TRANSVERSE ELECTROMAGNETIC WAVES IN PLASMAS

Propagate with cutoff at $\omega = \omega_p$

- At high frequency ions are immobile

Maxwell Equations For transverse waves of weak intensity

$$\nabla \times \mathbf{H} = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} - e n_o \mathbf{v}$$

$$\nabla \times \mathbf{E} = - \mu_0 \frac{\partial \mathbf{H}}{\partial t}$$

Maxwell-Euler equations describe the waves

$$\frac{\partial n_e}{\partial t} + n_o \nabla \cdot \mathbf{v} = 0$$

$$m \frac{\partial \mathbf{v}}{\partial t} = - \frac{kT_e}{n_o} \nabla n_o - e \mathbf{E}$$

- All non-linear terms can be neglected

- Only $J_T$ contributes to wave

- $(\nabla \rightarrow ik$ longitudinal)
Develop a wave equation

\[ \nabla \times \mathbf{H} = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} - \varepsilon_0 \mathbf{v} \]

\[ \nabla \times \mathbf{E} = -\varepsilon_0 \frac{\partial \mathbf{H}}{\partial t} \]

\[ \nabla \cdot \mathbf{E} = 0 \text{ and } \varepsilon_0 \mathbf{v} = \frac{\partial \mathbf{E}}{\partial t} \]

\[ \nabla \times (\nabla \times \mathbf{E}) = \nabla (\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} \]

\[ \nabla (\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{v}}{\partial t} \]

For \( \nabla \cdot \mathbf{E} = 0 \) and

\[ \frac{\partial \mathbf{v}}{\partial t} = -\frac{e \mathbf{E}}{m} \]

\[ \frac{\partial^2 \mathbf{E}}{\partial t^2} + \frac{e \varepsilon_0}{\varepsilon_0} \left( \frac{e \mathbf{E}}{m} \right) - \frac{1}{\varepsilon_0 \mu_0} \nabla^2 \mathbf{E} = 0 \]
TRANSVERSE ELECTROMAGNETIC WAVES IN PLASMAS (continuation)

\[
\frac{\partial^2 E}{\partial t^2} + \frac{en_o}{\varepsilon_o m} \left( \frac{eE}{m} \right) - \frac{1}{\varepsilon_o \mu_o} \nabla^2 E = 0
\]

Defining

\[
\omega_p^2 = \frac{e^2 n_o}{\varepsilon_o m}
\]
\[
c^2 = \frac{1}{\varepsilon_o \mu_o}
\]

Wave equation for transverse wave in a plasma

\[
\left( \frac{\partial^2}{\partial t^2} + \omega_p^2 - c^2 \nabla^2 \right) E(r, t) = 0
\]

For a plane wave

\[
E(r, t) = E_0 e^{-i(\omega t - k \cdot r)}
\]

Dispersion relation

\[
\omega^2 = \omega_p^2 + k^2 c^2
\]
TRANSVERSE ELECTROMAGNETIC WAVES IN PLASMAS (continuation)

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TRANSVERSE ELECTROMAGNETIC WAVES IN PLASMAS (continuation)

**Dispersion relation in plasma**

\[ \omega^2 = \omega_p^2 + k^2 c^2 \]

\[ k = \frac{\sqrt{\omega^2 - \omega_p^2}}{c} \]

**Underdense plasma**

\( \omega > \omega_p \)

\( k \) real

Wave propagates with phase velocity:

\[ v_\phi = \frac{\omega}{k} = \frac{c}{\sqrt{1 - \frac{\omega^2}{\omega_p^2}}} = \frac{c}{\sqrt{1 - n_e/n_c}} \]

Refractive index

\[ n \approx \frac{c}{v_\phi} \]

\[ n = \sqrt{1 - \frac{\omega_p^2}{\omega^2}} \] or

\[ n = \sqrt{1 - \frac{n_e}{n_c}} \]

**Overdense plasma**

\( \omega < \omega_p \)

\( k \) imaginary

Wave cannot propagate

For highly overdense

\( \omega^2 \ll \omega_p^2 \)

\[ k = i \frac{\omega_p}{c} \]

Penetration depth

\[ l = \frac{c}{\omega_p} \]
The plasma is dispersive

phase velocity

\[ v_\phi = \frac{\omega}{k} = \frac{c}{\sqrt{1 - \frac{\omega_p^2}{\omega^2}}} = \frac{c}{\sqrt{1 - \frac{n_e}{n_c}}} \]

- These velocities are not constant but change with \( \omega_p/\omega \) or \( n_e/n_c \)
- At low densities both \( v_\phi, v_g \) approach \( c \)
TRANSVERSE ELECTROMAGNETIC WAVES IN PLASMAS (continuation)

• Critical plasma density: frequency for which $\omega = \omega_p$ is the critical frequency

$$n_c \equiv \frac{\varepsilon_0 \omega^2 m}{e^2}$$

$$n_c = \frac{1.11 \times 10^{21} \text{ cm}^{-3}}{\lambda^2 (\mu\text{m})}$$

**TABLE 6.1.** Electron density, plasma frequency, critical photon energy for $\omega_c = \omega_p$, and critical wavelength for electromagnetic radiation.

