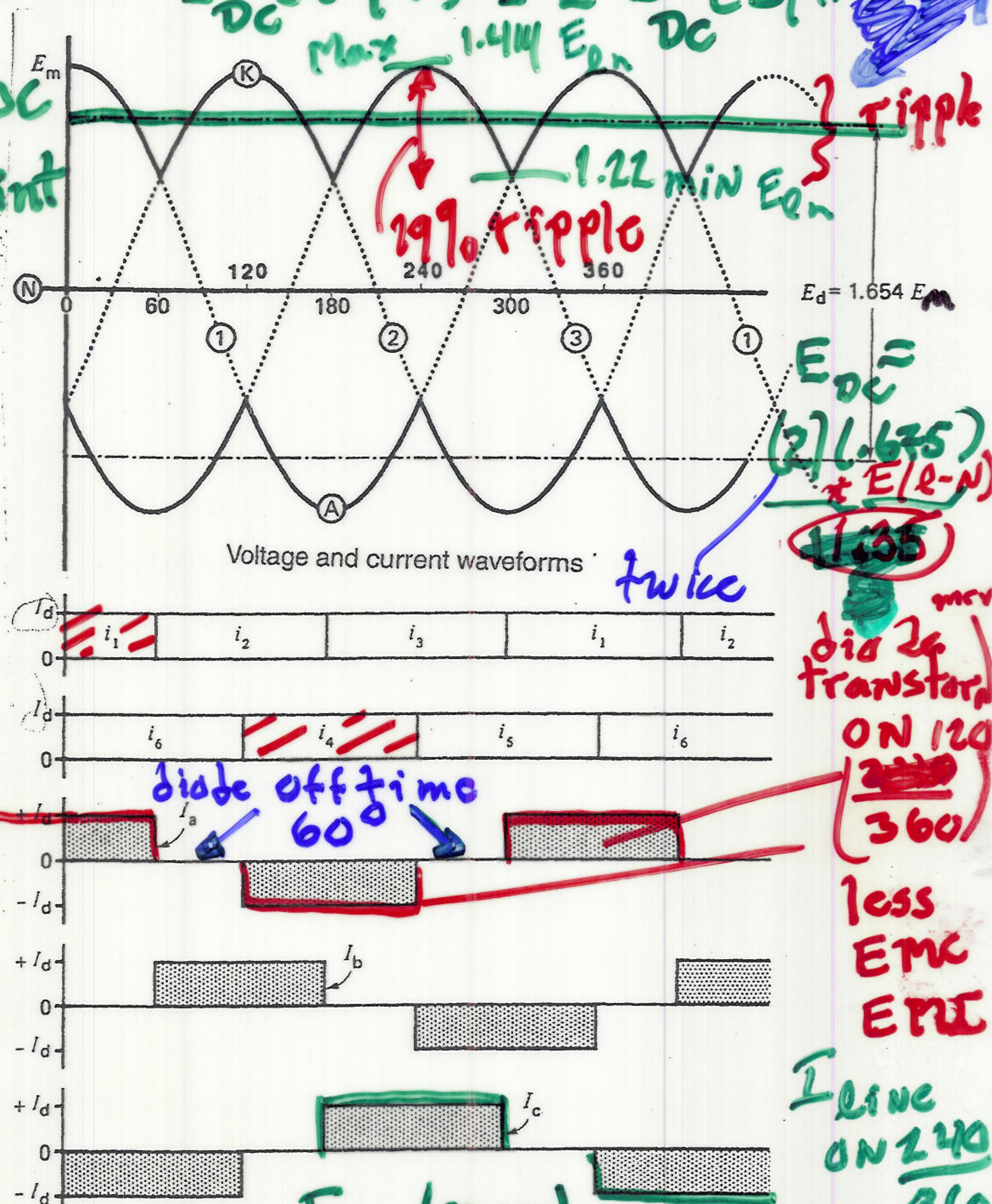
We find E_{DC}(6 pt.) = 2 E_{DC}(3pt)



Next E_{AC}(max) E_{AC}(min) } 6 point

CHAPTER 21

Fundamental Elements of Power Electronics

pg 17

21.0 Introduction

Electronic systems and controls have gained wide acceptance in power technology; consequently, it has become almost indispensable to know something about power electronics. Naturally, we cannot cover all aspects of this broad subject in a single chapter. Nevertheless, we can explain in simple terms the behavior of a large number of electronic power circuits, including those most commonly used today.

As far as electronic devices are concerned, we will first cover diodes and thyristors. They are found in all electronic systems that involve the conversion of ac power to dc power and vice versa. We then go on to discuss the application of more recent devices such as gate turn-off thyristors (GTOs), bipolar junction transistors (BJTs), metal oxide semiconductor field effect transistors (power MOSFETs), and insulated gate bipolar transistors (IGBTs). Their action on a circuit is basically no different from that of a thyristor and its associated switching circuitry. In power electronics all these devices act basically as high-speed switches, so much so, that much of power electronics can be explained by the opening and closing of circuits at precise instants of time. However, we should not conclude that circuits containing these components and devices are simple—they are not—but their behavior can be understood without having an extensive background in semiconductor theory.

old
10ms
100ms

21.1 Potential level

In Chapter 2, Sections 2.4 and 2.5, we described two ways of representing voltages in a circuit. We now introduce a third method that is particularly useful in circuits dealing with power electronics. The method is based upon the concept of potential levels.

To understand the operation of electronic circuits, it is useful to imagine that individual terminals have a potential level with respect to a reference terminal. The reference terminal is any convenient point chosen in a circuit; it is assumed to have zero electric potential. The potential level of all other points is then measured with respect to this zero reference terminal. In graphs, the reference level is shown as a horizontal line having a potential of 0 V.

Consider, for example, the circuit of Fig. 21.1, composed of an 80 V battery connected in series with an ac source E having a peak voltage of 100 V. Of the three possible terminals, let us choose terminal 1 as the reference point. The potential level of

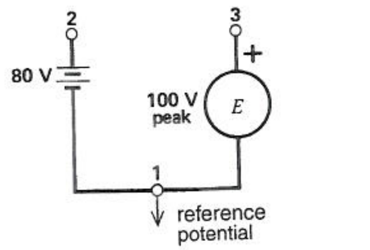


Figure 21.1
Potential level method of representing voltages.

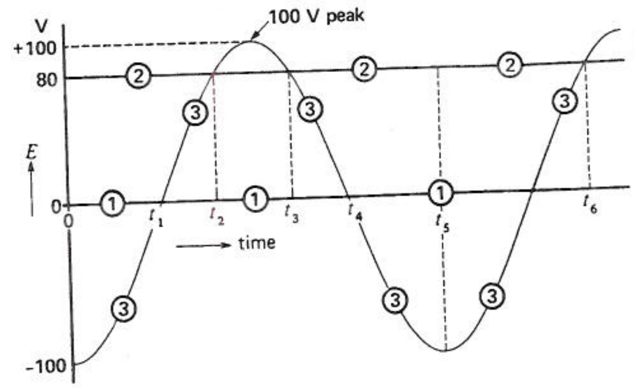


Figure 21.2
Potential levels of terminals 1, 2, and 3.

this terminal is therefore shown by a horizontal line, designated **1** in Fig. 21.2.

