ECE 555

03/23/2015

① Explain each term in the following equation.

$$\dot{\theta} = (J_e^{+}\dot{x}_e) + [J_0(I - J_e^{+}J_e)]^{+}(\dot{x}_0 - (J_0 J_e^{+}\dot{x}_e))$$

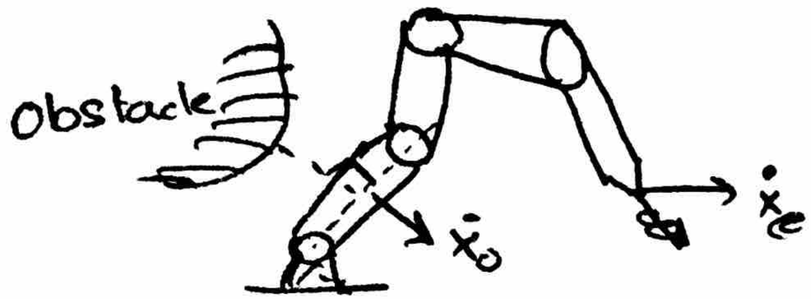
Why the terms are really needed? What is the significance of each bracket in the equation?

② Explain rank of matrix and pseudo inverse relation.

③ Mention 3 methods, that can be applied to minimize rank change effects? What are their limitations if any.

Home Work Question

Figure:



$\theta_1 = 45^\circ$

$\theta_2 = 10^\circ$

$\theta_3 = 90^\circ$

$\theta_4 = -30^\circ$.

$l_1 = l_2 = l_3 = l_4 = 2$

$\dot{x}_e = \begin{bmatrix} 0.5 \\ 0 \\ 0 \end{bmatrix}$ $\dot{x}_0 = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \end{bmatrix} \times \frac{2}{10}$

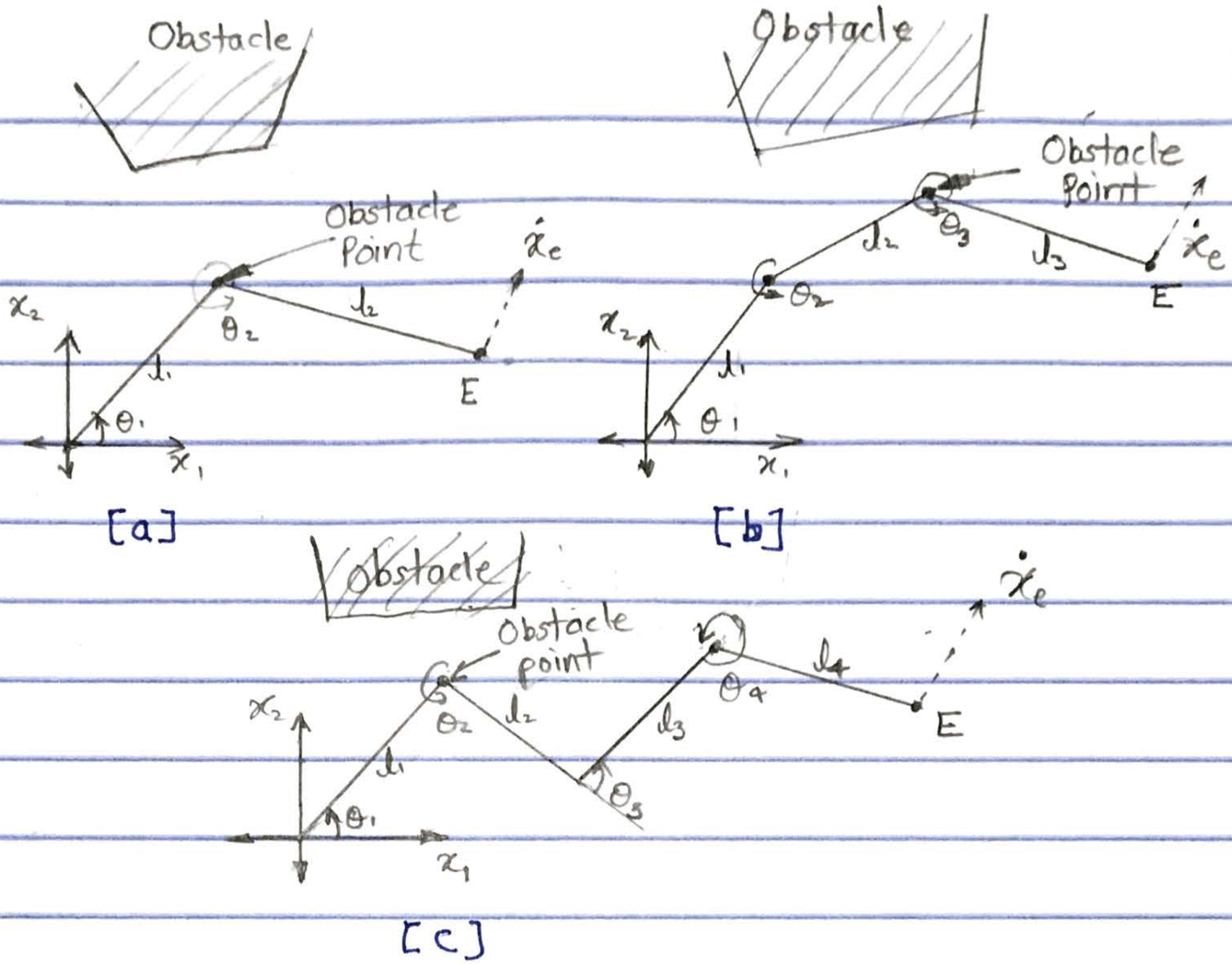
determine $\dot{\theta}$ for the above configuration.

