Digital Image Processing
Lectures 17 & 18

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Area 2: Image Enhancement

Goal:

To manipulate an image in order to improve its visual appearance or "quality" and prepare it for high level processing and analysis by an automated system or a human expert.

The process involves devising a sequence of operations for various tasks e.g., removing the effects of undesirable noise/interference, edge sharpening, pseudo-coloring, interpolation and magnification, multi-spectral processing, and geometrical operations.

The operations are ad-hoc i.e. that they do not consider any prior knowledge about the imaging systems. An acceptable result is typically obtained after several trials that may involve fine-tuning the parameters of a particular operation or even trying different algorithms. The criterion for converging to an acceptable result is based upon the subjective judgement of the human observers.

Point Operations

The operations used can be categorized into five general categories:

- Point operations
- Neighborhood or Spatial operations
- Transform-based operations
- Multi-spectral operations
- Geometric operations.

Point Operations

These memory-less operations are used for contrast enhancement by mapping the pixel intensities of an image. This is done irrespective of the spatial location of the pixels.

There are two general types of point operations namely grey-scale mapping and histogram modeling schemes.

Grey-Scale Mapping

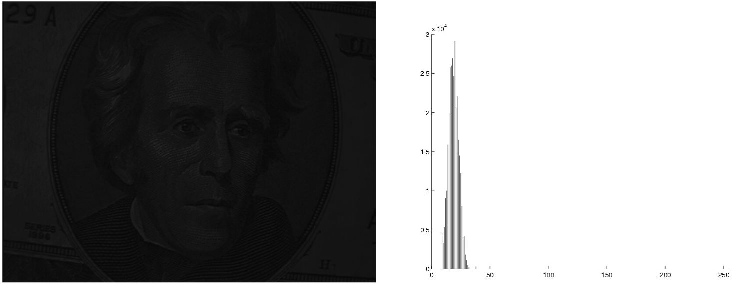
The grey-scale mapping transforms the original image pixel intensity $x = \{x_k, k \in [0, K-1]\}$ into the transformed image pixel intensity $y = \{y_l, l \in [0, L-1]\}$ using

$$y = f(x)$$

where $f(\cdot)$ is a *user-defined* (based upon the histogram of the original image) grey-scale mapping that can be a linear or nonlinear, one-to-one or many-to-one function. Since typical digital images have finite number of grey levels (e.g., K=256), this mapping can easily be implemented using a look-up table operation.

Figures 1(a) and 1(b) show a low contrast image taken under poor lighting condition and its corresponding histogram, respectively. The largest intensity in this image is $x_K = 51$ which explains why the image is predominantly dark. To stretch the contrast we use a simple linear mapping

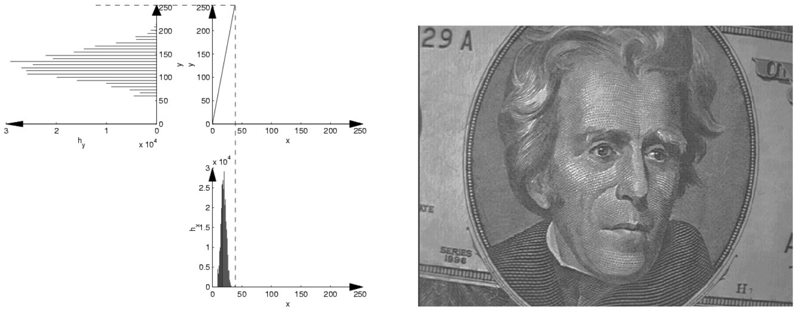
$$y = ax, \qquad a > 1$$



(a) (b)

Figure 1

The effect of this simple mapping, for a=5 (chosen to make mapping one-to-one), is shown on the histogram in Figure 2(a). Figure 2(b) shows the resultant image that clearly exhibits better visual appearance than that in Figure 1(a). Although histogram is stretched into a wider grey level range, there are a large number of bins with zero values.