Consider now the potential level of terminal 2. The difference of potential between terminals 1 and 2 is always 80 V, and terminal 2 is positive with respect to terminal 1. The level of this terminal is therefore indicated by a second horizontal line **2** placed 80 V above line **1.**

Now consider terminal 3. Voltage E between terminals 1 and 3 is alternating and we assume that its initial value is 100 V, with terminal 3 negative with respect to terminal 1. Because E is alternating, the potential of terminal 3 is first negative, then positive, with respect to terminal 1. The changing level is shown by curve **3.** Thus, during the interval from 0 to t_1, the level of point **3** is below the level of point **1,** which indicates that terminal 3 is negative with respect to terminal 1. During the interval t_1 to t_4, the polarity reverses, and so the level of curve **3** is now above line **1.** Terminal 1 is therefore negative with respect to terminal 3, because line **1** is below curve **3.**

This potential-level method now enables us to determine the instantaneous voltages between any two terminals in a circuit, as well as their relative polarities. For example, during the interval from t_2 to t_3, terminal 3 is positive with respect to terminal 2, because curve **3** is above line **2.** The voltage between these terminals reaches a maximum of 20 V during this interval. Then, from t_3 to t_6, terminal 3 is negative with respect to terminal 2 and the voltage between them reaches a maximum value of 180 V at instant t_5.

We could have chosen another terminal as a reference terminal. Thus, in Fig. 21.3 we chose terminal 3 and, as before, we represent the zero potential of this reference terminal by a horizontal line **3** (Fig. 21.4). Knowing that E is an alternating voltage and that terminal 3 is initially 100 V negative with respect to terminal 1 (as in Fig. 21.2), we can draw curve **1.**

To determine the level of terminal 2, we know that it is always 80 V positive with respect to terminal 1. Consequently, we draw curve **2** so that it is always 80 V above curve **1.** By so doing, we automatically establish the level of terminal 2 with respect to terminal 3.

Figs. 21.2 and 21.4 do not have the same appearance; however, at every instant, the relative polarities and potential differences between terminals are identical. From an electrical point of view, the two figures are identical. We invite the reader to check by comparing the voltages and their relative polarities at various instants in the two figures.

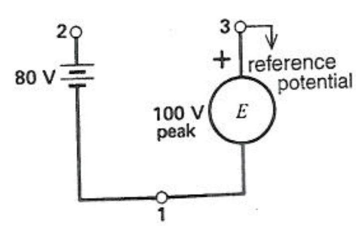


Figure 21.3
Changing the reference terminal.

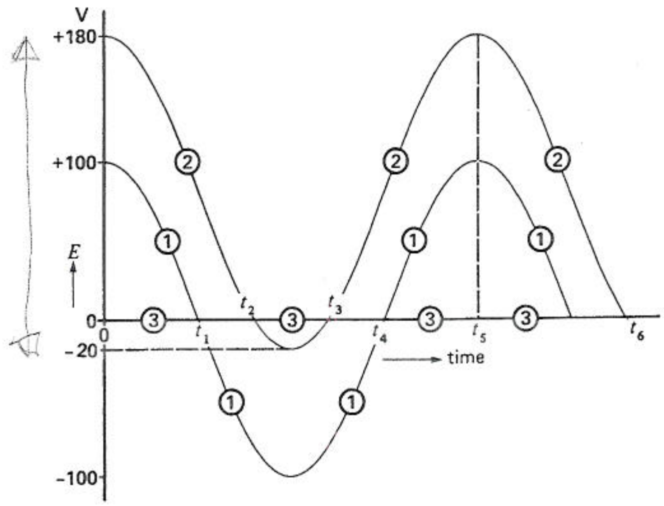


Figure 21.4
The relative potential levels are the same as in Fig. 21.2.

In analyzing electronic circuits the reference terminal may be selected anywhere; it should be easy to follow the voltages we are interested in.

21.2 Voltage across some circuit elements

Let us first look at the voltage levels that appear across some active and passive circuit elements commonly found in electronic circuits. Specifically, we examine sources, switches, resistors, coils, and capacitors.

1. Sources. By definition, ideal ac and dc voltage sources have zero internal impedance. Consequently, they impose rigid potential levels; nothing that happens in a circuit can modify these levels. On the other hand, ac and dc *current* sources have infinite internal impedance. Consequently, they deliver a constant current, and the voltage levels in the circuit must adapt themselves accordingly.

2. Potential Across a Switch. When a switch is open (Fig. 21.5), the voltage across its terminals depends exclusively upon the external elements that make up the circuit. On the other hand, when the switch is closed the potential level of both terminals must be the same. Thus, if we happen to know the level of terminal 2, then the level of terminal 1 is also known. This simple rule also applies to *idealized* thyristors and diodes, because they behave like perfect (albeit one-way) switches.

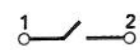


Figure 21.5
Potential across a switch.

3. Potential Across a Resistor. If no current flows in a resistor, its terminals 3, 4 must be at the same potential, because the *IR* drop is zero (Fig. 21.6). Consequently, if we happen to know the potential level of one of the terminals, the level of the other is also known. On the other hand, if the resistor carries a current *I*, the *IR* drop produces a corresponding potential difference between the terminals. For example, if current actually flows in the direction

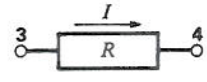


Figure 21.6
Potential across a resistor.

$V_L = \Delta$ for $di/dt \geq 0$ ⇒ Max min of i

shown in Fig. 21.6, the potential of terminal 3 is above that of terminal 4, by an amount equal to IR.

4. Potential Across a Coil or Inductance. The terminals of a coil are at the same potential only during those moments when the current is not changing. If the current varies, the potential difference is given by

$$E = L\,(\Delta I/\Delta t) \qquad (2.27)$$

Thus, if the current in Fig. 21.7 is *increasing* while flowing in the direction shown, the potential level of terminal 5 is *above* that of terminal 6 by an amount equal to $L\,\Delta I/\Delta t$. Conversely, if I is *decreasing* while flowing in the direction shown, the potential of terminal 5 is *below* that of terminal 6.

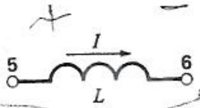


Figure 21.7
Potential across an inductor.

5. Potential Across a Capacitor. The terminals of a capacitor are at the same potential only when the capacitor is completely discharged. Furthermore, the potential difference between the terminals remains unchanged during those intervals when the current I is zero (Fig. 21.8).

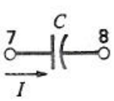


Figure 21.8
Potential across a capacitor.

6. Initial Potential Level. A final rule regarding potential levels is worth remembering. Unless we know otherwise, we assume the following initial conditions:

a. All currents in the circuit are zero and none are in the process of changing.
b. All capacitors are discharged.

These assumed starting conditions enable us to analyze the behavior of any circuit from the moment power is applied.

THE DIODE AND DIODE CIRCUITS

21.3 The diode

A diode is an electronic device possessing two terminals, respectively called anode (A) and cathode (K) (Fig. 21.9). Although it has no moving parts, a diode acts like a high-speed switch whose contacts open and close according to the following rules:

Rule 1. When no voltage is applied across a diode, it acts like an open switch. The circuit is therefore open between terminals A and K (Fig. 21.9a).

Rule 2. If we apply an *inverse voltage* E_2 across the diode so that the anode is *negative* with respect to the cathode, the diode continues to act as an open switch (Fig. 21.9b). We say that the diode is reverse biased.

depends on stored charge

Rule 3. If a momentary *forward voltage* E_1 of 0.7 V or more is applied across the terminals so that anode A is slightly *positive* with respect to the cathode, the terminals become short-circuited. The diode acts like a closed switch and a current I immediately begins to flow from anode to cathode (Fig. 21.9c). We say that the diode is forward biased.

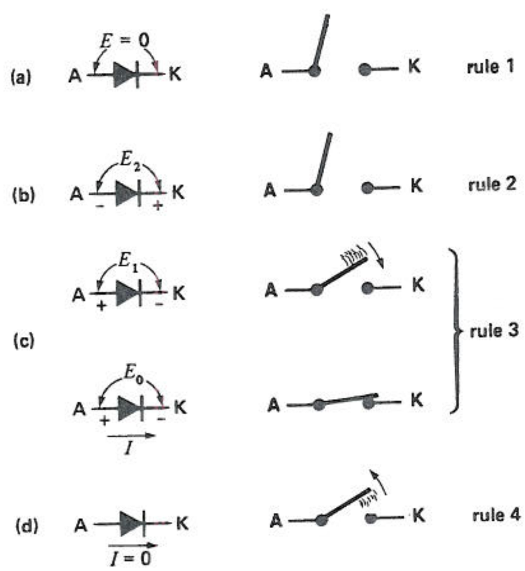


Figure 21.9
Basic rules governing diode behavior.

