrying 1A in each wire turn is magnetically coupled by the core to a 2-turn secondary carrying 4A in each wire turn. We assume that the 8 turns of primary wire, when properly sized for the required rms current cannot fit into the limited window breadth of the core, so it is configured in two layers.

As expected, there is an 8 Ampere-turn MMF field stretched across the entire window. If at the operating frequency of the AC current the conductor thickness is much greater than the skin depth, the H field cannot fully penetrate the conductor, and current flow is confined to near the conductor surfaces in the dimension w but extends fully in the dimension *l*. This is illustrated by the dot and cross current flows in the wires. Note that the inner secondary winding and the outer primary winding have surface currents just like our first case of single layer windings discussed above.

**A strange thing happens, however, to current profiles in the w direction at the interface between the primary and secondary wire windings only**. Here the first primary wire carries a net 1A but 2A in the cross-direction are on one wire surface facing the primary and 1A in the dot direction flows on the surface of the outer primary wire. The interface termination of currents via associated H fields is associated with mirror currents in the opposite sides of the respective wires of magnitudes as described below. The two wires involved with mirror currents are only the first turn of the primary winding and the second term of the secondary. The second term of the primary and the first tern of the secondary have no mirror currents on opposite surfaces just as the single layer case.

We first deal with the surface currents at **the inner surface of the first turn of the primary and the outer surface of the second turn of the secondary.** Since the H field cannot fully penetrate the wire conductor, an 8A driven H field terminates at the inner surface of the first primary turn and at the outer surface of the second turn of the secondary wires as shown above. Adding the dots and crosses in the outer turn of the secondary we end up with a net current of 4 A in this turn as seen from outside the wire. Likewise the total current from outside the wire in the first primary winding is 4A in the cross-direction, with 8A in the cross-direction at the inner surface and 4A at the outer surface in the dot direction for a net 4A. The two current density segments are flowing on opposite surfaces of the same wire. This occurs only for the turns layer of the primary that face the secondary winding and vice versa. The far left primary winding and the far right secondary winding carry current only in the cross direction and the dot direction respectively..

The 8A and the 4A current flows in the outer secondary winding are physically separated from each other. This means for $I^2R$ calculations in the second turn of the secondary winding or in the first of the primary we have to add the loss effect from each surface current, even though the net current is only 4A. This requires a total of 8 Ampere-turns in the cross direction at the inner surface of the first four primary windings -- 2A per wire. However, 4A in the dot direction also flow in the first turn of the primary at he outer surface. The H field between the primary and secondary windings can be terminated by H or MMF lines only and flows between opposing currents in the opposing wires of 8A magnitude. Again, on the inside surface of the outer layer of the secondary, there is a 4A flow represented by crosses which cancels out 4 A of the 8 A dot direction currents on the opposing surface.

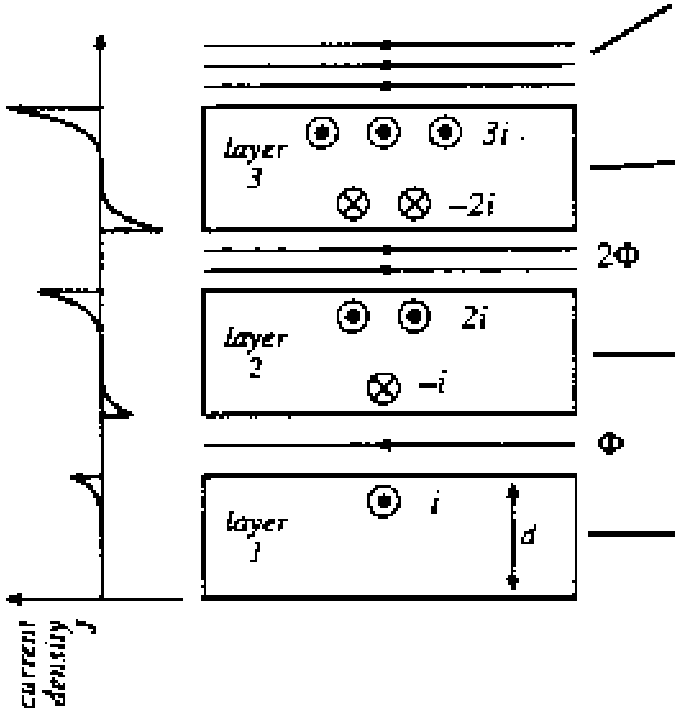
For the primary winding of two layers the current flow as seen outside the wire is 1A for both turns. The net current remains 1A in all series wires in both layers of the primary but the total $I^2$ R losses seen by the first turn of the primary is different from the second turn of the primary because of the role of surface mirror currents. Thus the first or inner primary turn layer of turns with 2A localized on its inner surface and 1A in the opposite direction localized on its outer surface behaves as follows. Since loss is proportional to $I^2$, the loss in the inner layer is $1^2 + 2^2 = 5$ times larger than the loss in the outer layer of the primary, where only the net 1A flows on its inner surface and no mirror current flows.