(a) (b)

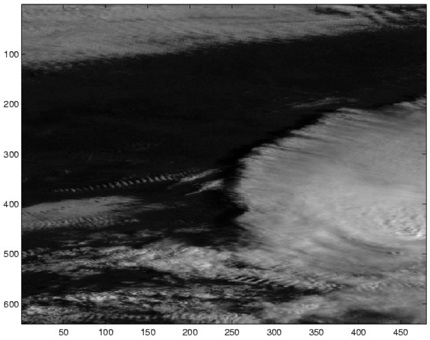
Figure 2

To generalize this idea we use piece-wise linear grey-scale transformations for a wide variety of tasks including: contrast stretching, clipping and thresholding, segmentation and digital negative operations.

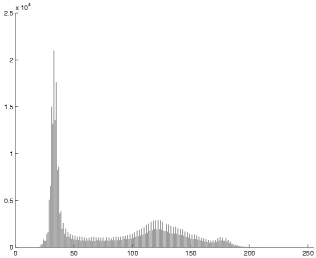
A typical piece-wise linear mapping with three segments with slopes a, b and c is

$$y = \begin{cases} ax & 0 \leq x < x_{k_1} \\ b(x - x_{k_1}) + l_1 & x_{k_1} \leq x < x_{k_2} \\ c(x - x_{k_2}) + l_2 & x_{k_2} \leq x \leq x_K \end{cases}$$

where x_K is the maximum intensity in the original image. The number of line segments is decided based upon the shape and number of "modes" of the histogram. For instance, Figure (1) represents has a unimodal histogram while the GOES 8 visible satellite image in Figure 3(a) has a bimodal histogram (see Figure 3(b)) corresponding to two separate intensity regions. The values x_{k_1} and x_{k_2} are typically placed at the valleys of the histogram or boundaries of the modes.



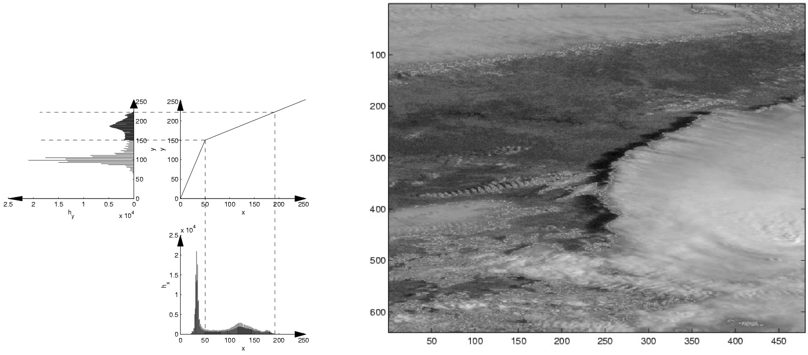
(a)



(b)

Figure 3

Line segments with slopes greater than one result in contrast stretching in the corresponding region while slope values less than one lead to compression of the intensity region (see Figure 4(a)). The resultant image is shown in Figure 4(b) which exhibits a better definition of cloud shadow areas and low intensity features.



(a) (b)

Figure 4

Effect on Histogram

The relationship between the histograms of original image, $h_X(x)$, and mapped image, $h_Y(y)$, as a function of transformation, $f(\cdot)$, can be determined by viewing these images as discrete random variables (r.v.) X and Y with possible values x_k and y_k, respectively.

For discrete r.v., X, the histogram is

$$h_X(x) = \sum_{k=0}^{K-1} N_k \delta(x - x_k)$$

where N_k is the number of pixels with intensity x_k. After the mapping

$$y_l = f(x_k)$$

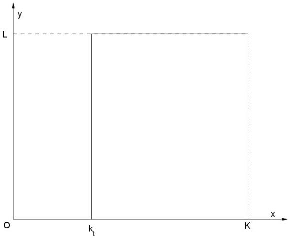
For one-to-one mapping $N_k = N_l$ since all the pixels with intensity x_k in image X are mapped to those with intensity y_l in image Y. Thus, the histogram of the enhanced image becomes