Power

In practice, while the diode conducts, a small voltage drop appears across its terminals. However, the voltage drop has an upper value of about 1.5 V, so it can be neglected in most electronic circuits. It is precisely because the voltage drop is small with respect to other circuit voltages that we can assume the diode is essentially a closed switch when it conducts.

Careful

Rule 4. As long as current flows, the diode acts like a closed switch. However, if it stops flowing for even as little as 10 μs, the ideal diode immediately returns to its original open state (Fig. 21.9d). Conduction will only resume when the anode again becomes slightly positive with respect to the cathode (Rule 3).

In conclusion, a perfect diode behaves like a normally open switch whose contacts close as soon as the anode *voltage* becomes slightly positive with respect to the cathode. Its contacts only reopen when the *current* (not the voltage) has fallen to zero. This simple rule is crucially important to an understanding of circuits involving diodes and thyristors.

Symbol For a Diode. The symbol for a diode (Fig. 21.9) bears an arrow that indicates the direction of conventional current flow when the diode conducts.

21.4 Main characteristics of a diode

Peak Inverse Voltage. A diode can withstand only so much inverse voltage before it breaks down. The *peak inverse voltage* (PIV) ranges from 50 V to 4000 V, depending on the construction. If the rated PIV is exceeded, the diode begins to conduct in reverse and, in many cases, is immediately destroyed.

Maximum Average Current. There is also a limit to the average current a diode can carry. The maximum current may range from a few hundred milliamperes to over 4000 A, depending on the construction and size of the diode. The nominal current rating depends upon the temperature of the diode, which, in turn, depends upon the way it is mounted and how it is cooled.

Maximum Temperature. The voltage across a diode times the current it carries is equal to a power loss which is entirely converted into heat. The resulting temperature rise of the diode must never exceed the permissible limits, otherwise the diode will be destroyed. Most silicon diodes can operate satisfactorily provided their internal temperature lies between −50°C and +200°C. The temperature of a diode can change very quickly, due to its small size and small mass. To improve heat transfer, diodes are usually mounted on thick metallic supports, called *heat sinks*. Furthermore, in large installations, the diodes may be cooled by fans, by oil, or by a continuous flow of deionized water. Table 21A gives the specifications of some typical diodes. Fig. 21.10 shows a range of low power to very high power diodes.

Diodes have many applications, some of which are found again and again, in one form or another, in electronic power circuits. In the sections that follow, we will analyze a few circuits that involve only diodes. They will illustrate the methodology of power circuit analysis while revealing some basic principles common to many industrial applications. Sections 21.5 to 21.14 cover the following topics:

21.5 Battery charger with series resistor
21.6 Battery charger with series inductor
21.7 Single-phase bridge rectifier
21.8 Filters
21.9 Three-phase, 3-pulse diode rectifier
21.10 Three-phase, 6-pulse diode rectifier
21.11 Effective line current; fundamental line current
21.12 Distortion power factor
21.13 Displacement power factor
21.14 Harmonic content

21.5 Battery charger with series resistor

The circuit of Fig. 21.11a represents a simplified battery charger. Transformer T, connected to a 120 V ac supply, furnishes a sinusoidal secondary volt-

AC Power or Mains

TABLE 21A PROPERTIES OF SOME TYPICAL DIODES

Relative power	I_0 [A]	E_0 [V]	I_{cr} [A]	E_2 [V]	I_2 [mA]	T_J[°C]	d [mm]	l mm
low	1	0.8	30	1000	0.05	175	3.8	4.6
medium	12	0.6	240	1000	0.6	200	11	32
high	100	0.6	1 600	1000	4.5	200	25	54
very high	1000	1.1	10 000	2000	50	200	47	26

I_0 – average dc current
E_0 – voltage drop corresponding to I_0
I_{cr} – peak value of surge current for one cycle ≈ (10 / 30) × I_{Av}
E_2 – peak inverse voltage
I_2 – reverse leakage current corresponding to E_2
T_J – maximum junction temperature (inside the diode)
d – diameter
l – length

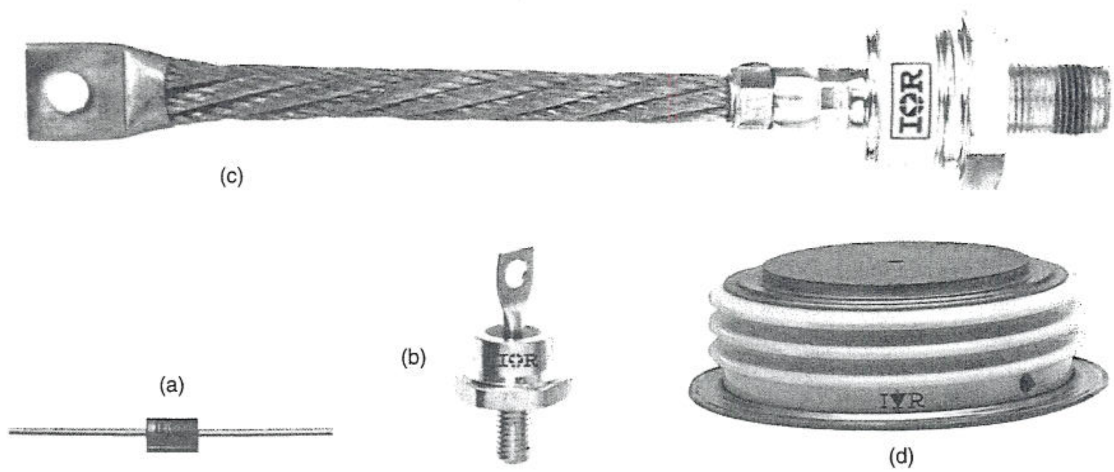
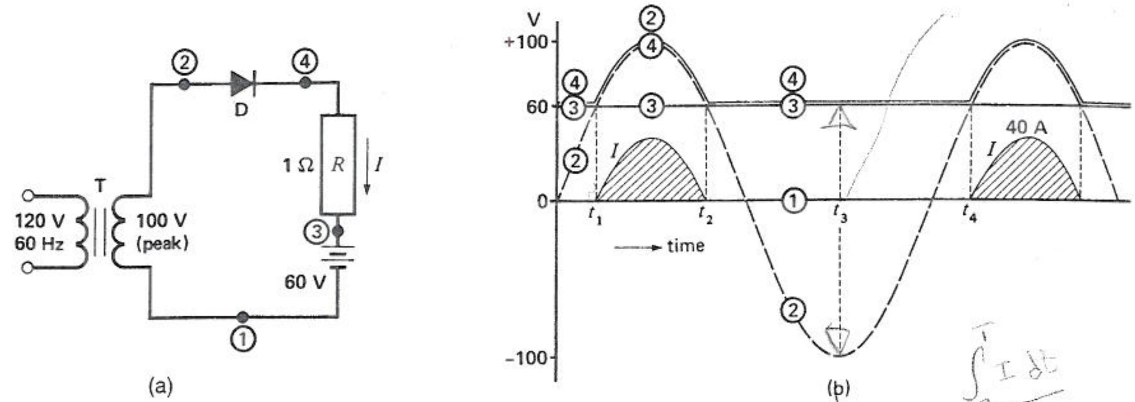


Figure 21.10
a. Average current: 4 A; PIV: 400 V; body length: 10 mm; diameter: 5.6 mm.
b. Average current: 15 A; PIV: 500 V; stud type; length less thread: 25 mm; diameter: 17 mm.
c. Average current: 500 A; PIV: 2000 V; length less thread: 244 mm; diameter: 40 mm.
d. Average current: 2600 A; PIV: 2500 V; Hockey Puk; distance between pole-faces: 35 mm; diameter: 98 mm.
(*Photos courtesy of International Rectifier*)

age having a peak of 100 V. A 60 V battery, a 1 Ω resistor, and an ideal diode D are connected in series across the secondary.