**For HW# 6 show that the ratio of the loss in the two turns of the secondary also differs by a factor five.**

Not only is the $I^2R$ loss larger because the mirror currents are confined to the surface, the loss also increases rapidly as the number of layers of turns employed increases as we show below. This is because the field intensity increases progressively toward the inside of the winding and PEAKS at the primary to secondary interface. Since the field cannot penetrate the conductors, surface currents must also increase progressively in the inner layers. For example, if there were 6 wire layers, all wires in series carrying 1A, then each wire in the inner layer will have 6A flowing on its inner surface (facing the secondary winding) and 5A in the opposite direction on its outer surface. The loss in the inner layer is $6^2 + 5^2 = 61$ times large than in the outer layer which has only the net 1A flowing on its inner surface!

All of the above surface mirror current flow was predicated on the wire diameter being much greater than the skin depth at the operating frequency of the current in the wire**. If the wire diameter is reduced to that approaching the skin depth, the + and - currents on the inner and outer surfaces of each wire start to merge, partially canceling and thereby reducing $I^2R$ losses in the wires located at the interface of primary and secondary**. The H field now partially penetrates through the conductor. When the wire diameter is much less than the penetration depth, the field penetrates completely, the opposing currents at the surfaces completely merge and cancel, and the 1A current flow in the primary is distributed throughout each wire of the primary uniformly.

Calculation of the $I^2R$ loss in the multi-layer turn coil where the wire diameter well exceeds the skin depth is very complex depending primarily on the number of layers in each winding section. In summary, we find for multi-layered windings the power loss,$P_M$, for each layer,m, goes as shown on page 22.




$P_m = P_1\,[(m-1)^2 + m^2]$

$P_3 = I_3^2(rms)\,R_{ac}\,\}$
$(2^2+3^2) = 13P_1$

$P_2 = I_2^2(rms)\,R_{ac}\}$
$(2^2+1^2) = 5\,P_1$

$P_1 = I_{1rms}^2\ R_{ac}$

We repeat, in stark contrast, for wire diameters and operating wire current frequencies such that $d \sim \delta$, in a coil of M layers each layer carrying the same total current $P_{total} = MP_1$

One can show that for wires and operating frequencies such that $d >> \delta$ the total loss of all layers together is:

$$P_{total} = \sum_{j=1}^{M} P_j = \frac{M}{3}\,(2M^2+1)\,P_1 \qquad \text{Only for } d >> \delta$$

The most difficult case to model is for $d \sim skin.depth$ in each wire with M layers of windings the $I^2R$ loss has two factors when normalized to the DC loss case which we state but do not derive:

$$\frac{P_{AC}}{P_{DC}} = F_R = \left(\frac{d}{d}\right)\left[\frac{2M^2+1}{3}\right]$$

↓ ↓

The single wire factor $d/\delta$ varies from 2-8. The second factor is not negligible and can easily be a factor of *10 even *100

Consider the term | $\underline{M}$ | $\dfrac{2m^2+1}{3}$

Depending on M we get for example. M=1 we get a factor of 1 but for M=3 we could get a factor of 6, while for M=10 a factor of 60 may occur.

