ECE507 - Plasma Physics and Applications

Lecture 11

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Now that we have seen how particles are created and lost, and how the plasma "connects" to the walls, can we analyze a simple discharge? The answer is yes! And we have already started.

DC discharges – positive column. Calculation of T_e

We have seen the particle balance before (steady state):

$$-\nabla^2 n\, D_a = \nu_{iz} n; \qquad \nu_{iz} \equiv \text{ionization rate } n_g < \sigma v >$$

Planar

$$\frac{d^2 n}{dx^2} = -\frac{\nu_{iz}}{D_a} n$$

$$n = n_0 \cos \beta x$$

$$\beta = \sqrt{\frac{\nu_{iz}}{D_a}} = \frac{\pi}{l}$$

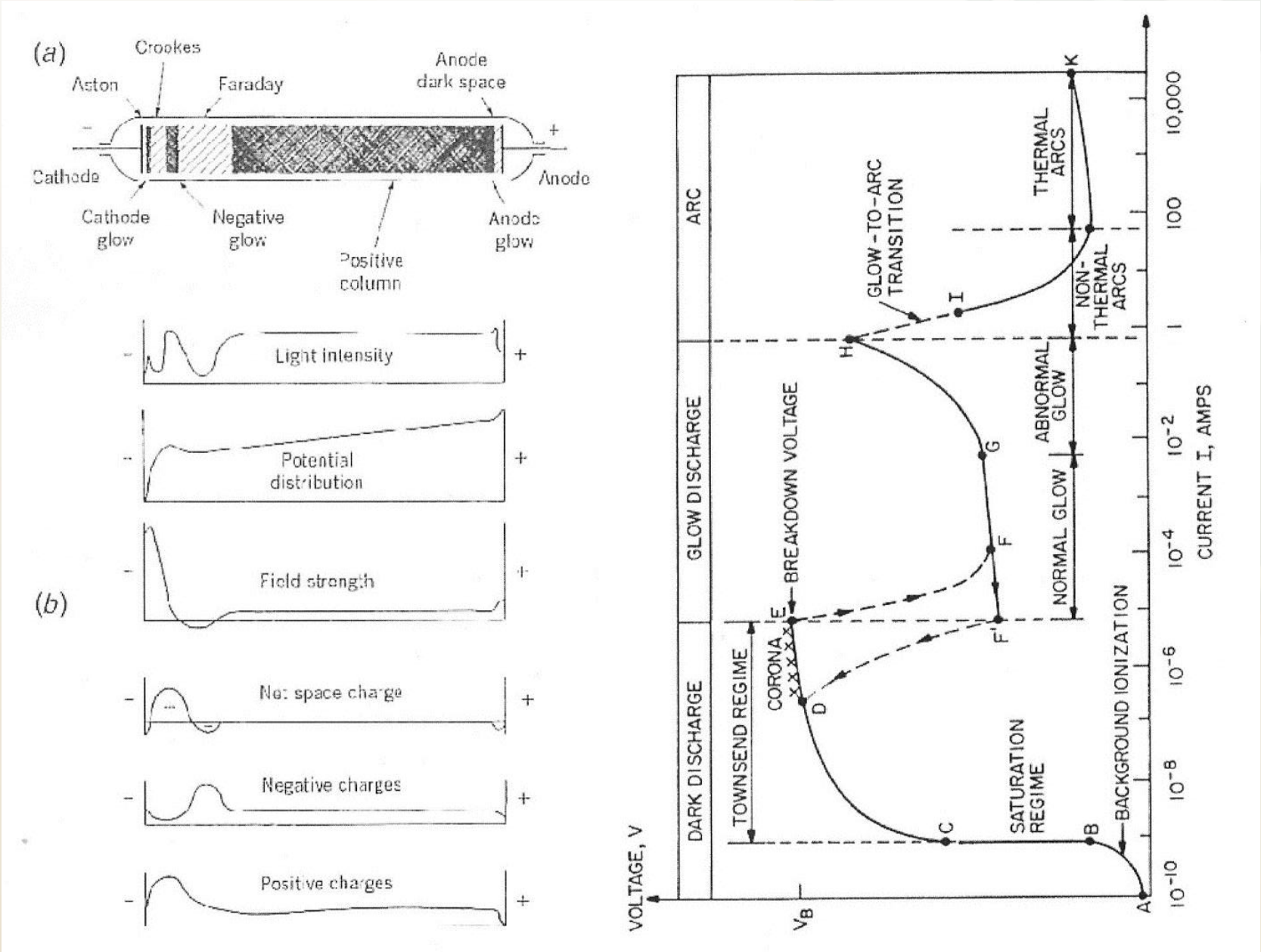
Cylindrical

$$\frac{d^2 n}{dr^2} + \frac{1}{r}\frac{dn}{dx} + \frac{\nu_{iz}}{D_a} n = 0$$

$$n = n_0 J_0(\beta r)$$

$$\beta = \sqrt{\frac{\nu_{iz}}{D_a}} = \frac{2.4}{r}$$

First zero of J_0


(a)
Crookes
Aston
Faraday
Anode dark space
Cathode
Anode
Cathode glow
Negative glow
Positive column
Anode glow
(b)
Light intensity
Potential distribution
Field strength
Net space charge
Negative charges
Positive charges
VOLTAGE, V
CURRENT I, AMPS
DARK DISCHARGE
GLOW DISCHARGE
ARC
TOWNSEND REGIME
CORONA
BREAKDOWN VOLTAGE
GLOW-TO-ARC TRANSITION
SATURATION REGIME
BACKGROUND IONIZATION
NORMAL GLOW
ABNORMAL GLOW
NON-THERMAL ARCS
THERMAL ARCS
V_B
A
B
C
D
E
F
F'
G
H
I
K

We already know that this is not a completely self-consistent solution, but it gives reasonable values for T_e. Why is this? Because ν_{iz} is a very sensitive function of T_e, so T_e depends weakly on all parameters except ε_{iz}. Well, let's do the numbers:

$$\nu_{iz} = n_g < \sigma_{iz} v >$$

Assume Maxwellian distribution, one-step ionization