To explain the operation of the circuit, let us choose point **1** as the reference terminal. The potential of this terminal is, therefore, a straight horizontal line. The potential of terminal 2 swings sinusoidally above and below point **1**, according to whether 2 is positive or negative with respect to 1. The level of terminal 3 is always 60 V above terminal 1, because the battery voltage is constant. The potential levels are shown in Fig. 21.11b.

160 V



$\frac{\int_0^T I\,dt}{T}$

Figure 21.11
a. Simple battery charger circuit.
b. Corresponding voltage and current waveforms.

Circuit analysis

a. Prior to $t = 0$, we assume that all currents are zero. The potential of point **4** is, therefore, at the same level as point **3,** and **2** is at the level of point **1.**
b. During the interval from 0 to t_1, anode 2 is negative with respect to cathode 4; consequently, the diode cannot conduct (Rule 2).
c. At instant t_1, terminal 2 becomes positive with respect to 4, and the diode begins to conduct (Rule 3). From this moment on, the ideal diode acts like a closed switch.
Consequently, point **4** must now be at the same level as point **2.** The voltage E_{43} across the resistor gradually builds up, reaching a maximum of 40 V before it falls to zero at instant t_2.
d. From t_1 to t_2 the current in the circuit is given by

$$I = E_{43}/1\ \Omega$$

The current reaches a peak of 40 A when $E_{43} = +40$ V. As long as current flows, the diode behaves like a closed switch;
e. At instant t_2 the current is zero, and the diode immediately opens the circuit (Rule 4). From this moment on, point **4** must follow point **3.**
f. From t_2 to t_4, point **2** is negative with respect to point **3.** Because point **4** follows point **3** (no *IR* drop in the resistor), the PIV across the diode reaches a maximum of 160 V at instant t_3.
g. Finally, at instant t_4 the cycle (a to f above) repeats itself. The resulting current is a truncated sine wave having a peak value of 40 A. Its calculated average value during one cycle is 7.75 A. The pulsating current always flows into the positive terminal of the battery. Consequently, the latter receives energy and progressively charges up.

21.6 Battery charger with series inductor

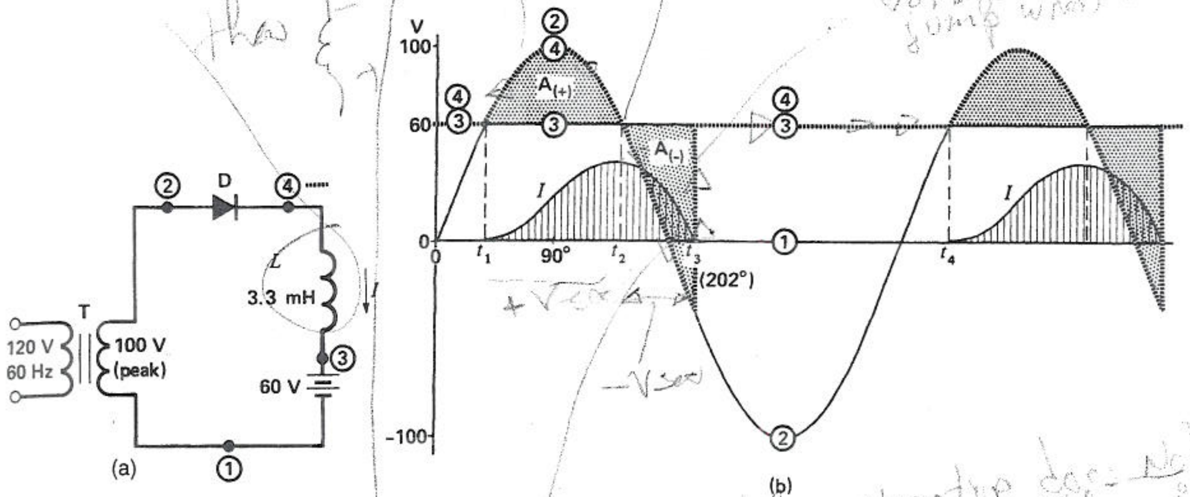
The current flow in the battery charger of Fig. 21.11 is limited by resistor *R.* Unfortunately, this produces I^2R losses and a corresponding poor efficiency. We can get around the problem by replacing the resistor by an inductor, as shown in Fig. 21.12a. Let us analyze the operation of this circuit, bearing in mind the behavior of an inductor, previously explained in Section 2.31.

a. As in the example of Fig. 21.11, the ideal diode begins to conduct at instant t_1 when anode 2 becomes positive with respect to cathode 4. From this moment on, point **4** follows point **2,** and voltage E_{43} appears across the inductor (Fig.

P436

Vsec I is used to shut the diode off. Not with L

discontinuous voltage jump when D off



Vs = 0 then Negative does Not shut diode off. Rather L turns the diode off

Figure 21.12
a. Battery charger using a series inductor.
b. Corresponding voltage and current waveforms.

21.12b). The latter begins to accumulate volt-seconds, and the current increases gradually until it reaches a maximum given by

$$I_{max} = A_{(+)}/L \tag{2.28}$$

where

$A_{(+)}$ = dotted area between t_1 and t_2 [V·s]
L = inductance [H]

Note that the current reaches its peak at instant t_2, whereas it was zero at this moment when a resistor was used. This is consistent with the fact that current through an inductor is no longer changing (has reached a maximum) because the voltage E_{43} across it is zero.

b. After t_2 the voltage E_{43} across the inductor becomes negative and so the inductor discharges volt-seconds. Consequently, the current decreases between t_2 and t_3, becoming zero at t_3 when dotted area $A_{(-)}$ is equal to dotted area $A_{(+)}$.
c. As soon as the current is zero, the diode opens the circuit, whereupon point **4** must jump from point **2** to the level of point **3.** It stays at this level until instant t_4, whereupon the whole cycle repeats.

This is an interesting example of the use of an inductor to store and release electrical energy. During the interval from t_1 to t_2, the inductor stores energy and, from t_2 to t_3, it returns it again to the circuit (see Section 2.13).

Example 21-1

The coil in Fig. 21.12 has an inductance of 3.3 mH and the battery voltage is 60 V. Calculate the peak current if the line frequency is 60 Hz.

Solution

a. To calculate the peak current, we must find the value of area $A_{(+)}$. This can be done by integral calculus, but we will employ a much simpler graphical method. Thus, referring to Fig. 21.12c, we have redrawn the voltage levels using graph paper. The 60 Hz voltage cycle is divided into 24 equal parts, each representing time interval Δt equal to

$$\Delta t = (1/24) \times (1/60) = 1/1440 \text{ s}$$

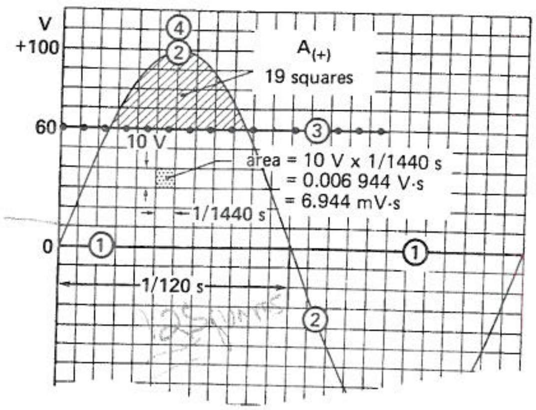


Figure 21.12c
See Example 21-1.