## 3. The Wire Parameter f and Losses

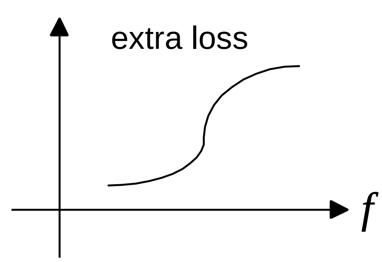
In practice it is impossible to calculate the wire losses each time we meet a new transformer or inductor with a unique wiring configuration. Rather we will utilize loss nomographs to get the loss quickly. To better obtain accurate wire losses from empirical or theoretical loss plots we define a new parameter, ϕ, that includes both individual wire porosity and skin effects we discussed earlier:

1. Wire porosity $\rightarrow \mathrm{d}_{\mathrm{eff}} \approx \sqrt{\mathrm{n}}\ \mathrm{d}$
2. Skin Effect on one wire $\rightarrow \dfrac{\mathrm{d}}{d}$

**The new parameter is called the wire parameter f**

We employ it as the x-axis in wire loss plots with the y-axis being the total increase in wire loss over the DC case

$$f\ \left(\begin{matrix}\text{effective}\\ \mathrm{d}/\ d\ \text{ratio}\end{matrix}\right) \equiv \sqrt{h}\left(\frac{\mathrm{d}}{d}\right)$$




$\downarrow$

This ϕ factor will be used in all empirical wire loss plots to determine extra wire losses due to proximity effects from layers of turns acting cooperatively

We will see in later lectures that this plot is only useful to see the increased loss due to proximity effects for AC currents. It does not determine the total $I^2R$ loss, which involves both the DC and the Ac losses.

With the large AC and low or zero DC current waveforms as is found in a transformer, $R_{AC}/R_{DC}$ of 1.5 is generally considered the optimum wire loss goal. Of course poorly designed transformers can have loss factors of 10 –100 as seen from any loss nomograph plot whose vertical axis extends to 100 times DC loss.

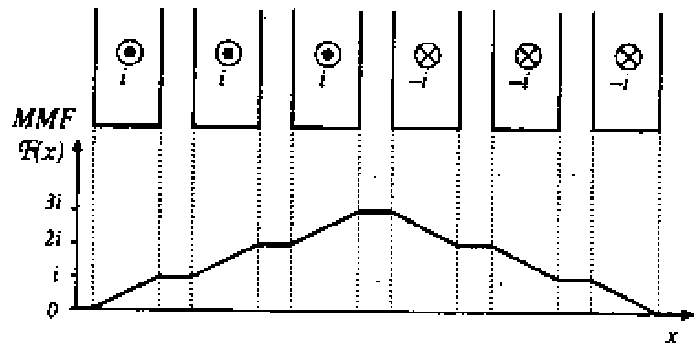
Achieving a lower $R_{AC}/R_{DC}$ ratio requires finer wire to be employed in the windings, and the wire insulation and voids between wires further reduces the amount of copper. The end result is while we decrease AC losses we end up with in higher dc losses. In a typical filter inductor with small ac ripple current component, a much larger $R_{AC}/R_{DC}$ can be tolerated and we do not want to increase DC losses as the DC component of current often exceeds the AC component.