$$h_Y(y) = \sum_{l=0}^{L-1} N_l \delta(y - y_l)$$

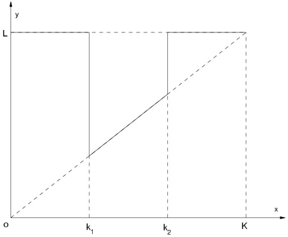
where $L = K$. In this case, the histogram of the contrast stretched image has the same number of bins and amplitudes as in that of the original image. However, the bins are placed at $y = y_l$, which are obviously determined by mapping $f(\cdot)$. When contrast compression or clipping occurs, the number of bins reduces in the compressed region since the mapping becomes many-to-one.

Special Cases of Grey-Scale Mapping

Thresholding: Figure 5(a) shows the mapping for thresholding operation where threshold at x_{k_t}. Pixel intensity values below x_{k_t} are mapped to zero (black) while values above this level are transformed to 255 (white) or any other desired intensity. The choice of x_{k_t} is made based upon examining the histogram of the original image. Thresholding operation is used in many applications e.g., hand writing character recognition, signature and finger print identification. Figure 6(a) shows a finger print image and binary image obtained by thresholding this image at $x_{k_t} = 80$ is shown in Figure 6(b).

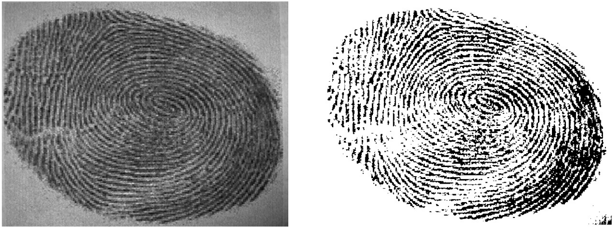


(a)



(b)

Figure 5

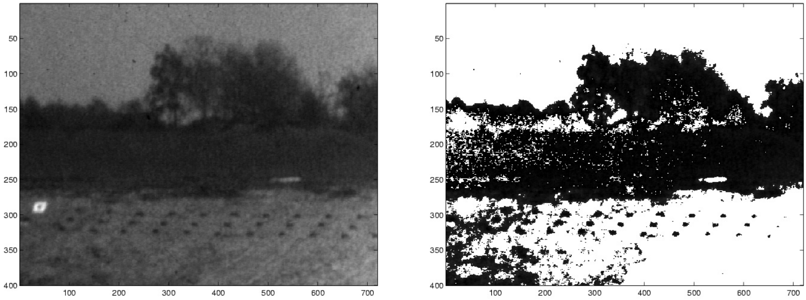


(a) (b)

Figure 6

Segmentation: Figure 5(b) shows a mapping for segmentation of pixel intensities in the range x_{k_1} to x_{k_2} from the rest of the grey levels. This mapping is useful when the desired features are known to lie in the segmented range. Examples of its applications include background removal and object detection and extraction.

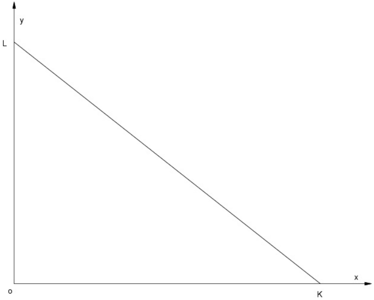
Figure 7(a) is an Infrared (IR) image of a minefield. Figure 7(b) shows its segmented version for $x_{k_1} = 50$ and $x_{k_2} = 80$ obtained from the histogram. As can be seen, the targets and some false detections are isolated from the background.



(a) (b)

Figure 7

Digital Negative: The digital negative image in Figure 8(b) is obtained by performing the grey scale mapping in Figure 8(a) to Lena image. This type of mapping is used in printing industry.