Similarly, the ordinates are scaled off in 10 V intervals. Consequently, each small square represents an area of (1/1440 s) $\times$ 10 V = 0.006944 V·s = 6.944 mV·s.

b. By counting squares, we find that $A_{(+)}$ contains approximately 19 squares; consequently, its area corresponds to

$$A_{(+)} = 19 \times 6.944 = 132 \text{ mV·s} = 0.132 \text{ V·s}$$

The peak current is, therefore,

$$I_{max} = A_{(+)}/L = 0.132/0.0033 = 40 \text{ A}$$

Thus, the current reaches the same peak with an inductor of 3.3 mH as it did with a resistor of 1 Ω. However, the big advantage of the inductor is that it has essentially no losses. The conversion of ac power to dc power is therefore much more efficient. ESR

21.7 Single-phase bridge rectifier

The circuit of Fig. 21.13a enables us to rectify both the positive and negative half-cycles of an ac source, to supply dc power to a load R. The four diodes together make up what is called a *single-phase bridge rectifier*. It is available in a single package.

The circuit operates as follows: When source voltage E_{12} is positive, terminal 1 is positive with respect to terminal 2 and current i_a flows through R by way of diodes A1 and A2. Consequently, point **3** follows point **1**, and point **4** follows point **2** during this conduction interval. Conduction ceases when i_a falls to zero at instant t_1 (Fig. 21.13b). The polarity then reverses and E_{21} becomes positive, meaning that terminal 2 is positive with respect to terminal 1. Current i_b now flows through R in the same direction as before, but this time by way of diodes B1 and B2. Consequently, point **3** now follows point **2** while point **4** follows point **1**. Voltage E_{34} across the load is, therefore, composed of a series of half-cycle sine waves that are always positive (Fig. 21.13c). The voltage pulsates between zero and a maximum value E_m equal to the peak voltage of the source. The average value of this rectified voltage is given by

$$E_d = 0.90\,E \quad \approx E_{DC} \tag{21.1}$$

where

E_d = dc voltage of a single-phase bridge rectifier [V]

rms

E = effective value of the ac line voltage [V]

0.90 = constant [exact value = $(2\sqrt{2})/\pi$]

Referring to Fig. 21.13b, in addition to drawing the curve E_{12} for the source voltage, we have also drawn curve E_{21}. This enables us to use the potential levels of either terminals 1 or 2 as zero reference potentials. Thus, we can select as reference level terminal 2 during the first half-cycle, terminal 1 during the second half-cycle, terminal 2 during the third half-cycle, and so forth, on alternate half-cycles. By using this technique, terminal 4 always remains at zero potential while terminal 3 follows the positive portion of the sine waves. It then becomes evident that E_{34} across the load is a pulsating dc voltage. Figure 21.13c shows the rectified voltage and current of the load.

In addition to its dc value, load voltage E_{34} contains an ac component whose fundamental frequency is twice the line frequency. In effect, the voltage across the load pulsates between zero and $+E_m$, twice per cycle. Consequently, the peak-to-peak *ripple* is equal to E_m.

In the case of a resistive load, the current has the same waveshape as the voltage; its average, or dc value, is given by

$$I_d = E_d/R$$

1Ø (.9)(707) Not .95 Eq 21.3 3Ø

3Ø .65 .64

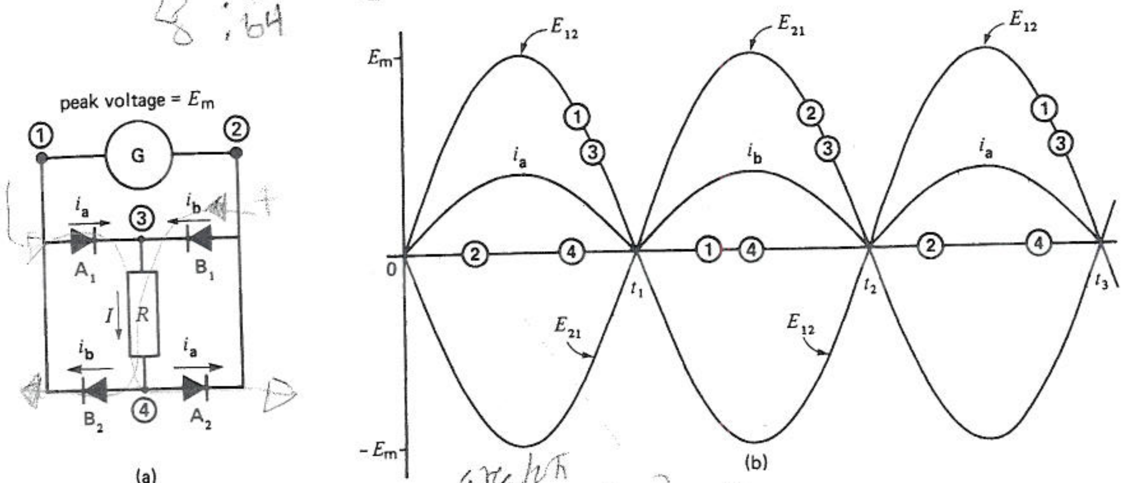


Figure 21.13
a. Single-phase bridge rectifier.
b. Voltage levels.

$V_{AV} \approx \left(\frac{2}{\pi}\right)\sqrt{2}$ peak $*\ V_{rms}$

check .9 E_{DC} 15

rms of f_0 reverse pg 529

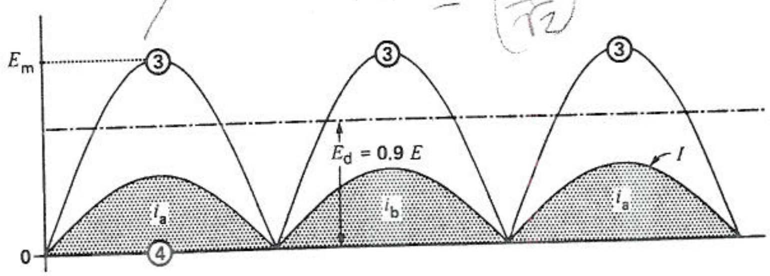


Figure 21.13c
Voltage and current waveforms in load R.

Example 21-2

The ac source in Fig. 21-13a has an effective voltage of 120 V, 60 Hz. The load draws a dc current of 20 A.

Calculate

a. The dc voltage across the load
b. The average dc current in each diode

Solution

a. The dc voltage across the load is given by Eq. 21.1:

$$E_d = 0.90\ E$$
$$= 0.90 \times 120$$
$$= 108\ \text{V}$$

rms

b. The dc current in the load is known to be 20 A, but the diodes only carry the current on alternate half-cycles. Consequently, the average dc current in each diode is:

$$I = I_d/2 = 20/2 = 10\ \text{A}$$

Think Solution

21.8 Filters

The rectifier circuits we have studied so far produce pulsating voltages and currents. In some types of loads, we cannot tolerate such pulsations, and *filters* must be used to smooth out the valleys and peaks. The basic purpose of a dc filter is to produce a smooth power flow into a load. Consequently, a filter must

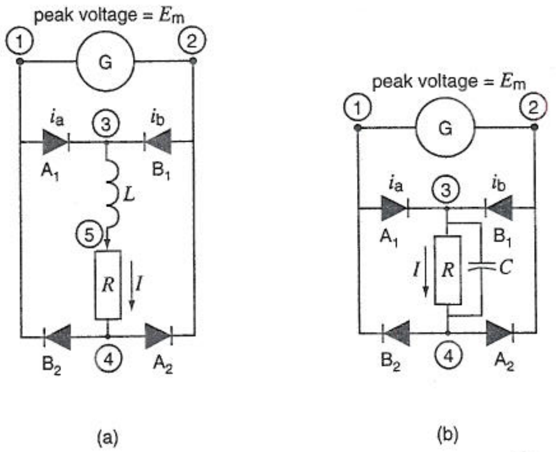


Figure 21.14
a. Rectifier with inductive filter.
b. Rectifier with capacitive filter.

absorb energy whenever the dc voltage or current tends to rise, and it must release energy whenever the voltage or current tends to fall. In this way the filter tends to maintain a constant voltage and current in the load.