# 4. Variation of Flux Strength from Stacked Windings: Non-interleaved versus Interleaved Windings

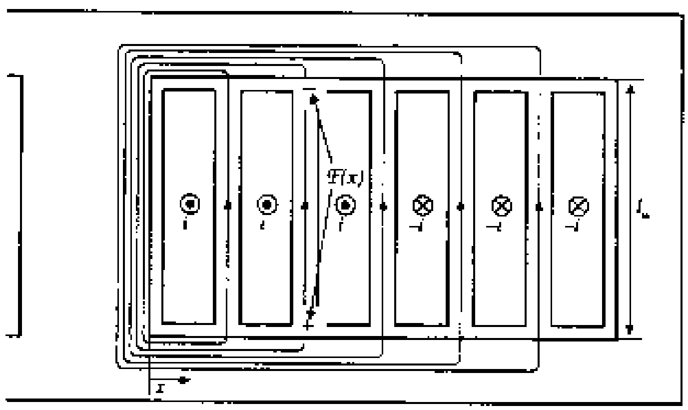
Windings need not be wired with the primary and then the secondary windings. We can interleave the windings with a portion of the primary followed by a portion of the secondary. This interleaving brings big benefits to the total losses as we will see below.

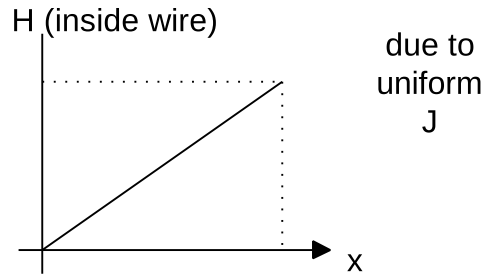
Consider a transformer-wiring stack placed along the x direction of the core-winding window. We have m (primary) coils wound first with current flow in one direction next to a series stack of n(secondary coils) with current in the opposite direction. Both stacks encircle a common magnetic core. If we plot mmf versus distance in the wire-winding window, x, from the interleaved windings we get very unique MMF plots as compared to the case without interleaving. We first look at non-interleaved for the two cases below for wire size larger (part 4b) and less than(part 4a) skin depth and vice versa. Then we address the case of interleaved windings in 4c. Finally in section 4d we calculate the losses for the interleaved case.

**a. Uniform J flowing in layered winding wires: D< δ(skin depth)**




The mmf vs. "x" varies in a ramp/step fashion as we add the coils and coil spacings respectively. [$m_{primary}$ - $m_{sec}$] i=F(x) the mmf in air region of the wire winding window

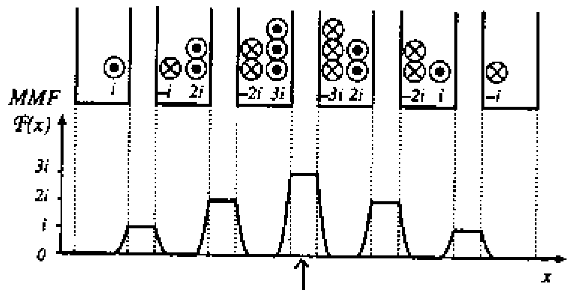


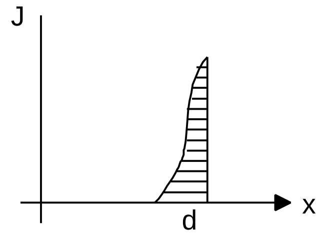




Using Ampere's Law $\int H \cdot dl = NI$ and noting to a first approximation for high permeability cores, at top and bottom of the core window H(core) = 0 as well as H = 0 on the left inside the core. Hence, in the core window we find : $\mathrm{H(vertical)} = \dfrac{\mathrm{F(x)}}{l_W}$ as shown. This is the leakage flux, which appears in the core window.

**b. Skin Effect on J distributions in various layered windings and associated mmf vs x. This is the case where D> δ(skin effect)**.

The currents in the second turn have one ⊗ directed current on the left hand side surface and two ⊙ directed currents on the right hand surface. All H fields inside wires are assumed zero with the J distributed only at the edge of the wire due to Ampere's law. This causes F(x) to vary in a trapezoidal bar graph fashion rather than in ramp/step fashion when J was uniform inside the wire.