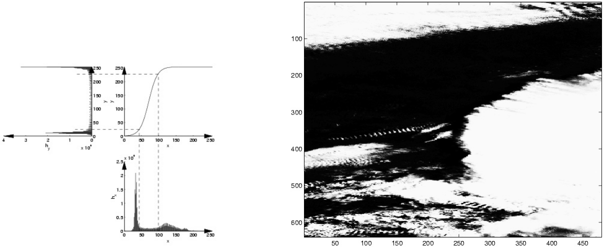


(a)

(b)

Figure 8

Nonlinear Mapping: Figure 9(a) shows the effect of a sigmoidal type non-linear mapping on the image of Figure 3(a) with bimodal histogram. Clearly, this mapping results in better separation of the modes or intensity regions though the contrast is reduced within each intensity region. The result of this non-linear mapping is shown in the image of Figure 9(b).



(a) (b)

Figure 9

A useful example of non-linear transformation is the logarithmic operation

$$y = a\log_{10}(b + |x|)$$

where a and b are some constants. This mapping compresses the dynamic range of the image by reducing the large intensity values and boosting the small intensities comparing to the large ones. A typical choice for b is 1 for zero intensity mapping.

Histogram Modeling

Histogram modeling methods slightly differ from grey-scale mapping methods in a way that the mapping functions are defined. Here, the user is not directly involved with the design of the explicit mapping, rather he/she desires to reshape the histogram to some predetermined distribution. These operations generally use nonlinear transformations that may not be explicitly *visible* to the user. However, the user sees the outcome based upon the specified desired histogram. Two types of histogram modeling methods are: histogram equalization and histogram specification (general).

Histogram Equalization

The goal of histogram equalization is to devise a mapping that yields an image with a uniformly distributed histogram.

If we assume that image intensity is continuous represented by continuous r.v., X, then, the cumulative distribution function (*CDF*) of the original image can be used as the mapping function to yield a uniformly distributed image.

If the probability density function (*PDF*), or the normalized histogram, $p_X(x)$ is given, then the *CDF*, $P_X(x)$ given by

$$y = f(x) = P_X(x) = \int_0^x p_X(\alpha)d\alpha$$

is the solution to this mapping problem (Note this mapping function is single-valued and monotonically increasing hence satisfies all the necessary conditions). To verify check

$$p_Y(y) = \frac{p_X(x)}{|f'(x)|}\Big|_{x=f^{-1}(y)} = 1$$

A similar idea can be applied to digital images. The *CDF* of the digital image is

$$P_X(x) = \int_0^x p_x(\alpha)d\alpha = \sum_{k=0}^{K-1} \frac{N_k}{N} u(x - x_k)$$

where $u(x)$ represents a unit step function. Clearly, the *CDF* for discrete r.v. is a stair-step function which starts from 0 and goes to its final value of 1, since $\sum_{k=0}^{K-1} \frac{N_k}{N} = 1$.

Now, if we use this *CDF* as the grey-scale mapping, for $x \in [x_i, x_{i+1})$ the intensity levels of the mapped image are obtained using

$$y'_i = f(x) = P_X(x) = \sum_{k=0}^{i} \frac{N_k}{N}$$

There are two problems. First, since $0 \leq P_X(x) \leq 1$ the mapped values y'_i are normalized and do not represent the actual intensity levels. Also, since $P(X = x_i) = P(Y' = y'_i)$, the PDF of the mapped image is a scaled version of the original PDF with the same number of bins and same amplitudes (*i.e.* relative frequencies) but with different spacing between the bins that depend on $P_X(x)$ mapping.

These problems can be overcome by performing a uniform quantization mapping of the form

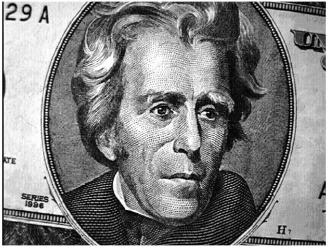
$$y_l = Int\left[\frac{(y'_l - y'_{min})}{1 - y'_{min}}(y_L - y_{L_0}) + y_{L_0} + 0.5\right]$$

where y'_{min} is the smallest value of y'_l, y_{L_0} and y_L are the smallest and largest intensity allowed in the mapped image y and $Int[x]$ represents integer part of quantity x.

