ECE507 - Plasma Physics and Applications

Lecture 2

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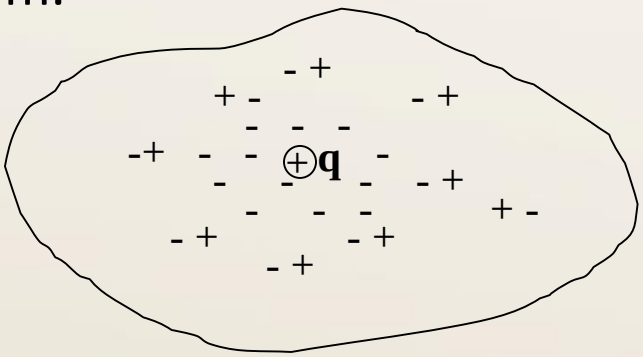
Debye Shielding

(see for example F. Chen, page 10)

A fundamental characteristic of a plasma is its ability to shield out electric potentials applied to it. A charge displacement occurs that shields the perturbation.

Let us assume that a positive test charge ⊕ q is introduced in a uniformly charged neutral medium.



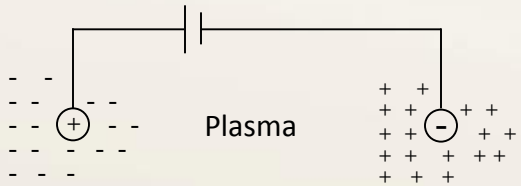
The coulomb potential near the test charge is: $V(r) = \dfrac{q}{4\pi\,\varepsilon_0\, r}$

In an ionized medium the field will not be strictly coulombic since the electrons will tend to move towards the test charge and "shield" it from the rest of the medium.

Debye Shielding

A fundamental characteristics of a plasma is its ability to shield out electric potentials that are applied

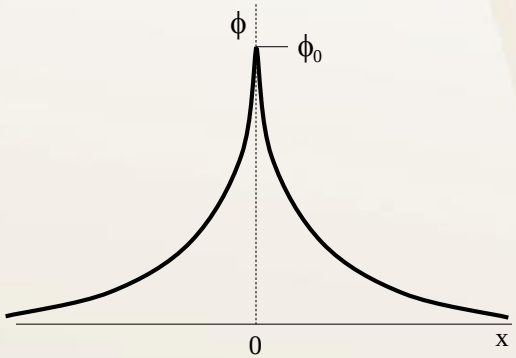
Assume an idealized plasma in which we introduce an electric field by inserting two charge balls connected to a battery



- If the plasma was cold and there would be no thermal motion, there would be as many particles in the cloud as in the balls and the field would be perfectly shielded (no field outside the clouds).
- Instead, if temperature is finite, particles at the edge of cloud, where the field is weak, have enough thermal energy to scape.
- The "edge" of the cloud occurs where the potential energy is approximately equal to the thermal energy kT.
- Shielding is not complete and potentials of the order kT/e can leak into the plasma.

Debye Shielding

To compute the approximate thickness of the cloud, assume a potential ϕ held by a plasma grid



For simplicity assume ions are motionless $\frac{M}{m_e} = \infty$

Poisson's equation in one dimension is:

$$\varepsilon_0 \nabla^2 \varphi = \varepsilon_0 \frac{d^2\varphi}{dx^2} = -e\left(N_i - N_e\right) \quad (z = 1) \qquad 1.19$$

If density far away $\longrightarrow$ $N_i = N_\infty$

Debye Shielding

- In the presence of a potential energy q ϕ the electron distribution function is:

$$f(v) = A \exp\left[\left(-\tfrac{1}{2} m v^2 + q\varphi\right) / kT_e\right] \qquad 1.20$$

This equation says there are fewer particles where the potential is large, since not all particles have enough energy to get there

- Integrating *f(v)* over *v*, setting *q = -e* , and noting $N_e(\phi \to 0) = N_\infty$

$$\left.\begin{aligned} N_e &= N_\infty \exp\left(e\varphi / kT_e\right) \\ N_i &= N_\infty \end{aligned}\right\}$$

$$\varepsilon_0 \frac{d^2\varphi}{dx^2} = \overbrace{-e\left(N_i - N_e\right)} = e N_\infty \left\{\left[\exp\left(\frac{e\varphi}{kT_e}\right)\right] - 1\right\} \qquad 1.21$$

Debye Shielding

- In regions where $\left[e\varphi / kT_e\right] << 1$

$$\varepsilon_0 \frac{d^2\varphi}{dx^2} = e N_\infty \left[\frac{e\varphi}{kT_e} + \tfrac{1}{2} v \left(\frac{e\varphi}{kT_e} \right)^2 + \ldots \right] \qquad 1.22$$

- No simplification is possible near grid where field is large, but this region is thin and does not contribute much to the cloud width. Keeping only linear terms we obtain

$$\varepsilon_0 \frac{d^2\varphi}{dx^2} = \frac{N_\infty e^2}{KT_e} \varphi \rightarrow \varphi = \varphi_0 \exp\left(- \frac{|x|}{\lambda_D} \right) \qquad 1.23$$

$$\boxed{\lambda_D = \left[\frac{\varepsilon_0 kT_e}{N_e e^2} \right]^{1/2}} \quad \text{Debye length} \qquad 1.24$$

Debye Shielding

$$\boxed{\lambda_D = \left[\frac{\varepsilon_0 \, kT_e}{N_e \, e^2}\right]^{1/2}}$$ Debye length

Notice:

