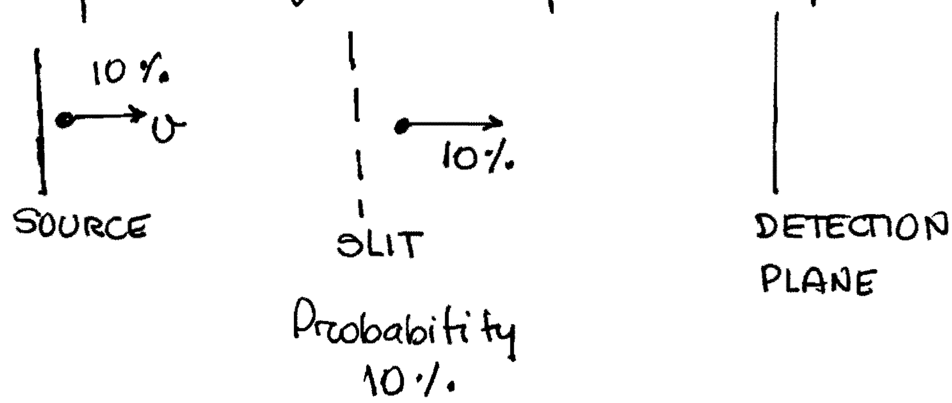
Homework # 1

Hints on problems solution

Conceptual questions from Chapter 2

1.



Particle is tunneling through slit -
Slit is invisible in Part.

If the particle makes it to the slit, then the wavefunction satisfies $|\psi|^2 = 1$ at slit -
Then there is 10 % probability of the particle reaching the detector plane -

2. C3 Electrons in different states have different quantum numbers therefore can have same or different spins

Example: $1s^1 2s^1$

$s \Rightarrow \ell$

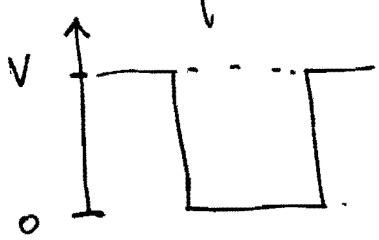
	$n=1$	$n=2$
	$\ell=0$	$\ell=1$
How many different ways can the spins be arranged?	$m=0$	$m=1,0,-1$
Spin	$\uparrow$	$\uparrow$
	$\downarrow$	$\downarrow$

M	1	0	-1
0	↑ ↑↓	↑ ↓↑	↑↓↓
0	↓ ↑↓	↓ ↑↓	↓↑↓

Each box - 2 arrangements

Total posibilities : 12 configurations

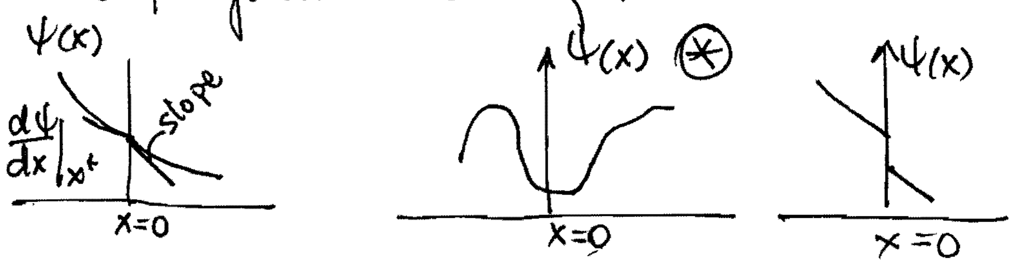
3. C6 Wavelength of an election in a 1D potential for $E = V$



The particle can either be in the well or outside the well. The state at the well / continuum interface does not have physical meaning -