The most common filters are inductors and capacitors. Inductors store energy in their magnetic field. They tend to maintain a constant current; consequently, they are placed *in series* with the load (Fig. 21.14a). Capacitors store energy in their electric field. They tend to maintain a constant voltage; consequently, they are placed *in parallel* with the load (Fig. 21.14b).

The greater the amount of energy stored in the filter, the better is the filtering action. In the case of a bridge rectifier using an inductor, the peak-to-peak ripple in percent is given by

ΔI

$$\text{ripple} = 5.5 \frac{P}{f W_L} \tag{21.2}$$

%

where

ripple = peak-to-peak current as a percent of the dc current [%]

W_L = dc energy stored in the smoothing inductor [J]

P = dc power drawn by the load [W]

f = frequency of the source [Hz]

5.5 = coefficient to take care of units

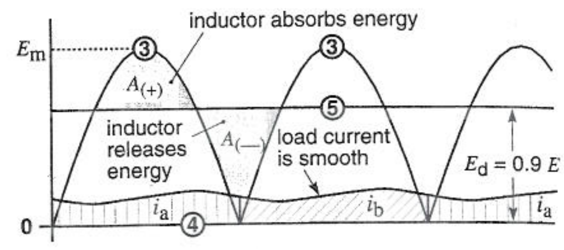


Figure 21.15
Current and voltage waveforms with inductive filter.

Key

The load current in Fig. 21.14a is much more constant than in Fig. 21.13a. The voltage between terminals 3 and 4 pulsates between zero and E_m as before, but the voltage E_{54} across the load is very smooth (Fig. 21.15). The dc voltage across the load is still given by Eq. 21.1. This is to be expected because the dc IR drop across the inductor is negligibly small.

Bridge rectifiers are used to provide dc current for relays, electromagnets, motors, and many other magnetic devices. In most cases the self-inductance of the coil is sufficient to provide good filtering. Thus, although the voltage across a coil may pulsate very strongly, the dc current can be smooth. Consequently, the magnetic field pulsates very little.

Low Chatter

Example 21-3

We wish to build a 135 V, 20 A dc power supply using a single-phase bridge rectifier and an inductive filter. The peak-to-peak current ripple should be about 10%. If a 60 Hz ac source is available, calculate the following:

a. The effective value of the ac voltage
b. The energy stored in the inductor
c. The inductance of the inductor
d. The peak-to-peak current ripple

Solution

a. The effective ac voltage E can be derived from Eq. 21.1:

derive for HW

$$E_d = 0.9\,E$$
$$135 = 0.9\,E$$
$$E = 150\ \text{V}$$

– rms

How to guarantee $L \neq f(I)$
L doesn't saturate

b. The dc power output of the rectifier is

$$P = E_d I_d$$
$$= 135 \times 20 = 2700 \text{ W}$$

The energy to be stored in the inductor or "choke" is given by

$$W_L = \frac{5.5\ P}{f \text{ ripple}} \qquad (21.2)$$
$$= \frac{5.5 \times 2700}{60 \times 10}$$
$$= 24.75 \text{ J}$$

Consequently, to obtain a peak-to-peak current ripple of 10 percent, the inductor must store 24.75 J in its magnetic field.

c. The inductance of the choke can be calculated from

$$W_L = \frac{1}{2} L I_d^2 \qquad (2.8)$$

?

$$24.75 = \frac{1}{2} L (20)^2$$

$$L = 0.124 \text{ H}$$

d. The peak-to-peak ripple is about 10 percent of the dc current:

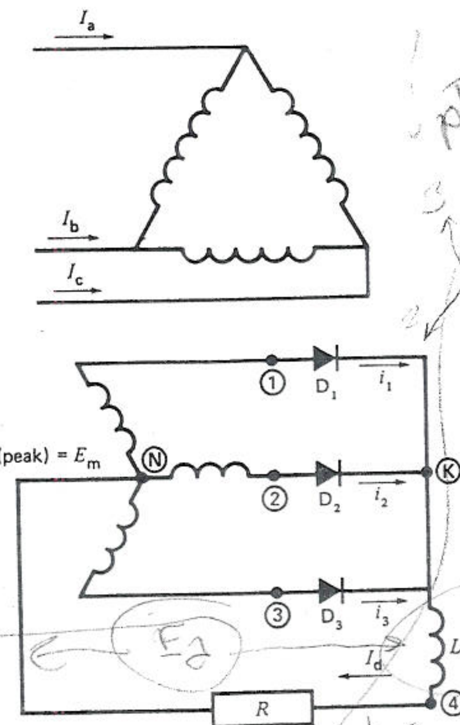
$$I_{\text{peak-to-peak}} = 0.1 \times 20 = 2 \text{ A}$$

The dc output current therefore pulsates between 19 A and 21 A.

21.9 Three-phase, 3-pulse diode rectifier

The simplest 3-phase rectifier is composed of three diodes connected in series with the secondary windings of a 3-phase, delta-wye transformer (Fig. 21.16). The line-to-neutral voltage has a peak value E_m. A large filter inductance L is connected in series with the load, so that current I_d remains essentially ripple-free. Although the load is represented by a resistance R, in reality it is always a useful energy-consuming device and not a heat-dissipating resistor. Thus, the load may be a dc motor, a large magnet, or an electroplating bath. This simple rectifier has some serious drawbacks, but it provides a good

AE
DC
pinned ?



Ieff set up in SS

small ripple

Figure 21.16
Three-phase, 3-pulse rectifier with inductive filter fed by a 3-phase transformer.

introduction to 3-phase rectifiers in general. We now analyze its behavior.

1. Voltage Across the Load. By choosing the transformer neutral as the zero potential reference point, the secondary terminals follow the voltage levels 1, 2, 3 shown in Fig. 21.17. These potential levels are rigidly fixed by the ac source and they successively reach a peak value E_m.

Before the transformer is energized, points **K, 4, N** are at the same level because I_d is zero. However, the moment we apply power, voltages E_{1N}, E_{2N}, E_{3N} appear. Consequently, at $t = 0$ the potential of point **1** suddenly becomes positive with respect to **K.** This immediately initiates conduction in diode D1 (Section 21.3, Rule 3). Current i_1 increases rapidly, attaining a final value I which depends upon load R. During this interval **K** is at the same level as point **1** because the diode is conducting. $\tau = \frac{L}{R}$

E = linear ramp

e wave $\frac{E_m}{2} \rightarrow E_m$? Why is a □-wave

Ripple as a fraction Erms

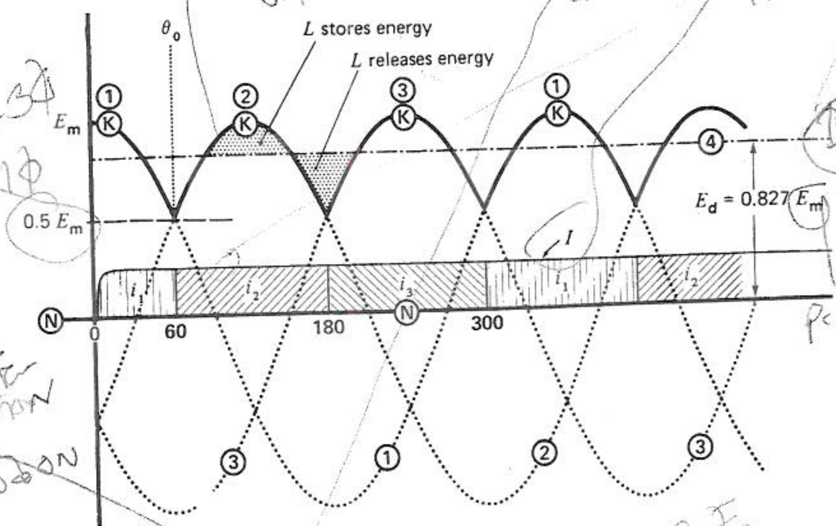


Figure 21.17
Voltage and current waveforms in a 3-phase, 3-pulse rectifier.