- As N_e increases λ_D decreases, as expected since each layer will contain more charge
- As kT_e increases λ_D increases, as without thermal agitation the cloud would collapse into infinitely thin layer.
- Electrons are the more mobile particles – they generally do the shielding by moving and creating a surplus of deficit of negative charge

$$\lambda_D\,(m) = 7430\left(\frac{kT_e}{N_e}\right)^{1/2}$$ Useful form (kT_e in eV)

Example:

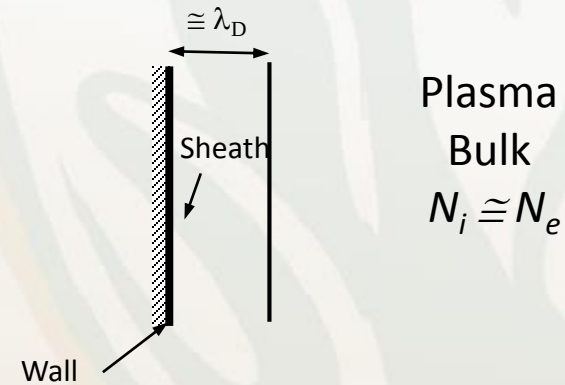
$$10\ eV \quad N_e = 10^{13}\ cm^{-3} = 10^{19}\ m^{-3} \longrightarrow \lambda_D = 7.43\ 10^3\ (10\,10^{-19})^{1/2} = 7.43\,10^3\ 10^{-9}\ m =$$

$$= 7.43\ 10^{-6} = 7.43\ \mu m$$

Quasi neutrality

If the dimensions *L* of a system are $L >> \lambda_D$, then whenever local concentrations of charge arise or external potentials are introduced, they are shielded in a distance short compared with *L*, leaving the bulk of the plasma free of large electric fields.

Outside of the sheath on the wall or on an obstacle, $\nabla^2 \phi$ is small and $N_i \cong N_e$ in typically, better in one part in 10^6 !



This is, the plasma bulk is "quasi-neutral"

For the shielding to be effective, there must be a large number of particles within the Debye sphere:

$$N_D = N_e \frac{4}{3}\pi\lambda_D^3 = 1.38\,10^6 \frac{T^{3/2}}{N_e^{1/2}}$$

Where $[T]$ = K, $[N_e]$ = m^{-3}

Example:

- Glow discharge

$$T_e = 10\ eV \quad N_e = 1\,x\,10^{13}\ cm^{-3} \Rightarrow N_D = 1.7\ 10^4 >> 1$$

$$T_e = 1000\ eV \quad N_e = 1\,x\,10^{21}\ cm^{-3} \Rightarrow N_D = 1.7\ 10^3 >> 1$$

Quasi neutrality

Notice that we arrived to the definition of λ_D assuming a Maxwellian distribution. However in some types of plasmas, such as in a solid Electrons obey Fermi-Dirac statistics and the screening length is the Thomas-Fermi screening length

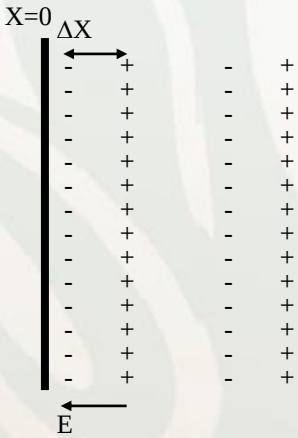
$$\lambda_{TF} = \left(\frac{\pi}{3\,N_e}\right)^{1/6} \frac{\mathrm{h}}{2\,\varepsilon_0\,m_e^{1/2}}$$

which can be many orders of magnitude larger than λ_D, and thus the electrons tend to decouple from the ions and do not screen the inter-ion coulombic potential. The Fermi degeneracy parameter determines the type of statistics that will describe the electrons in the plasma.

$$\theta = \frac{2\,m_e\,kT_e}{\mathrm{h}^2}\left(3\pi^2\,N_e\right)^{-2/3}$$

Fermi degeneracy parameter. $\theta << 1$ implies a degenerate case and the electrons are quantum particles subject to Fermi statistics.

Oscillation of plasma electrons: the plasma frequency



We can estimate a typical time for the response of a plasma to on external field

If at a given time all the electrons are shifted by a distance $\Delta x = x$ to the left, this creates an electric field whose strength is given by the Poisson equation:

$$\frac{d^2V}{dx^2} = \frac{\rho}{\varepsilon_0} \longrightarrow \frac{dE}{dx} = \frac{(N_i - N_e)}{\varepsilon_0} e \longrightarrow E = \frac{e\, N_e\, x}{\varepsilon_0}$$

The equation of motion for the electrons

$$m_e \frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -e\,E = -\frac{e^2 N_e}{\varepsilon_0}\, x \longrightarrow \frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -\frac{e^2 N_e\, x}{\varepsilon_0\, m_e} = -\omega_p^2\, x$$

The equation of motion predicts an oscillatory motion with frequency ω_p

$$\omega_p = \left[\frac{N_e\, e^2}{\varepsilon_0 m_e}\right]^{1/2}$$

where ω_p is called the Plasma Frequency, or Langmuir Frequency.

Plasma frequency

The time response for the plasma to an external signal is

$$\tau = \omega_p^{-1}$$

Thus the two fundamental parameters of ideal quasi-neutral plasmas are:

$$\lambda_D = \left(\frac{kT_e \, \varepsilon_0}{N_e \, e^2} \right)^{1/2}$$

Debye length

$$\omega_p = \left[\frac{N_e}{\varepsilon_0 m_e} \, e^2 \right]^{1/2}$$

Plasma frequency