4. C7 Which of the wavefunctions does not have physical meaning?

$\frac{d\psi}{dx}\Big|_{x^-} \neq \frac{d\psi}{dx}\Big|_{x^+}$



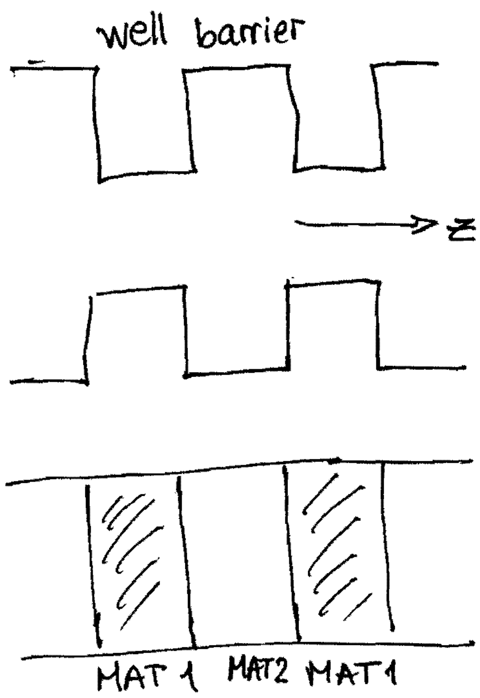
For a wavefunction to ~~have~~ be solution of Schrwedinger's equation, it has to be continous + its first derivative has to be continous. The only function that satisfies this is ⊛

5 -

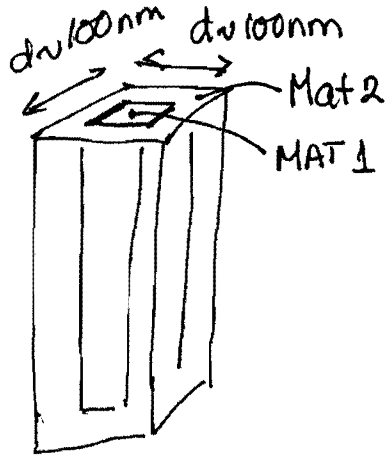
Electrons in a crystal	Photons
$\psi = A(\bar{r})\, e^{ik\bar{r}}$	$\psi = \bar{E}\, e^{i\bar{k}\cdot\bar{r}}$
$p = \hbar k$	$p = \hbar k$
	Have polarization
$H = E + V(r)$	No potential in H -

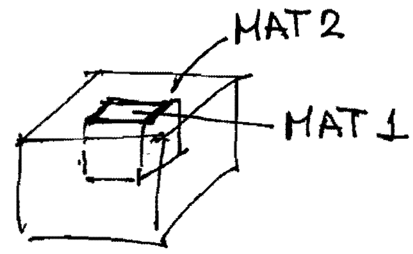
6 - Confinement for an electron is realized by designing structures in which the potential the electron sees is discontinuous in one or more directions

Quantum well



Bandgap Mat 1 < Bandgap Mat 2





Quantum wire

electron confined in two directions

Quantum dot

electron confined in three directions

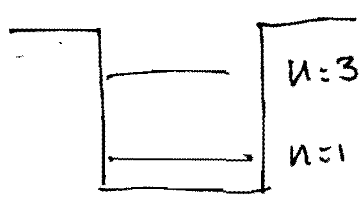
Problems

1.) $E_{ion} = 13.6\,eV$

$$h\nu = E_{ion} \implies \nu = \frac{E_{ion}}{h}$$

$$\frac{hc}{\lambda} = E_{ion} \implies \lambda = \frac{hc}{E_{ion}}$$

2.)



Solve Schroedinger's equation

$$H = \frac{p^2}{2m}$$

obtain eigen-energies and eigen-functions

Homework 2

1) Schroedinger's equation

Crystal

$$H = \frac{p^2}{2m^*} + V(r)$$

↑ lattice potential

$$\psi = A(r)\, e^{i\vec{k}\cdot r}$$

↑ planewaves

Hydrogen atom

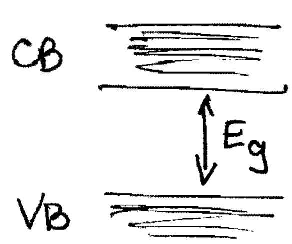
$$H = \frac{p^2}{2m} - \frac{e}{r}$$

↑ Coulomb potential

$$\psi = \rho(r)\, Y_{lm}(\theta, \varphi)$$

↑ polynomials ↑ cos-like

2) - · When electrons in two nearby atoms interact, the electronic structure of each of the e^- is modified. The 'new' configuration is described by two new states (bonding + anti-bonding). In a crystal there are 10^{23} atoms, therefore the interaction of the nearby valence electrons give rise to multiple allowed energy state. This collection of states are referred to as 'BANDS'. Top most filled band is called: VALENCE BAND. NEXT UNFILLED BAND is called. CONDUCTION BAND -