As points **K** and **1** move together in time, they eventually reach a critical moment, corresponding to an angle θ_0 of 60° (Fig. 21.17). The moment is critical because immediately later, terminal 2 becomes positive with respect to **K** and **1.** According to Rule 3, this initiates conduction in diode D2, so that *it* begins to carry current *I*. At the same time that conduction starts in diode D2, it ceases in diode D1. Consequently, beyond 60°, point **K** follows the level of point **2.**

The sudden switchover from one diode to another is called *commutation*. When the switchover takes place automatically (as it does in our example), it is called *natural commutation*, or *line commutation*. In this book we prefer the term *line commutation*, because it is the line voltage that forces the transfer of current from one diode to the next.

Commutation from one diode to another does not really take place instantaneously, as we have indicated. Owing to transformer leakage reactance, the current gradually increases in diode D2 while it decreases in diode D1. This gradual transition continues until all the load current is carried by diode D2. However, the commutation period is very short (typically less than 2 ms on a 60 Hz system) and, for our purposes, we will assume it occurs instantaneously.

The next critical moment occurs at 180°, because terminal 3 then becomes positive with respect to point **2** (and point **K**). Commutation again takes place as the load current switches from diode D2 to diode D3. Point **K** therefore follows the positive peaks of waves 1, 2, and 3, and each diode carries the full-load current for equal intervals of time (120°). The diode currents i_1, i_2, i_3 have rectangular waveshapes composed of positive current intervals of 120° followed by zero current intervals of 240°.

Voltage E_{KN} across the load and inductor in Fig. 21.17 pulsates between 0.5 E_m and E_m. The ripple voltage is therefore smaller than that produced by a single-phase bridge rectifier (Fig. 21.15). Moreover, the fundamental ripple frequency is three times the supply frequency, which makes it easier to achieve good filtering. The dc voltage across the load is given by

$$E_d = 0.675\,E \qquad (21.3)$$

where

E_d = average or dc voltage of a 3-pulse rectifier [V]

E = effective ac line voltage [V]

0.675 = a constant [exact value = $3/(\pi\sqrt{2})$]

explain why ∧ in to L-R graph

Key harmonics ↑

AE DC light rectifie

Note that if we reverse the diodes in Fig. 21.16, the rectifier operates the same way, except that the load current reverses. Voltage E_{KN} becomes negative and point **K** follows the negative peaks of waves 1, 2, and 3.

2. Line Currents Currents i_1, i_2, i_3 that flow in the diodes also flow in the secondary windings of the transformer. As we have seen, these currents have a chopped rectangular waveshape which is quite different from the sinusoidal currents we are familiar with. Furthermore, the currents flow for only one-third of the time in a given winding. Due to this intermittent flow, the maximum possible dc output power is less than the nominal rating of the transformer. For example, if the transformer in Fig. 21.16 has a rating of 100 kVA, we can show that it can only deliver 74 kW of dc power without overheating.

The chopped secondary currents are reflected into the primary windings, with the result that the line currents feeding the transformer also change very abruptly. The sudden jumps in currents I_a, I_b, and I_c produce rapid fluctuations in the magnetic field surrounding the feeder. These fluctuations can induce substantial voltages and noise in nearby telephone lines.

Because of these drawbacks, we try to design rectifiers so that the transformer windings carry current for more than one-third of the time. This is achieved by using 3-phase, 6-pulse rectifier.

more than 120°

21.10 Three-phase, 6-pulse rectifier*

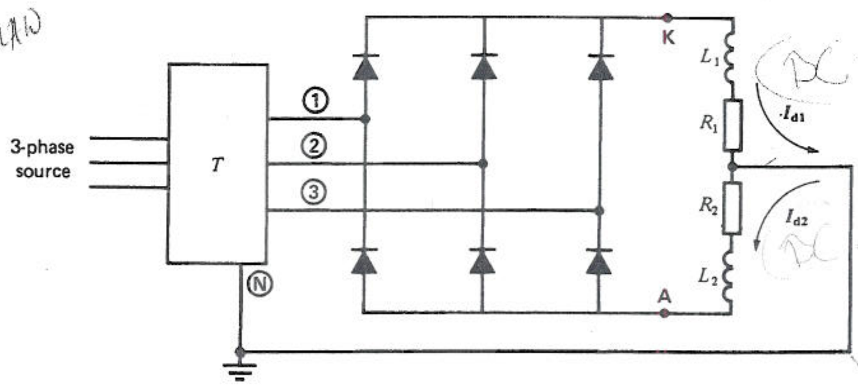
Consider the circuit of Fig. 21.18 in which a transformer T (identical to the one shown in Fig. 21.16), supplies power to 6 diodes and their associated dc loads R_1 and R_2. The upper set of diodes together with inductor L_1 and load R_1 are identical to the 3-phase, 3-pulse rectifier we have just studied. Thus, load current I_{d1} flows in the neutral line, as shown. The lower set of diodes, together with R_2 and L_2, also constitute a 3-phase, 3-pulse rectifier but with the polarity reversed. The corresponding load current I_{d2} flows in the neutral, as shown. The two 3-phase rectifiers operate quite independently of each other, **K** following the positive peaks of points **1, 2, 3** while A follows the negative peaks. All diodes conduct during 120° intervals.

If we make $R_1 = R_2$, then $I_{d1} = I_{d2}$ and the dc current in the neutral becomes zero. Consequently, we can remove the neutral conductor, yielding the circuit of Fig. 21.19. The two loads and the two inductors are simply combined into one, shown as R and L, respectively. The 6 diodes constitute what is called a 3-phase, 6-pulse rectifier. It is called *6-pulse* because the currents flowing in the 6 diodes start at 6 different moments during each cycle of the line frequency. However, each diode still conducts for only 120°.

$\frac{120}{360} = \frac{1}{3}$

* Also called a 3-phase bridge rectifier.

Previously 74% due to
- harmonics
- dutycycle



Net $I_{DC} = 0$

Figure 21.18
Dual 3-phase, 3-pulse rectifier.

i flows 240/360

See harmonics

why 95 kW at DC

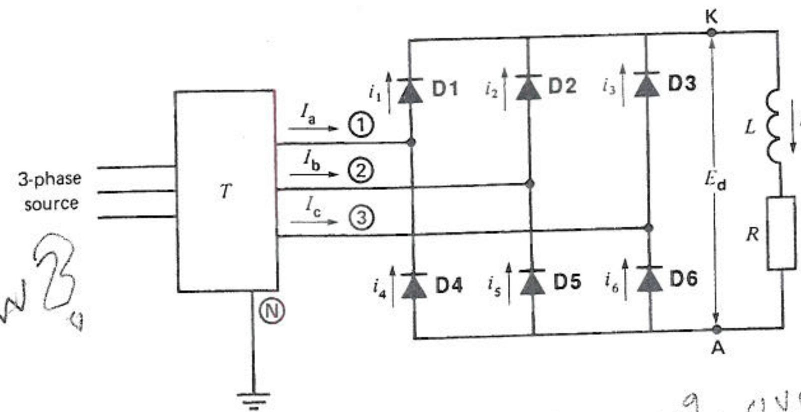


Figure 21.19
Three-phase, 6-pulse rectifier with inductive filter.