1. Projection operator $(I - J^{+}J)$, is used to describe redundancy of the system. This operator is used in the obstacle avoidance technique. Prove,

$$(I - J^{+}J) = \sum_{i=r+1}^{n} \hat{v}_i \hat{v}_i^T$$

r = rank of matrix, n = Degrees of freedom, $\hat{v}_i$ unit vector resulting from singular value decomposition of the Jacobian J

Q.



Consider the figures (a, b, c) drawn above, showing 3 configuration for 3 different robot manipulators.

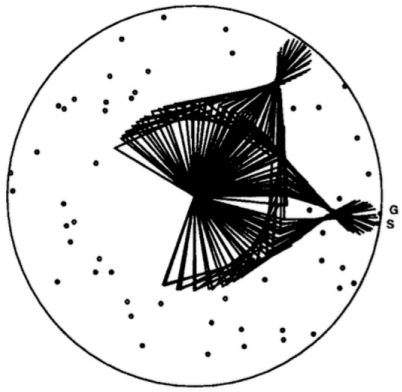
a) Analyze all three cases [a, b, and c] and state if it is possible to achieve the secondary goal of obstacle avoidance while ensuring that the primary goal of having desired end-effector velocity ($\dot{x}_e$) is being achieved. Explain every case with a reason.

b) Draw the specified obstacle avoidance point velocity (approximately) given by $\dot{x}_o$ for the cases where it is possible to achieve both primary and secondary goal simultaneously.

Problems:

1. Why we use Jacobian theories to compute joint angles when the end-effector position and orientation are known instead of the inverse kinematics procedure? What are the physical meanings of Jacobian and transpose Jacobian?

2. What is the difference between numerical instability and ill conditioning, and does the manipulator in following figure have ill conditioning problem, why? What are the three distinct cases in the physical interpretation of the pseudo-inverse solution? Why do the pseudo-inverses provide the best possible approximation?

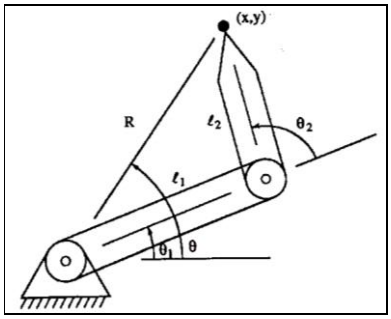


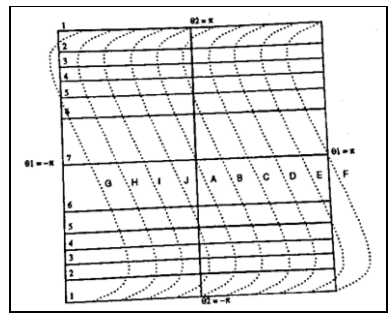
3. Can a robot avoid a random obstacle if it does not have redundant degrees of freedom? If the Jacobian matrix is not square of of full rank, can we use the pseudo-inverse to obtain the desirable end-effector motion and the obstacle velocities in the same way? If not, what method should be used?