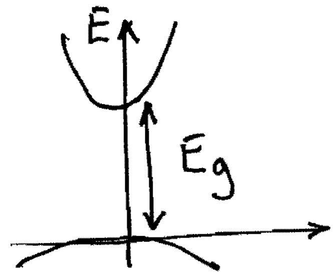


$E_g = 0$ _ metal

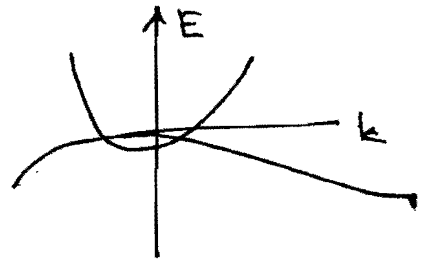
$E_g \simeq 0.5 - 5$ or so eV _ semiconductor

$E_g \gtrsim 5$ eV - insulator

In terms of a band picture, in the parabolic approximation and assuming direct gap materials



SEMICONDUCTOR
or
INSULATOR



METAL.

3. Hole represents the collective behavior of electrons in the valence band. When there are empty k-states, the electrons can occupy these states ~~[illegible]~~ if there are external forces acting upon them. The motion of the empty state is opposite to that of the e^-. Therefore equivalent to a +e.

4. This problem is not phrased correctly. The density of states for CB or VB is a function. Therefore there is no meaning to calculating a value.

$$g_c(E) = \frac{1}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}(E-E_g)^{1/2}$$

HW #2-3

To calculate the total # of electrons/vol at $T = 100K$ for an excess energy $\Delta E = 60\,meV$

$$N = \int_{E_g}^{E_g + \Delta E} g_c(E)\, f(E)\, dE$$

where $$f(E) = \frac{1}{1 + \exp[(E - E_F)/k_B T]}$$

This integration is performed in the Boltzmann limit because $\frac{E_C - E_F}{k_B T} \sim \frac{E_g}{2 k_B T} \gg 1$

$$N = N_c \exp\left[\frac{-(E_C - E_F)}{k_B T}\right]$$ when integ from E_g to ∞.

where $$N_C = 2\left(\frac{2\pi m_n^* k_B T}{h^2}\right)^{3/2}$$

Ge $\quad \frac{m_c^*}{m_0} \sim 0.55 \qquad E_C - E_F \simeq \frac{E_g}{2}$

$\cancel{E_F \simeq \frac{E_g}{2}}$

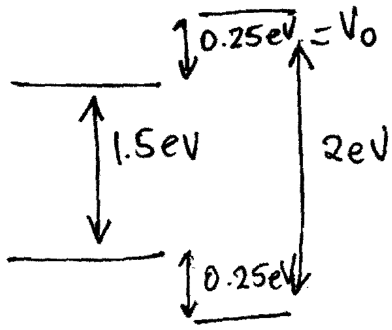
$E_g = 0.66\,eV.$

5 - There are many possible solutions to this problem. Need to use the semiconductor roadmap. Choose a substrate ; choose well and barrier material. Well bandgap should be ~ 1.4 - 1.5 eV

Barrier bandgap ~ 2eV

Choose well width a

50/50 conduction / valence band split means that 1/2 of the bandgap difference is accomodated in the CB, the rest in the valence band



Energies of confined states are

$$E_n = \frac{\hbar^2 n^2 \pi^2}{2ma}$$

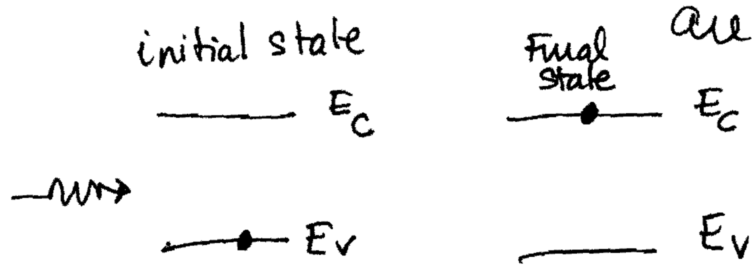
There should be 1 or 2 state confined

6 - Variation of bandgap energy in a quantum dot material with dot size:

$$E = E_{bandgap} + \frac{\hbar^2 \pi^2}{2\mu a^2} \quad \text{(for CB states)}$$