$$\sigma_{iz} \cong a(\varepsilon - \varepsilon_i), \quad \varepsilon > \varepsilon_i \rightarrow \nu_{iz} = n_g < (\varepsilon - \varepsilon_i) v >$$

We have seen that

$$f(\varepsilon) = \frac{2}{\sqrt{\pi}(kT_e)^{3/2}} \sqrt{\varepsilon} e^{-\varepsilon / kT_e}$$

Rocca, Lecture #1, page 18

So

$$\nu_{iz} = \int_{\varepsilon_i}^{\infty} \underbrace{\frac{2}{\sqrt{\pi}} \frac{\sqrt{\varepsilon}}{(kT_e)^{3/2}} e^{-\varepsilon / kT_e}}_{f} \underbrace{a n_g (\varepsilon - \varepsilon_i)}_{\sigma} \underbrace{\sqrt{\frac{2\varepsilon}{m}}}_{v} d\varepsilon$$

Integrating…

$$\nu_{iz} = \frac{4\sqrt{2}}{\sqrt{\pi}} a n_g \frac{(kT_e)^{3/2}}{\sqrt{m}} e^{-\varepsilon_i / kT_e} \left(1 + \frac{\varepsilon_i}{2kT_e} \right)$$

For typical glow discharges $kT_e << \varepsilon_i$ (the electrons on the tail do all the work!)

$$\nu_{iz} = \left(\frac{2.4}{R}\right)^2 D_a \; ; \qquad D_a \cong \frac{kT_e}{e}\mu_i \qquad T_e >> T_i$$

$$\Rightarrow \frac{4\sqrt{2}}{\sqrt{\pi}} a n_g \frac{(kT_e)^{3/2}}{\sqrt{m}} e^{-\varepsilon_i / kT_e}\left(1 + \frac{\varepsilon_i}{2kT_e}\right) = \frac{kT_e}{e}\mu_i \left(\frac{2.4}{R}\right)^2$$

Rearranging we get

$$\frac{e^{\varepsilon_i / kT_e}\sqrt{\varepsilon_i / kT_e}}{1 + \frac{\varepsilon_i}{2kT_e}} = \frac{0.55e}{\sqrt{m}}\left[\frac{a\sqrt{\varepsilon_i}}{\mu_i N}\right] \underset{n_g}{N^2} R^2$$

Notice that for a given gas the electron temperature is a unique function of the density-radius ore pressure-radius product! It does <u>not</u> depend on electron density or current. Remember we are assuming Maxwellian distributions and one-step ionization. The tail may be depleted by high speed electrons escaping through the sheath.