Question:

Refer to the manipulator as below

1.) θ_1 = ?
2.) θ_2 = ?
3.) What does the configuration space look like when R and θ are constants (R = 5 , θ = 37 ° and ℓ_1 = 4, ℓ_2 = 3). The example figure as below.





This question uses Paper 6, Path Planing and the Topology of Configuration Space

Q. 1: ..... 1 point
Why is it important to know object connectivity in configuration space?

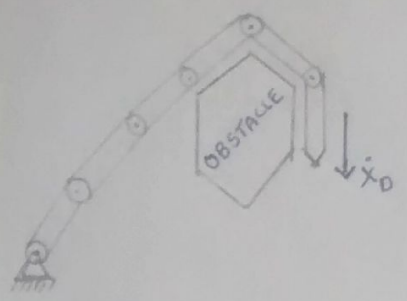
Q. 2: ..... 1 point
Given a 2 link manipulator with $l_1 = l_2$ and the arm pointed along the $+x_1$ axis when $\theta_1 = 0, \theta_2 = 0$, do the 2 obstacles (given in cartesian space) connect in configuration space? If so, in what configuration?

- Obstacle 1 $= [1, 0]^T$, Obstacle 2 $= [0.13397, 0.5]^T$
- Obstacle 1 $= [0.8, 0.7071]^T$, Obstacle 2 $= [0.13397, 0.5]^T$

a) What do the terms task abort distance, unity gain distance and sphere of influence mean? Explain with the help of diagram of a manipulator arm and obstacle.

b) There are different methods of obstacle avoidance. What method (in terms of property of the manipulator) is the basis for obstacle avoidance mentioned in the paper “Obstacle Avoidance for Kinematically Redundant Manipulators in Dynamically Varying Environments”? Explain.

A) Consider the configuration of the robot in the diagram below.



Primary Goal : Achieve end-effector velocity ($\dot{x}_D$) as shown in the diagram

Secondary Goal : Avoid obstacle.

- Assume you are using the formulation given below to achieve the primary and secondary goal for the robot configuration shown in the diagram.

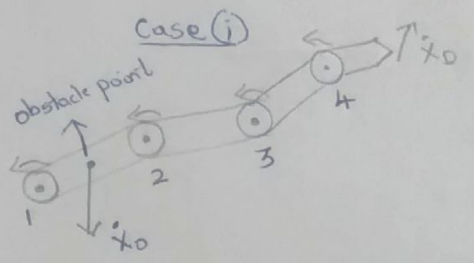
$$\dot{\theta} = J_E^{+}\dot{x} + \left[J_0 \sum_{i=r+1}^{n} \hat{v}_i \hat{v}_i^{T}\right]^{+} (\dot{x}_0 - J_0 J_E^{+} \dot{x}_E)$$

Describe what would happen to the robot shown in the diagram if you are using the above formulation

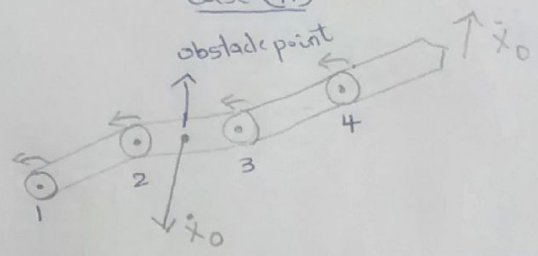
B) Describe an alternative formulation to the above one, which can work better. Justify your answer on how this new formulation works.

c) Consider the robot configuration below. This is a PLANAR robot.

Case (i)



Case (ii)



Assume that the obstacle point is at a distance <u>less than</u> d_{TA} to the obstacle. Can the formulation you used in part (B) move the obstacle point EXACTLY along $\dot{x}_0$ (as shown in diagram). Consider case (i) and case (ii) separately and give your answers for each case.