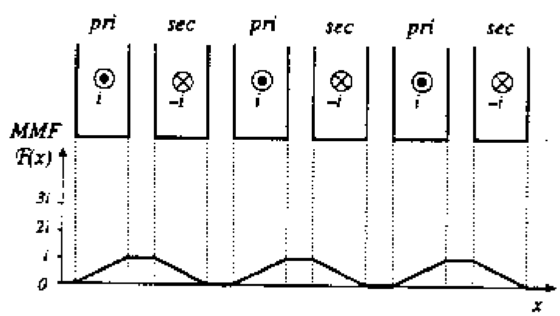



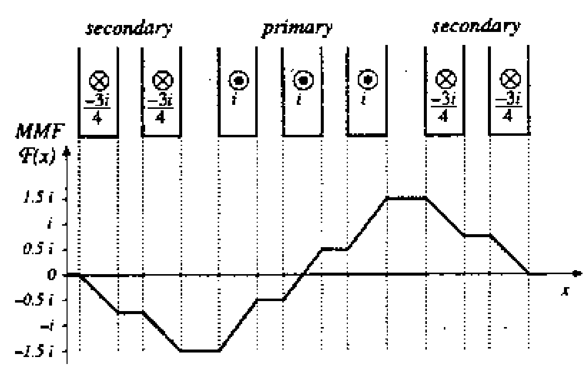

$$\int \mathbf{H} \bullet d\mathbf{l} \equiv \int \vec{J} \bullet d\vec{A}$$
$= I$

H exists <u>only</u> at the outside of the wire where ? J? dA is non-zero

Peak F(x) at 3*I just at the primary-secondary interface

c. **Interleaving the primary and secondary winding turns** acts to reshape mmf vs x patterns in the core wire winding window as shown below. Two interleaving wiring patterns are shown below. Note that fractional NET M values of ½ or 3/2 etc are now possible, whereas in the non-interleaved windings only integer values were possible.




For alternating primary/secondary winding windings. $F_{max} = i$

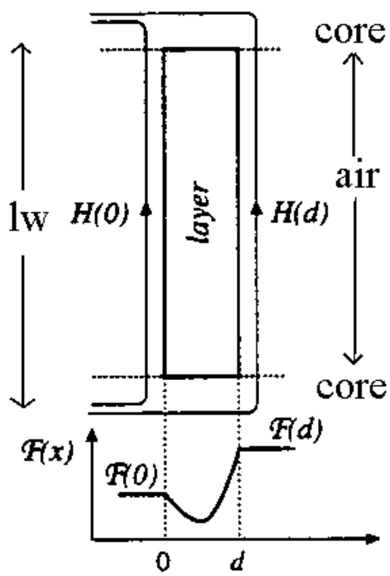
For a 2-3-2 alternating pattern. $F_{max} = 1.5\ i$

Note we can both set $(\mathrm{mmf})_{max}$ and tailor its spatial profile by choice of interleave patterns of the wire windings.

Spatial location of the zeros of F(x) may also be tailored as to precise <u>x spatial location</u> they occur by tailoring net $(m_p - m_s) = 0$ locations as shown above.

## d. $J^2 R_{ac}$ Losses in the Windings

This is not a real derivation only an outline of one. Full details are considered beyond this introductory course. Note that H(x) in the core winding window on either side of a layer of turns of wire is in general non-zero but inside the wire H =0. Inside one single turn of width D Both $H(x=0) \neq 0$ and $H(x=d) \neq 0$.