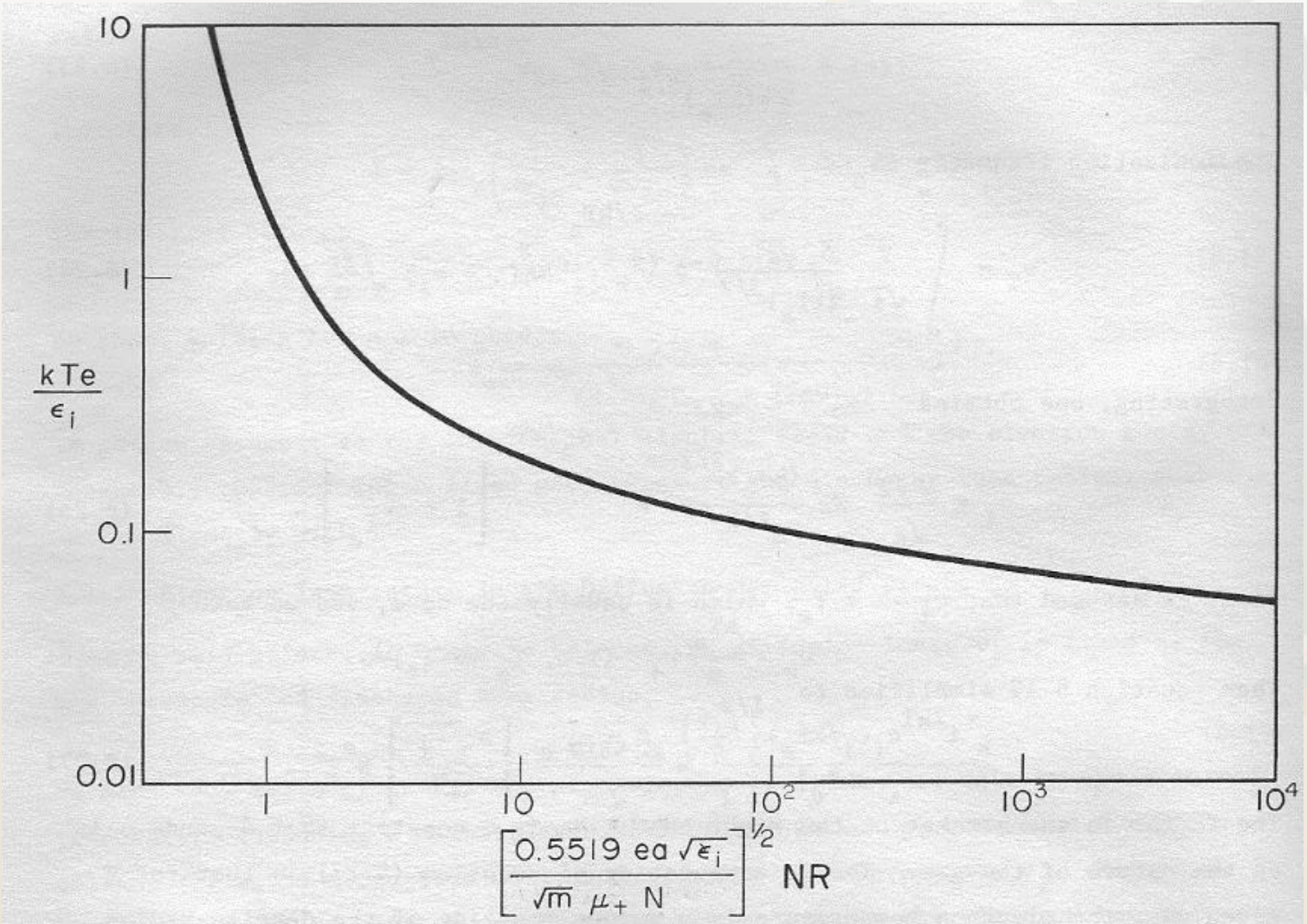


Fig. 8.1. Normalized electron temperature vs normalized pressure density product

B. Cherrington, Gaseous electronics and gas lasers

How do we calculate electric field and density? Balancing energy gain from the ***E*** field with energy losses:

$$P_{absorbed} = 2\pi \int_0^R \mathbf{J}\cdot\mathbf{E}\, r dr\,; \qquad P_{lost} = 2\pi R \Gamma e \varepsilon_T$$

Where Γ is the radial particle flux and ε_T is the total energy carried out per e-ion pair created

$$\underbrace{e n_0 \mu_e E^2}_{J = e n \mu_e E\,,\quad E \cong \text{const}} 2\pi \int_0^R J_0(\beta r) dr = 2\pi R \underbrace{\sqrt{D_a \nu_{iz}}\, n_0 J_1(\beta R)}_{\Gamma|_{r=R} = -\nabla n|_{r=R} D_a} e \varepsilon_T \qquad n = n_0 J_0(\beta r)$$

Integrating we get

$$E = \sqrt{\frac{\nu_{iz}\varepsilon_T}{\mu_e}} = \sqrt{\frac{m}{e}\nu_{iz}\nu_m \varepsilon_T} \qquad \left(\mu_e = \frac{e}{m\nu_m}\right)$$

so

$$\left(\frac{E}{N}\right)^2 = \frac{m}{e}\langle\sigma_{iz} v\rangle\langle\sigma_c v\rangle \varepsilon_T$$

This is a function of gas and T_e only, and T_e is determined by *NR*, so $(E/N)^2 = f(NR)$!

