Now that we have seen how particles are created and lost, and how the plasma “connects” to the walls, can we analyze a simple discharge? The answer is yes! And we have already started.

**DC discharges – positive column.** Calculation of $T_e$

We have seen the particle balance before (steady state):

$$-\nabla^2 n \frac{D_a}{n} = \nu_{iz} n; \quad \nu_{iz} \equiv \text{ionization rate } n_g < \sigma v$$

**Planar**

$$\frac{d^2 n}{dx^2} = -\nu_{iz} \frac{n}{D_a}$$

$$n = n_0 \cos \beta x$$

$$\beta = \sqrt{\frac{\nu_{iz}}{D_a}} = \frac{\pi}{l}$$

**Cylindrical**

$$\frac{d^2 n}{dr^2} + \frac{1}{r} \frac{dn}{dx} + \nu_{iz} \frac{n}{D_a} = 0$$

$$n = n_0 J_0(\beta r)$$

$$\beta = \sqrt{\frac{\nu_{iz}}{D_a}} = \frac{2.4}{r}$$

First zero of $J_0$
We already know that this is not a completely self-consistent solution, but it gives reasonable values for $T_e$. Why is this? Because $v_{iz}$ is a very sensitive function of $T_e$, so $T_e$ depends weakly on all parameters except $\varepsilon_{iz}$. Well, let’s do the numbers:

\[
v_{iz} = n_g < \sigma_{iz} v >
\]

Assume Maxwellian distribution, one-step ionization

\[
\sigma_{iz} \cong a(\varepsilon - \varepsilon_i), \quad \varepsilon > \varepsilon_i \rightarrow v_{iz} = n_g < (\varepsilon - \varepsilon_i) v >
\]

We have seen that

\[
f(\varepsilon) = \frac{2}{\sqrt{\pi} (kT_e)^{3/2}} \sqrt{\varepsilon} e^{-\varepsilon/kT_e}
\]

Rocca, Lecture #1, page 18

So

\[
v_{iz} = \int_{\varepsilon_i}^{\infty} \frac{2}{\sqrt{\pi}} \frac{\sqrt{\varepsilon}}{(kT_e)^{3/2}} e^{-\varepsilon/kT_e} \sigma \left( \frac{2\varepsilon}{m} \right) d\varepsilon
\]

Integrating...

\[
v_{iz} = \frac{4\sqrt{2}}{\sqrt{\pi}} an_g \frac{(kT_e)^{3/2}}{\sqrt{m}} e^{-\varepsilon_i/kT_e} \left( 1 + \frac{\varepsilon_i}{2kT_e} \right)
\]
For typical glow discharges $kT_e << \varepsilon_i$ (the electrons on the tail do all the work!)

$$v_{iz} = \left(\frac{2.4}{R}\right)^2 D_a ; \quad D_a \equiv \frac{kT_e}{e} \mu_i \quad T_e >> T_i$$

$$\Rightarrow \frac{4\sqrt{2}}{\sqrt{\pi}} a_n g \frac{(kT_e)^{3/2}}{\sqrt{m}} e^{-\varepsilon_i/kT_e} \left(1 + \frac{\varepsilon_i}{2kT_e} \right) = \frac{kT_e}{e} \mu_i \left(\frac{2.4}{R}\right)^2$$

Rearranging we get

$$e^{\varepsilon_i/kT_e} \frac{\sqrt{\varepsilon_i/kT_e}}{1 + \frac{\varepsilon_i}{2kT_e}} = 0.55e \left[ a \sqrt{\varepsilon_i} \right] N^2 R^2$$

Notice that for a given gas the electron temperature is a unique function of the density-radius or pressure-radius product! It does not depend on electron density or current. Remember we are assuming Maxwellian distributions and one-step ionization. The tail may be depleted by high speed electrons escaping through the sheath.
Fig. 8.1. Normalized electron temperature vs normalized pressure density product

B. Cherrington, Gaseous electronics and gas lasers
How do we calculate electric field and density? Balancing energy gain from the \( E \) field with energy losses:

\[
P_{\text{absorbed}} = 2\pi \int_0^R \mathbf{J} \cdot \mathbf{E} \, r \, dr ; \quad P_{\text{lost}} = 2\pi R \Gamma e \varepsilon_T
\]

Where \( \Gamma \) is the radial particle flux and \( \varepsilon_T \) is the total energy carried out per e-ion pair created

\[
en_0 \mu_e E^2 2\pi \int_0^R J_0(\beta r) \, dr = 2\pi R \sqrt{D_a \nu_{iz} n_0 J_1(\beta R) e \varepsilon_T} \quad n = n_0 J_0(\beta r)
\]

\[
J = en\mu_e E , \quad E \approx \text{const}
\]