From Article 5, we know there are two methods get get the obstacle avoidance solution the first is $\begin{bmatrix} J_e \\ \cdots \\ J_o \end{bmatrix} \dot{\theta} = \begin{bmatrix} \dot{x}_e \\ \cdots \\ \dot{x}_o \end{bmatrix}$ and the second one is $\dot{\theta} = J_e^+ \dot{x}_e + \alpha_\eta [J_0 (I - J_e^+ J_e)]^+ (\alpha_0 \dot{x}_0 - J_0 J_e^+ \dot{x}_e)$,

(1) please explain one of the short coming of method one
(2) Suppose we know J_e J_o $\dot{x}_e$and $\dot{x}_o$ please give a brief derivation about how to get the second equation $\dot{\theta} = J_e^+ \dot{x}_e + \alpha_\eta [J_0 (I - J_e^+ J_e)]^+ (\alpha_0 \dot{x}_0 - J_0 J_e^+ \dot{x}_e)$,.

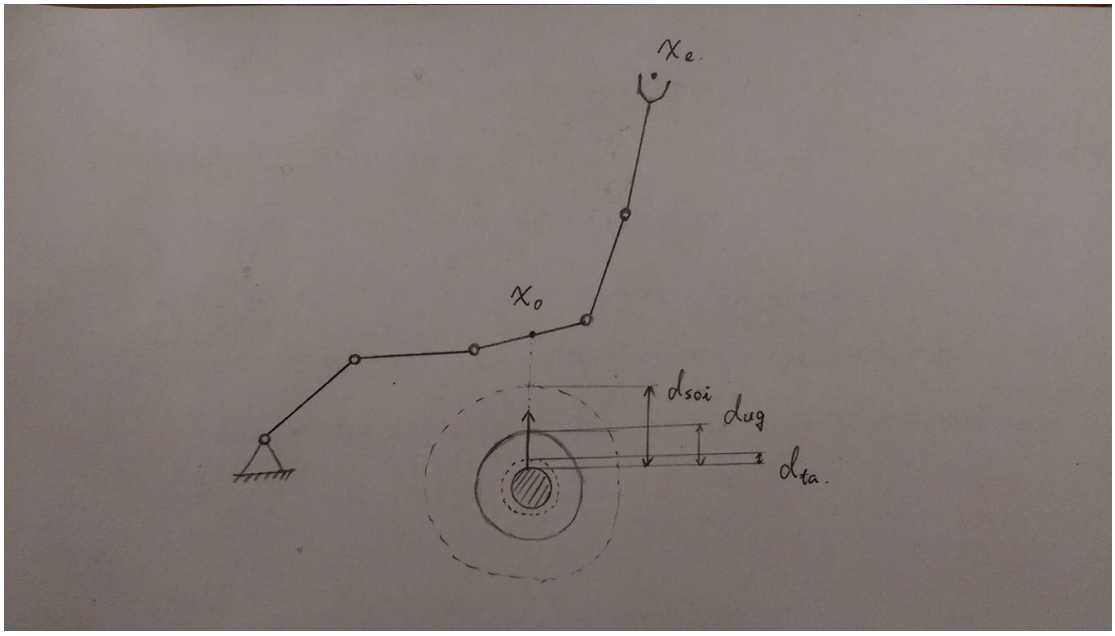
1. Write out the equation used for DLS (Equation form: MIN()). If there is a preferred joint velocity $\dot{\theta}^*$, what should be this equation like? Write out the solution for obstacle avoidance without gain terms and explain every term in this solution.

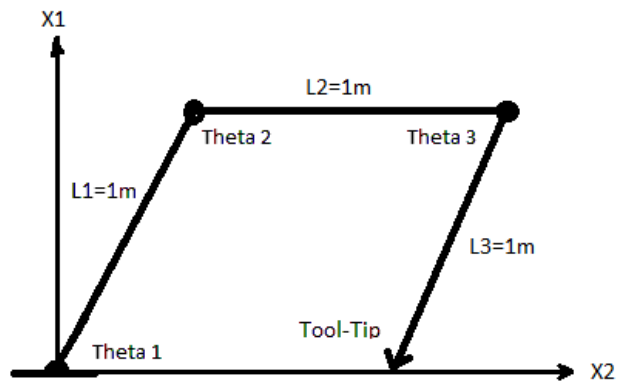
If we have a preferred joint velocity, explain how we can take that into account while maintaining proper tracking of the end effector.