<table>
<thead>
<tr>
<th>$n_e (e/cm^3)$</th>
<th>$\omega_p/2\pi$</th>
<th>$\hbar \omega_c$ (eV)</th>
<th>$\lambda_c$</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.00 \times 10^6$</td>
<td>8.94 MHz</td>
<td>33.5 m</td>
<td></td>
<td>Between AM and FM radio</td>
</tr>
<tr>
<td>$1.00 \times 10^{14}$</td>
<td>89.4 GHz</td>
<td>3.35 mm</td>
<td></td>
<td>Microwaves</td>
</tr>
<tr>
<td>$1.00 \times 10^{19}$</td>
<td></td>
<td></td>
<td>10.6 $\mu$m</td>
<td>CO$_2$ laser</td>
</tr>
<tr>
<td>$1.00 \times 10^{21}$</td>
<td>1.17 $\mu$m</td>
<td>1.06 $\mu$m</td>
<td></td>
<td>Nd laser</td>
</tr>
<tr>
<td>$1.60 \times 10^{22}$</td>
<td>4.86 mm</td>
<td>266 nm</td>
<td></td>
<td>4$\omega$ of Nd laser</td>
</tr>
<tr>
<td>$4.60 \times 10^{24}$</td>
<td>80.0 nm</td>
<td>15.5 nm</td>
<td></td>
<td>Ne-like Y laser</td>
</tr>
</tbody>
</table>
**Collisional Absorption:**

Now include collision between oscillating electrons and ions in momentum equation

\[
\frac{\partial v}{\partial t} = -eE - m\nu_{ei}v
\]

Where \(\nu_{ei}\) is the electron-ion collision frequency, \(mv\) is the electron momentum

For \(\nu_{ei} \ll \omega\) the dispersion relation becomes:

\[
\omega^2 = \omega_p^2 \left(1 - i \frac{\nu_{ei}}{\omega}\right) + k^2 c^2
\]

\[
\omega = \omega_r + i \omega_i
\]

\[
\omega_r^2 = \omega_p^2 + k^2 c^2
\]

\[
\omega_i \approx -\frac{\nu_{ei} \omega_p^2}{2\omega^2} = -\frac{n_e}{2n_c} \nu_{ei}
\]

The negative sign indicates damping

\[
k = k_r + ik_i
\]

\[
k_r = \frac{\sqrt{(\omega^2 + \omega_p^2)}}{c}
\]

\[
k_i = \frac{\nu_e \omega_p^2}{2v_g \omega^2}
\]
Collisional Absorption:

\[ I = E^2 = \left[ E_0 e^{-i(\omega t - kx)} \right]^2 \]

Then

\[ l_{abs} = \frac{1}{2k_i} = \frac{\omega^2 v_g}{\omega_p v_{ei}} = \frac{n_e v_g}{n_c v_{ei}} \]

Electron-ion collision frequency (Dawson et al)

\[ \nu_{ei} = \frac{e^4 Z n_e \ln \Lambda}{3(2\pi)^{3/2} \varepsilon_0 m^{1/2} (kT_e)^{3/2}} \]

Example: \( kT_e = 1\) keV, \( \lambda = 1.06 \) µm, \( Z = +14 \), \( n_e = \frac{1}{2} n_c \) \( \rightarrow l_{abs} = 130 \) µm
Resonance Absorption

It is possible to directly excite plasma waves through the resonance $\omega = \omega_p$ at the critical density.

- Occurs for p-polarized radiation.

$E$ field has component in direction of $\Delta n_e$ that tunnels into critical region.

Absorption peaks at $\approx 50\%$ at

$$(k_i \lambda)^{1/3} \sin \theta = 0.8$$

$\lambda$ = density scale length

For normal incidence ($\theta=0$) there is no axial component of $E$ to drive the resonance.

For glancing incidence tunneling field is weak.
Plasma Waves: Collective longitudinal waves that propagate naturally

- electron-acoustic waves (high frequency)
  (ion too massive to participate)
- ion-acoustic waves (low frequency)

From Maxwell-Euler equations we obtain the longitudinal wave equation for $n_e$ fluctuations

$$\frac{\partial^2}{\partial t^2} + \omega_p^2 - \alpha_e^2 \nabla^2 \right) n_e (\mathbf{r}, t) = 0$$

Electron sound speed

$$a_e = \left( \frac{\gamma kT_e}{m} \right)^{1/2}$$

Natural frequency of oscillation $\omega_p$

$$\omega_p = \left( \frac{e^2 n_o}{\varepsilon_o m} \right)^{1/2}$$
Plasma Waves

Assuming a solution

\[ n_e (r, t) = n_e e^{-i(\omega t - k \cdot r)} \]

for the electron wave equation

\[ \omega^2 = \omega_p^2 + k^2 a_e^2 \]

\( a_e \): electron sound speed

Dispersion diagram for naturally occurring waves in isotropic plasma

Fig. 6.7 Attwood
Non-Linear Processes in Plasmas

- In the Maxwell-Euler equations there are several terms involving the products of fields

\[
\nabla \times H = \varepsilon_0 \frac{\partial E}{\partial t} + \sum_j (n q v)_j \\
\nabla \times E = \mu_0 \frac{\partial H}{\partial t} \\
\frac{\partial n_j}{\partial t} + n_o \nabla \cdot (n_j v) = 0 \\
m_j n_j \left( \frac{\partial}{\partial t} + v \cdot \nabla \right) v = -\nabla P_j + q_j n_j (E + v \times B)
\]

- These non-linear terms create possibility of non-linear growth and frequency mixing.

