EE507 – Plasma Physics and Applications

Homework #2

1. The ionosphere (altitude > 80 km) is composed mostly of a proton-electron plasma immersed in the Earth's magnetic field of about $3x10^{-5}$ T. How fast is the gravitational drift for each species on the lowest ionospheric layer?

2. An ion engine has a 1-T magnetic field, and a hydrogen plasma is to be shot out at an ***E*** x ***B*** velocity of 1000 km/sec. How much internal electric field perpendicular to ***B*** must be present in the plasma?

3. Suppose that a so-called Q machine has a uniform field of 0.2 T and a cylindrical plasma with $kT_e = kT_i = 0.2$ eV. The density profile is found experimentally to be of the form

$$n = n_0 \exp[\exp(-r^2/a^2) - 1]$$

   Assume the density obeys the Boltzmann relation $n = n_o \exp(e\phi/kT_e)$.
   a) Calculate the maximum v_e if a = 1 cm. (Hint: first find the radial electric field by finding the electric potential and calculating its gradient)
   b) Compare this with vg due to the earth's gravitational field.
   c) To what value can B be lowered before the ions of potassium (A=39, Z=1) has a Larmor radius equal to a?

4. An electron lies at rest in the magnetic field of an infinite straight wire carrying a current I. At $t = 0$, the wire is suddenly floated to a potential ϕ without affecting I. The electron gains energy from the electric field and begins to drift.
   a. Draw a diagram showing the orbit of the electron and the relative position of I, B, v_e, v_{grad} and v_{curv}.
   b. Calculate the magnitude of these drifts at a radius of 1 cm if I = 500 A, ϕ = 460V, and the radius of the wire is 1 mm. Assume that ϕ is held at 0 V on the vacuum chamber walls 10 cm away.

5. A 2-keV electron is injected into a magnetic field B = 1 mT at an angle of 87° with respect to ***B***, resulting in a helical path. Find
   a. Helix radius
   b. Distance between turns of helix
   c. Circular orbital velocity of the electron
   d. Axial velocity of the electro
   e. Electron's orbital frequency

6. If a particle is injected into a uniform magnetic field ***B*** at an angle $\theta = 90°$ with respect to ***B***, the particle will move in a circular path of radius *r*. However, if the particle has a component velocity in the direction of ***B*** (θ less than 90°), it will move in a helical path of constant radius with axis in the ***B*** direction (see previous problem). For a uniform B of infinite extent, the particle will continue to move along the helical path indefinitely. Suppose now that ***B*** is in the *z* direction and increases with increasing distance. This means that ***B*** not only has a B_z component, but also a radial component B_r. The Lorentz force $\boldsymbol{v} \times \boldsymbol{B}$ of the orbital velocity v_ϕ of the particle and the radial field B_r exerts a braking effect on the particle that results in a reduction of its velocity v_z until the particle stops and reverses direction or bounces back. The point at which the particle turns around is called the mirror point. Show that a particle with radius *r*, orbital velocity v_ϕ and axial velocity v_z, at a given instant, will stop and turn around in a distance

$$d = \frac{m}{er}\frac{v_z^2}{v_\phi}\frac{1}{(\partial B/\partial z)}$$

Where *e* and *m* are the charge and mass, and B_z is the axial component of the magnetic field.

7. If an electron in a helical path of 100 m radius, axial velocity $4\text{x}10^5$ m/s, and orbital velocity 10^6 m/s encounters a magnetic field that increases by 10^{-13} T/m, find
   a. The distance, and
   b. The time to the mirror point.