$\oint H \bullet dl$ around paths enclosing a layer of turns. Note we choose the path so that H(core) ~ 0 at the top and bottom horizontal paths.

$$H(0)l_w - H(d)l_w = \int \vec{J} \bullet dA \Rightarrow \text{Yields } J(x)$$

$\downarrow$ Net Hdl $\qquad$ $\downarrow$ Net I inside the layer of turns

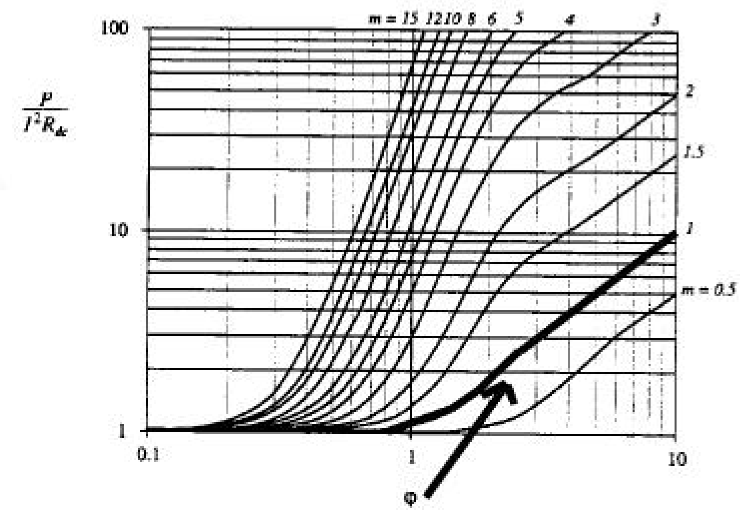
Total $J^2$ copper loss

One finds that the resistive wire loss depends on the mmf values at x = 0 and x = d in a complex way we only hint at here.

$$P = R_{dc} \frac{j}{n_l^2} \left[ \left( F^2(d) + F^2(0) \right) G_1(j) - 4F(d)F(0)G_2(j) \right] \qquad f \equiv \sqrt{h}\, \frac{d}{d}$$

We will consider the derivation for P curves on the nomograph of wire loss beyond ourcapability. But the result is a complex graphical solution to the multiplicative factor for <u>increased</u> power loss. In the coil of n turns employing wire with a characteristic factor $\phi$, we may to vary the loss term from 1 to 100 * dc power loss. This range of additional loss depends on applied frequency as well as both wire type as well as the **net number of stacks of of turns m**. For example, m=1 is the interwoven windings case with one turn of primary followed by one turn of secondary. This causes the square-wave like mmf pattern versus distance shown on page 26. In general, in wire loss plots m is the ratio of the mmf, F(d), to the layers ampere turns $n_l i$. and is the scaling parameter as shown below. The factor $\phi$ is used for the x-axis and the Ac loss to DC loss ratio is the y-axis. Note again $f = \sqrt{h} \ \frac{d}{d}$

At $\varphi = 10$ in the winding loss plot there is strong proximity effect observed since $d/\delta = 10$ and skin effects will cause a big increase in R depending on the NET # of layers of turns m as shown below.
$P/(I^2 R_{dc})$ = (Actual Loss) / (expected dc loss)
Clearly m=1 or the interleaved primary and secondary windings is a desired low loss case as compared to m=10. For low loss wire windings we need to heed the interleaving versus non-interleaved.



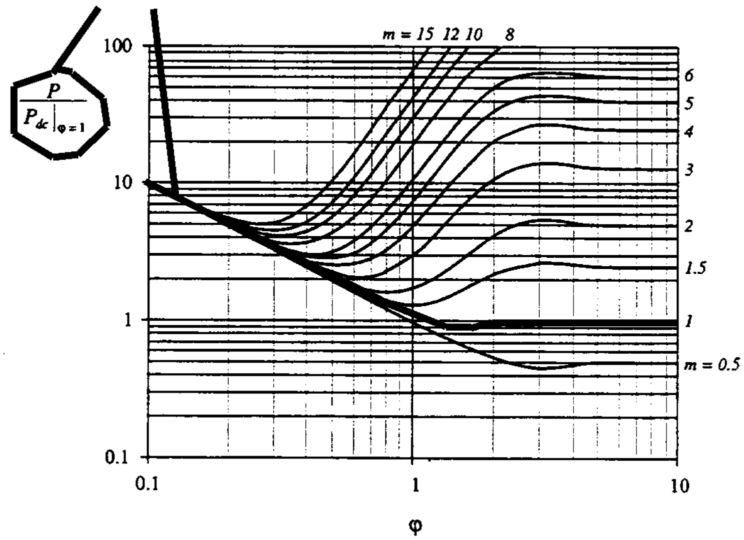

In calculating the increased loss we ask.

