Assignment 4 (Due in two weeks):

**Problem 1:**

For an image \( x = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \) and orthogonal transformation matrix \( A = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \). Find: (a) the basis images for this transform, (b) the transformed image, and (c) show the inverse mapping yields the original image.

**Problem 2:**

Find the basis images of a 2-D DCT and size N=2. Describe your results. You can verify your results using MATLAB if needed.

**Problem 3:**

Let \( x(n) \) be a zero-mean wide sense stationary (WSS) random process (rp) with covariance function \( r_s(m) = \mathbb{E}[x(n)x(n-m)] = \sigma^2 \rho^{|m|} \) with \( |\rho|<1 \), and let \( \mathbf{x} = [x(n) \ x(n-1)]^T \) be a 2x1 random vector of this rp. Determine

(a) Covariance matrix of \( \mathbf{x} \) and its eigenvalues and eigenvectors.

(b) Principal components of this vector process and comment on the results.

(c) If the signal is corrupted by noise i.e. \( y(n)=x(n)+\eta(n) \) where \( \eta \) is a zero-mean white Gaussian noise with unit variance, what are the changes to your results?

**Problem 4:**

The 1-D Hadamard transform (HT) of signal \{x(n)\} and its inverse are given by

\[
X(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x(n)(-1)^{b(k,n)} \quad , k \in [0, N-1]
\]

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\]

where \( b(k,n) = \sum_{i=0}^{p-1} k_i n_i \) and \( n_i \) and \( k_i \) are \( i^{th} \) bits in binary representations of indices \( n \) and \( k \), respectively, i.e. \( k = k_0 + 2k_1 + \ldots + 2^{p-1} k_{p-1} \) with \( k_0 \) being the LSB and \( k_{p-1} \) the MSB, and similarly for \( n \).

Represent this transform pair in the unified matrix-vector form and examine the properties (like those in Problem 1) of the Hadamard matrix, \( H \). If the signal vector \( \mathbf{x} \) is expressed in terms of the linear combination of some basis vectors with coefficients or weights \( X(k) \), what are these basis vectors for HT?