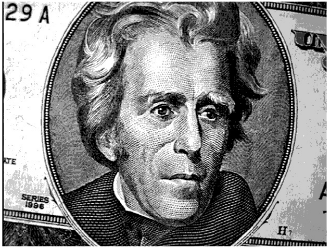
The number of bins in the histogram of y_L is determined by $(y_L - y_{L_0})$. By decreasing the number of bins, the histogram of the mapped image becomes closer to a uniform one. Of course, this leads to loss of grey level resolution which cannot be afforded in images with great details. The appropriate number of bins is typically determined experimentally by observing the visual appearance of the equalized images.

Figures 10(a),(b),(c), and (d) show equalized versions of the image in Figure 1(a) and their histograms for 60 and 8 number of bins, respectively. The improvement in the visual appearance of these images over the original one is clearly noticeable. Loss of grey level resolution leads to some patchiness distinguishable in the mapped image more so in Figure 10(b).

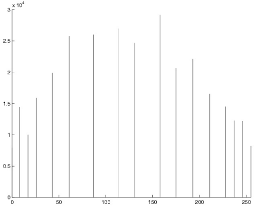
Notice that unlike the grey-scale mapping histogram equalization is more *automatic* i.e. it does not rely on user's involvement and fine tuning of the mapping parameters. As a consequence of this *non-interactive* property, user does not have the option of trying a variety of mapping in order to arrive at the most visually pleasing final image.



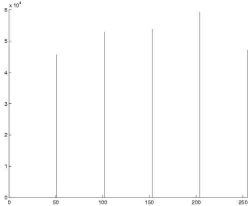
(a)



(b)



(c)



(d)

Figure 10

Histogram Specification

In this scheme the user can specify different types of desired histograms and interactively examine the effects of the corresponding transformations on the image. The types of the desired histogram are determined depending on the characteristics of the histogram of the original image and the specific goals of the enhancement procedure.

Again let us start with the continuous case and let X and Y be two continuous r.v.'s representing the intensity of the original and desired images, respectively. Now, given the original image, X, and its PDF, $p_X(x)$, and the desired PDF, $p_Y(y)$, our goal is to identify a grey-scale mapping that transforms the original image to a new image, Y, with the desired PDF (or histogram) $p_Y(y)$.

If we hypothetically assume that the desired image, Y, is available, then both the original and desired image can be histogram-equalized i.e.

$$w = f(x) = P_X(x) = \int_0^x p_X(\alpha)d\alpha$$

and

$$z = g(y) = P_Y(y) = \int_0^y p_Y(\beta)d\beta$$

The transformed images w and z that have identical uniformly distributed PDF's can be regarded as two realizations of the same process. However, in this case since x and y are related through an unknown transformation we can assume that $w = z$. As a result, the desired image y can be obtained by using inverse transformation

$$y = g^{-1}(f(x))$$

or

$$y = P_Y^{-1}(P_X(x))$$

Since $g(\cdot)$ is single-valued and monotonic, the above transformation gives the unique mapped values. This solution relies on the existence of analytical inverse function $g^{-1}(\cdot)$ which may not be available for certain functions e.g., Gaussian. However, this problem does not exist for the discrete case.

Let X and Y be discrete r.v.'s representing the original and desired images, respectively. Given the original and desired histograms we can write

$$w = P_X(x) = \sum_{k=0}^{K-1} \frac{N_k}{N} u(x - x_k)$$

and

$$z = P_Y(y) = \sum_{l=0}^{L-1} \frac{M_l}{N} u(y - y_l)$$

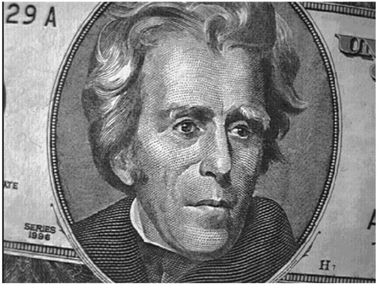
where N_k and M_l are number of pixels in images X and Y. Now the equivalent of inverse mapping is performed using the following simple algorithm.