Integrating we get

\[
E = \sqrt{\frac{\nu_{iz} \varepsilon_T}{\mu_e}} = \sqrt{\frac{m}{e}} \frac{\nu_{iz} \nu_m \varepsilon_T}{m \nu_m}
\]

so

\[
\left( \frac{E}{N} \right)^2 = \frac{m}{e} \left\langle \sigma_{iz} v \right\rangle \left\langle \sigma_c v \right\rangle \varepsilon_T
\]

This is a function of gas and \( T_e \) only, and \( T_e \) is determined by \( NR \), so \( (E/N)^2 = f(NR)! \)
The voltage drop is independent of the current through the discharge. What is the current? We just need to integrate the current density over the discharge cross section:

\[ I = \int JdA = 2\pi en_0 \left( \frac{R^2}{2.4} \right) J_1(2.4) \mu_e E \]

From this expression we can calculate the plasma density given a discharge current.

\[ en_0 \mu_e E^2 2\pi \int_0^R J_0(\beta r)dr = 2\pi R \sqrt{D_a \nu_{iz} n_0 J_1(\beta R)} e \epsilon_T \]

\[ J = en \mu_e E, \quad E \cong \text{const} \]

\[ \Gamma |_{r=R} = -\nabla n |_{r=R} D_a \]

\[ E = \sqrt{\frac{\nu_{iz} \epsilon_T}{\mu_e}} = \sqrt{\frac{m}{e} \nu_{iz} v_m \epsilon_T} \]

\[ \left( \frac{\mu_e}{m \nu_m} \right) \]

This is a function of gas and \( T_e \) only, and \( T_e \) is determined by \( NR \), so \( (E/N)^2 = f(NR) \)!
DC Discharges – an application example
Magnetron discharges

a) DC voltage: Almost all voltage drops across the cathode sheath. The secondary emission coefficient for Ar ions on aluminum is $\gamma_{SE} \sim 0.1$ for 200-1000V Ar$^+$:

$$\gamma_{EFF} < \gamma_{SE}, \quad \gamma_{EFF} \sim \frac{1}{2} \gamma_{SE}$$

In steady-state $\gamma_{EFF}N = 1$, so

$$V_{DC} \approx \frac{2\varepsilon_C}{\gamma_{SE}} \rightarrow V_{DC} \approx 600V$$

b) Are the electrons and ions magnetized?

$$r_{ce} = \frac{v_e}{\omega_{ce}} = \frac{1}{B_0} \sqrt{\frac{2m_eV_{DC}}{e}} \quad \left(\frac{1}{2}m_ev_e^2 = V_{DC}e\right)$$
\[ B_0 \approx 200\text{G} & \ V_{DC} = 600\text{V} \quad \rightarrow \quad r_{ce} = 5\text{mm} \]
\[ r_{ci} = \frac{1}{B_0} \sqrt{\frac{2m_i V_{DC}}{e}} \quad \rightarrow \quad r_{ci} \approx 1.3\text{m!} \]

c) Ion current density and sheath thickness – Child-Langmuir

\[ J_i = \frac{4}{9} \varepsilon_0 \left( \frac{2e}{m_i} \right)^{1/2} \frac{V_{DC}^{3/2}}{s^2} \]

also

\[ J_i = \frac{I_{DC}}{2\pi RW} \]

Width of the plasma ring

Typically \( I_{DC} = 5\text{ A} \), \( R = 5\text{ cm} \), \( W = 4\text{ cm} \), \( V_{DC} = 600\text{ V} \) so \( J_i \approx 40\text{ mA/cm}^2 \) and \( s = 0.56\text{ mm} \)
d) Plasma density – Bohm flux

\[ J_i = 0.61n_i e u_B \]

Assuming \( T_e \approx 3 \text{ eV} \) and \( J_i = 40 \text{ mA/cm}^2 \) we get \( n_i \approx 1.5 \times 10^{12} \text{ cm}^{-3} \)

e) Sputtering rate \( \gamma_{SPUT} = \# \text{ of sputtered atoms per ion} \)

\[ R_{SPUT} = \frac{J_i}{e \gamma_{SPUT}} \frac{1}{n_{AI}} \]

With \( n_{AI} \approx 6 \times 10^{22} \text{ cm}^{-3} \), \( \gamma_{SPUT} \approx 1 \), and \( J_i \approx 40 \text{ mA/cm}^2 \), we get \( R_{SPUT} = 4.1 \times 10^{-6} \text{ cm/s} \rightarrow 3.6 \text{ mm/24 hr}! \)

f) Discharge power: \( P_{ABS} = V_{DC} I_{DC} = 3 \text{ kW} \)