The voltage drop is independent of the current through the discharge. What the current? We just need to integrate the current density over the discharge cross section:

$$I = \int J dA = 2\pi e n_0 \left(\frac{R^2}{2.4} \right) J_1(2.4) \mu_e E$$

From this expression we can calculate the plasma density given a discharge current.

$$\underbrace{e n_0 \mu_e E^2}_{J = e n \mu_e E\,,\quad E \cong \text{const}} 2\pi \int_0^R J_0(\beta r) dr = 2\pi R \underbrace{\sqrt{D_a \nu_{iz}}\, n_0 J_1(\beta R)}_{\Gamma|_{r=R} = -\nabla n|_{r=R} D_a} e \varepsilon_T$$

$$E = \sqrt{\frac{\nu_{iz} \varepsilon_T}{\mu_e}} = \sqrt{\frac{m}{e} \nu_{iz} \nu_m \varepsilon_T} \qquad \left(\mu_e = \frac{e}{m \nu_m} \right)$$

$$\left(\frac{E}{N} \right)^2 = \frac{m}{e} \langle \sigma_{iz} v \rangle \langle \sigma_c v \rangle \varepsilon_T$$

This is a function of gas and T_e only, and T_e is determined by NR, so $(E/N)^2 = f(NR)$!

DC Discharges – an application example
Magnetron discharges

a) DC voltage: Almost all voltage drops across the cathode sheath. The secondary emission coefficient for Ar ions on aluminum is γ_{SE} ~ 0.1 for 200-1000V Ar^+:

electron-ion pairs created by each secondary electron

$$N \cong \frac{V_{DC}}{\varepsilon_C}$$

Energy lost per electron-ion created by secondary electrons, ~ 30V for 200 - 1000V secondaries

$$\gamma_{EFF} < \gamma_{SE}\,, \qquad \gamma_{EFF} \sim \frac{1}{2}\gamma_{SE}$$

$$\text{in steady-state } \gamma_{EFF} N = 1, \text{so} \quad V_{DC} \cong \frac{2\varepsilon_C}{\gamma_{SE}} \quad \rightarrow \quad V_{DC} \cong 600\text{V}$$

b) Are the electrons and ions magnetized?

$$r_{ce} = \frac{v_e}{\omega_{ce}} = \frac{1}{B_0}\sqrt{\frac{2m_e V_{DC}}{e}} \qquad \left(\frac{1}{2}m_e v_e^2 = V_{DC} e\right)$$

$$B_0 \cong 200\text{G} \;\&\; V_{DC} = 600\text{V} \quad \rightarrow \quad r_{ce} = 5\text{mm}$$

$$r_{ci} = \frac{1}{B_0}\sqrt{\frac{2m_i V_{DC}}{e}} \quad \rightarrow \quad r_{ci} \cong 1.3\text{m!}$$

c) Ion current density and sheath thickness – Child-Langmuir

$$J_i = \frac{4}{9}\varepsilon_0 \left(\frac{2e}{m_i}\right)^{1/2} \frac{V_{DC}^{3/2}}{s^2}$$

also

$$J_i = \frac{I_{DC}}{2\pi R W}$$

Width of the plasma ring

Typically I_{DC} = 5 A, R = 5 cm, W = 4 cm, V_{DC} = 600 V so $J_i \approx$ 40 mA/cm^2 and s = 0.56 mm

d) Plasma density – Bohm flux

$$J_i = 0.61 n_i e u_B$$

Assuming $T_e \approx 3$ eV and $J_i = 40$ mA/cm^2 we get $n_i \approx 1.5\text{x}10^{12}$ cm^{-3}

e) Sputtering rate $\gamma_{SPUT} = \#\text{ of sputtered atoms per ion}$

$$R_{SPUT} = \frac{J_i}{e} \gamma_{SPUT} \frac{1}{n_{Al}}$$

#ions/m^2

With $n_{Al} \approx 6\text{x}10^{22}$ cm^{-3}, $\gamma_{SPUT} \sim 1$, and $J_i \approx 40$ mA/cm2, we get $R_{SPUT} = 4.1\text{x}10^{-6}$ cm/s → 3.6 mm/24 hr!

f) Discharge power: $P_{ABS} = V_{DC} I_{DC} = 3$ kW