Let $w_i = \sum_{k=0}^{i} \frac{N_k}{N}$ and $z_i = \sum_{l=0}^{i} \frac{M_l}{N}$ be the equalized levels of W and Z images. For a chosen level w_i, let j be the smallest possible index for which

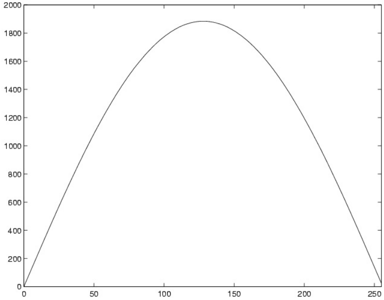
$$z_j \geq w_i$$

then the level of the desired image corresponding to the original grey level x_i is y_j *i.e.* all the pixels (N_i) with grey level x_i are mapped to intensity level y_j in the desired image. It is very important to note that this mapping can be many-to-one. In other words, several intensity levels in the original image can be mapped to one level in the desired image.

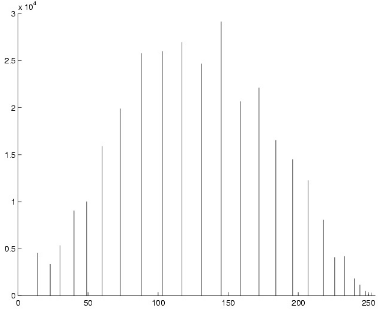
Figure 11(a) shows the histogram specified version of the image in Figure 1(a) for a specified histogram in Figure 11(b) which is $1/2$ cycle of a sinewave. The histogram of the resultant image in Figure 11(a) is given in Figure 11(c) which is similar in shape to the specified desired in Figure 11(b). Visual comparison of the image in Figure 11(a) with those in Figure 10(a) shows much better enhancement without introducing any undesirable artifacts.



(a)



(b)



(c)

Figure 11

Example:

Histogram of a continuous image, X, is given by $h_X(x) = Ae^{-x}$ where $x \in [0, b]$ and A is a normalization factor. We would like to map this image to a new image, Y, with the desired histogram $h_Y(y) = Bye^{-y^2}$ with range of values $y \in [0, b]$ and B is the normalization factor. Find the transformation function $y = T(x)$ that accomplishes this task.

Solution:

This is histogram specification problem. First, equalize both images.

$$w = f(x) = P_X(x) = \frac{1}{AR_1}\int_0^x Ae^{-\alpha}d\alpha = \frac{A}{AR_1}(1 - e^{-x})$$

where AR_1 is area under $h_X(x)$ i.e. $AR_1 = A\int_0^b e^{-\alpha}d\alpha = A(1 - e^{-b})$. Thus,

$$w = f(x) = \frac{(1 - e^{-x})}{(1 - e^{-b})}$$

For other image

$$z = g(y) = P_Y(y) = \frac{1}{AR_2}\int_0^y B\alpha e^{-\alpha^2}d\alpha$$

Let $\alpha^2 = \beta$ then

$$z = g(y) = \frac{1}{2AR_2}\int_0^{y^2} Be^{-\beta}d\beta = \frac{B}{2AR_2}(1 - e^{-y^2})$$

where $AR_2 = \frac{B}{2}\int_0^{b^2} e^{-\beta}d\beta = \frac{B}{2}(1 - e^{-b^2})$. Thus,

$$z = g(y) = \frac{(1 - e^{-y^2})}{(1 - e^{-b^2})}$$

Now, we should have $w = z$ which gives $y = T(x) = g^{-1}(f(x))$ or

$$y = \sqrt{ln\frac{1}{1 - z(1 - e^{-b^2})}}$$

But using $z = w = \frac{(1-e^{-x})}{(1-e^{-b})}$ we get

$$y = \sqrt{ln\frac{1}{1 - \frac{(1-e^{-x})(1-e^{-b^2})}{(1-e^{-b})}}}$$