- Two processes involving intense incident radiation are:
  - Stimulated Brillouin scattering (SBS)
  - Stimulated Raman scattering (SRS)
Non-Linear Processes in Plasmas

For example analyzing the electron current terms in:

\[ \nabla \times \mathbf{H} = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \sum_j (nqv)_j \]

\[ \mathbf{J}(\mathbf{r}, t) = -en_e(\mathbf{r}, t)\mathbf{v}(\mathbf{r}, t) \]

\[ J_e^{\omega_t - \mathbf{k}_1\cdot\mathbf{r}} = -e n_e^{\omega_2 t - \mathbf{k}_2\cdot\mathbf{r}} \mathbf{v} e^{\omega_3 t - \mathbf{k}_3\cdot\mathbf{r}} \]

A term to term match shows:

**Conservation of energy**

\[ \omega_1 = \omega_2 \pm \omega_3 \]

**Conservation of momentum**

\[ \mathbf{k}_1 = \mathbf{k}_2 \pm \mathbf{k}_3 \]

Fig. 6.10 Attwood
Non-linear Processes in Plasmas

Stimulated Scattering:

- An intense incoming wave $(\omega_1, \mathbf{k}_1)$ drives an existing plasma wave $(\omega_2, \mathbf{k}_2)$ out of the noise and stimulates it to grow.

- $(\omega_1, \mathbf{k}_1)$ scatters from $(\omega_2, \mathbf{k}_2)$ producing $(\omega_3, \mathbf{k}_3)$ (not all combinations satisfy conservation equation)

- when matching occurs the new scattered wave $(\omega_3, \mathbf{k}_3)$ interferes with $(\omega_1, \mathbf{k}_1)$ at $\omega_2=\omega_3-\omega_1$ $\mathbf{k}_2=\mathbf{k}_3-\mathbf{k}_1$ causing the initial noise $\omega_2, \mathbf{k}_2$ to grow in amplitude.

- The scattering and growth process continues.
Non-Linear Processes in Plasmas:

Stimulated Brillouin scattering (SBS)
- Scattering from low-frequency ion-acoustic wave
- Scattered wave \((\omega_B, k_B)\) at slightly shifted \(\omega\)

Stimulated Raman Scattering (SRS)
- Scattering from high-frequency electron acoustic wave
- Scattered wave \((\omega_R, F_R)\) at a substantially shifted \(\omega\).

Both "stimulated" because the electron or ion plasma wave is caused to grow out of noise through stimulation at the beat frequency \((\omega_3 - \omega_1)\),

Fig. 6.11 - Attwood
Threshold for Non-Linear Processes in Plasmas

- Nonlinear processes become significant when oscillation velocity \( v_{os} \) becomes important relative to random thermal velocity \( v_e \).

\[
\frac{v_{os}}{v_e} \geq \left( \frac{e^2 E^2}{m \omega^2 kT_e} \right) = \frac{1}{c} \frac{n_c kT_e}{n_c kT_e}
\]

Example: at \( \lambda = 1.06 \, \mu m \) (Nd-YAG laser), \( n_c = 1 \times 10^{21} \, cm^{-3} \)

\[
\Rightarrow \frac{v_{os}}{v_e} = 1 \quad \text{at} \quad I = 4.7 \times 10^{15} \, w/cm^2
\]

To avoid non-linear processes in laser-created plasmas use:

- short wavelengths
- low intensities
Laser wavelength trends in Laser-Created Plasmas

Plasma properties \((n_e, T_e, \nabla n_e, Z)\)

- Irradiation Wavelength
- Intensity
- Pulse length (throughout the absorption scale length \(l\))

Collisional Absorption favors shorter \(\lambda\)

\[ (\propto n_c, \quad n_c \propto \frac{1}{\lambda^2}) \]

- At high intensity ‘radiation pressure’ depresses sub-critical density, reducing absorption

\[ \Delta t \approx 1\text{ns} \]

0.1-1.5 keV

Attwood fig. 6.31 and 6.32
Laser wavelength trends in laser-created plasmas

- At high irradiation non-linear processes tend to dominate laser-plasma interaction
- Appearance of high energy suprathermal photons
- At very high intensities long \( \lambda \) results in increased stimulated scattering processes and in decrease absorption
- At very high intensities use of shorter wavelengths (2W, 3W) is preferred

Figure 6.30 Attwood