1. Where does Litz wire lie?
2. Where do losses from harmonic frequencies as for example from a square wave excitation lie?

The normalizing factor is $P_{dc}$ for $d = \delta$.

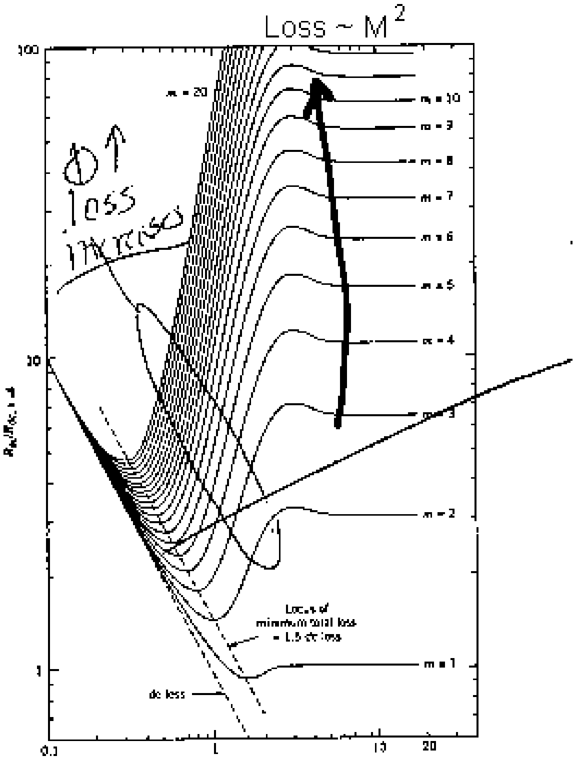
## For HW# 6 explain the physical situation for the wire windings that creates m=1/2

There is no benefit in pushing to an $R_{AC}/R_{DC}$ of 1.5 by arbitrarly making the $\phi$ factor on the x-axis smaller. If via the $\phi$ factor reduction for minimizing $R_{AC}$ the $R_{DC}$ is increased to two or three times we have not really saved anything except in transformers where $I_{DC}$ is zero. In an inductor where the DC and AC signals are comparable, because of the reduced copper area of the many fine wires involves we can create a DC resistance problem. By plotting P(total) as normalized for a DC conductor with D=$\delta$Skin depth), or $\phi$=1,we can better avoid this trap by using the loss plot below.




Note that 3 layers of windings, m=3, make for 10* higher wiring power loss than expected from dc. High $\phi$ asymptote is $m^2 + (m-1)^2$.

Note for $\phi$ = 1 or less is where minimum loss occurs in the plot on page 30 below. A practical goal is to find from the plotted wire loss vs $\phi$ for the given configuration of the interleaved or non-interleaved transformer windings and the choice of wire diameter. Wire diameter d(conductor) and NET # MMF/ni ratio, m, at a given ac driving frequency are also needed so that total <u>winding loss</u> is <u>minimum</u> due to the combination of both $I_{DC}$ and $I_{AC}$ as shown on page 30.




For 10 layers of turns, m, increased skin effect loss occurs. It's possible to get 100* greater loss. Loss ~ $m^2$

$\phi$(min) also varies with m.

Having the winding parameter $f = \sqrt{h}\ \frac{d}{d}$ optimum we can achieve as little as a 50% increase in loss over dc values

Harmonics of the fundamental square wave drive have increased loss as ? ? with frequency.

Use the above increase in $R_{ac}$ or loss versus $\phi$ to tailor a desired total wire winding loss to the levels compatible with the available heat removal for the core-winding system taken as a whole as we will discuss in Lecture 29.