Given the following, find $\dot{\theta}_{e_{n+1}}$ that attempts to minimize the change in acceleration in the arm without sacrificing tracking for $\dot{x_e}$. Hint: Use the desired $\dot{\theta}^*$ projected onto the nullspace of J_e.

$$\dot{\theta}_{e_n} = \begin{bmatrix} 4/3 \\ 2 \\ 2/3 \\ 3/7 \\ 1 \end{bmatrix} \quad \dot{x_e} = \begin{bmatrix} 12 \\ 13 \\ 14 \end{bmatrix} \quad J_e^{3x5} = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 \end{bmatrix}$$

As figure shown, the obstacle is moving close to the robot arm. Determine the values of α_η and α_0, and describe the movement of the robot arm with different distance between x_o and the top of the obstacle. (Hint: Figure 2 in paper 5)





Theta 1 = 1.0472 rad
Theta 2 = -1.0472 rad
Theta 3 = -2.0944 rad

Jacobian= $\begin{bmatrix} 0 & 0.866 & 0.866 \\ 1 & 0.5 & -0.5 \end{bmatrix}$

Tool Tip position = (1, 0) meter

a. Find below which Delta Theta $\Delta\theta$ is a Null-motion solution for the 3DOF planar robot.
(Which Delta Theta that keeps tool tip at the same position when $\Delta\theta$ added to Theta 1, 2 & 3)

1. $\Delta\theta$ = (-0.4 0.4 -0.4)
2. $\Delta\theta$ = (0.75 0.75 0.75)
3. $\Delta\theta$ = (1 -1 1)

b. Generate one Delta Theta $\Delta\theta$ that keeps tool tip at the same position (null motion) and briefly write the steps that you use to calculate $\Delta\theta$.

Q (i) What is the physical interpretation of the equation

$$\dot{\theta} = J_e^{+} \dot{x}_e + [J_0 (I - J_e^{+} J_e)]^{+} (\dot{X}_0 - J_0 J_e^{+} \dot{x}_e)$$

(ii) When can gaussian elimination be used to solve a part of the above equation?

(iii) Perform gaussian elimination to find x_1, x_2, x_3

$$\begin{bmatrix} 0 & 2 & 1 \\ 1 & -2 & -3 \\ -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -8 \\ 0 \\ 3 \end{bmatrix}$$

A redundant robot is required to satisfy three tasks with the order of priority. The first-priority subtask is to satisfy the specific end-effector velocity $\dot{x}_e$. The second-priority subtask is to satisfy the obstacle avoidance point velocity $\dot{x}_o$ to avoid collisions with obstacles. The third-priority subtask is to minimize a dexterity criterion—condition number κ.

Please following the steps in paper 5 (equation (2), (3), (4), (6), (7), (8), (9)) and extend the equation (9) to three subtasks. Finally you should give an equation like (9) to solve the inverse kinematics considering these three subtasks with the order of priority.

Homework Problem 5

$$\dot{\theta} = J_e^+ \dot{x_e} + [J_0(I - J_e^+ J_e)]^+ (\dot{x_0} - J_0 J_e^+ \dot{x_e}) \qquad (1)$$

$$\dot{\theta} = J_e^+ \dot{x_e} + \alpha_\eta [J_0(I - J_e^+ J_e)]^+ (\alpha_0 \dot{x_0} - J_0 J_e^+ \dot{x_e}) \qquad (2)$$

$$\dot{\theta} = J_e^+ \dot{x_e} + \alpha_1 \left(\frac{d_2}{d_1}\right) h_1 + \alpha_2 \left(\frac{d_2}{d_1}\right) h_2 \qquad (3)$$

(1). Brief explain the physical interpretation for each of the terms in the equation (1)
(2). What is the difference between the equation(1) and (2), how the equation(2) works?
(3). Why do we use the equation(3).

Answer 2 questions about obstacle avoidance below:

a) For the equation below that is used to describe early solution of redundant manipulators:

$$\dot{\theta} = J^{-1}\dot{x} + (I - J^{-1}J)z \qquad (1)$$

Compared with inverse kinematics jacobian equation:

$$\dot{\theta} = J^{-1}\dot{x} \qquad (2)$$

What is the function of the second term in equation (1)? Why is it necessary?

b) In order to get effective obstacle avoidance in the dynamical environment, equation (1) is developed into an equation as following:

$$\dot{\theta} = J_e^{-1}\dot{x}_e + [J_0(I - J_e^{-1}J_e)]^{-1}(\dot{x_0} - J_0 J_e^{-1}\dot{x_e}) \qquad (3)$$

How can you improve the equation (3) to make sure a continuity when the object moves from far to close? Briefly explain your answer.