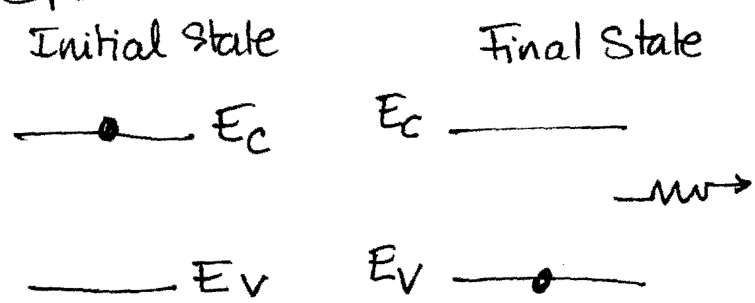
7- The interaction between an electron in a crystal and a photon is modelled by adding to H a term proportional to $-\bar{A}\cdot\bar{p}$ where $\bar{A}$ is the vector magnetic potential -
The Schroedinger equation $H = \frac{p^2}{2m} + V(r) - \bar{A}\cdot\bar{p}$ is solved by perturbation theory.

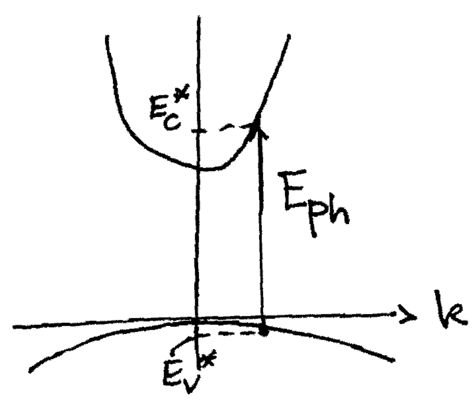
8- Conservation laws: Energy and momentum are conserved



ABSORPTION

Spontaneous Emission





$k_{ph} << k_e$
Therefore an absorption transition conserves k for the electron
$E_v^* + E_{ph} = E_c^*$

9 - Free electron density in Na = $2.65\ 10^{22}/cm^3$

Calculate k_F, V_F, E_F

$$E_F = \frac{\hbar^2 \pi^2}{2mL^2}\left(\frac{3N}{\pi}\right)^{2/3}$$ for a 3D material

$$L^2 = V^{2/3}$$

$$E_F = \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3}$$

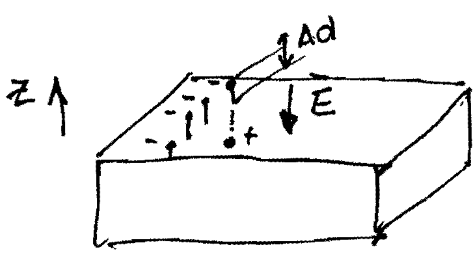
$\frac{N}{V}$ = # electrons per unit volume -

k_F = is obtained from $\rightarrow E_F = \frac{\hbar^2 k_F^2}{2m_0}$

V_F is obtained from $p = \hbar k_F = m v_F$

Fermi Energy is related to pop of electrons in a system; variations in E_F represent variations in N.

10 - Exercise 4



$E \propto E_0 e^{i\omega t}$

Charge density n

motion of displaced charge

$F = ma$

$-eE\cancel{z} = m\frac{d^2z}{dt^2}$ ← harmonic oscillator

$$\omega = \sqrt{\frac{k}{m}}$$

11 - Exercise # 8

Charging energy of
Fe atom $(r = 1.25\,\mathring{A})$
100 nm - QD -

$$\Delta V = \frac{e}{8\pi \varepsilon \varepsilon_0 a} \quad \text{Volts}$$

a = radius -