The line currents I_a, I_b, I_c supplied by the transformer are given by Kirchhoff's law:

See i of trf

$$I_a = i_1 - i_4$$
$$I_b = i_2 - i_5$$
$$I_c = i_3 - i_6$$

They consist of three identical rectangular waves that are out of phase by 120° (Fig. 21.20). The currents now flow for two-thirds of the time in the secondary windings. As a result, it can be shown that a 100 kVA transformer can deliver 95 kW of dc power without overheating.

Figs. 21.18 and 21.19 reveal that the average dc output voltage is twice that of a 3-phase, 3-pulse rectifier. Its value is given by 2x

$$E_d = 1.35\,E \tag{21.4}$$

where

E_d = dc voltage of a 6-pulse rectifier [V]
E = effective line voltage [V]
1.35 = a constant [exact value = $3\sqrt{2}/\pi$]

The instantaneous output voltage is equal to the intercept between levels K and A in Fig. 21.20. However, it is much easier to visualize the waveshape of E_{KA} by using terminal A as a reference point. Thus, in Fig. 21.21, we show the line voltages, E_{12}, E_{23}, E_{31} (and E_{21}, E_{32}, E_{13}) rather than the line-to-neutral voltages used in Fig. 21.20. The level of K follows the tops of the successive sine waves while A remains at zero potential. The output voltage fluctuates between 1.414 E and 1.225 E, where E is the effective value of the line voltage. The average value of E_{KA} is 1.35 E, as given by Eq. 21.4.

Key 30% overvoltage DC

The peak-to-peak ripple is only (1.414 − 1.225) E = 0.189 E and the fundamental ripple frequency is six times the line frequency. Consequently, the ripple is much easier to filter. The approximate peak-to-peak current ripple in percent is given by

20%

$$\text{ripple} = 0.17\,\frac{P}{fW_L} \tag{21.5}$$

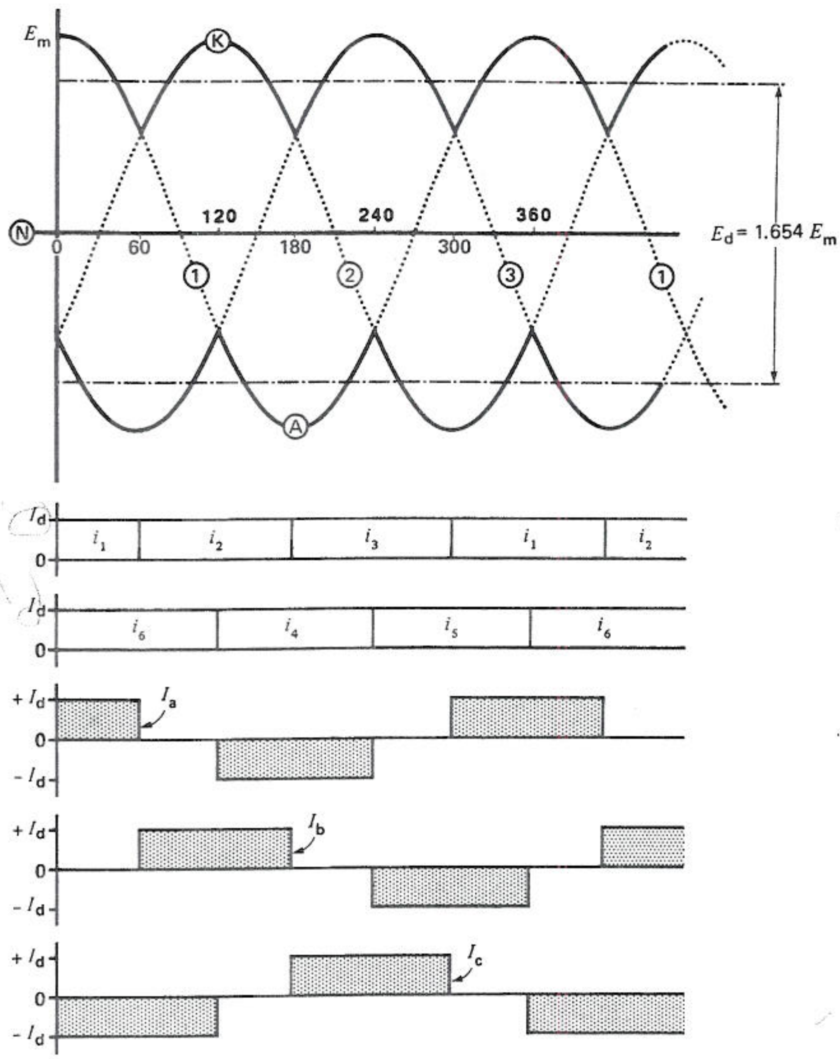
where

ripple = peak-to-peak current as a percent of the dc current [%]
W_L = dc energy stored in the inductor [J]
P = dc power drawn by the load [W]
f = frequency of the 3-phase, 6-pulse source [Hz]

Fig. 21.21 shows that the inductor stores energy whenever the rectifier voltage exceeds the average value E_d. This energy is then released during the brief interval when the rectifier voltage is less than E_d.

The peak inverse voltage across each diode is equal to the peak value of the line voltage, or $\sqrt{2}\,E$.

pg 484 derive



Diode Currents ON each i for 120° BUT Secondary Trf / Line currents ON for 240°

Figure 21.20
Voltage and current waveforms in Fig. 21.19.

Key line-line segment Not l-N

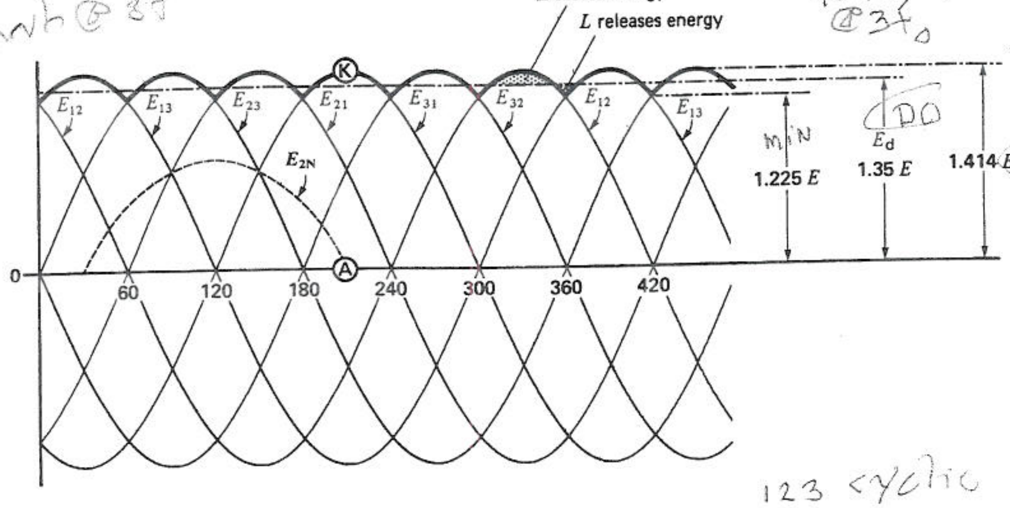
The 3-phase, 6-pulse rectifier is a big improvement over the 3-phase, 3-pulse rectifier. It constitutes the basic building block of most large rectifier installations.

Another way of looking at the 3-phase bridge rectifier is to imagine the diodes to be in a box (Fig. 21.22). The box is fed by three ac lines and it has two output terminals K and A. The diodes act like automatic switches that successively connect these terminals to the ac lines. The connections can be made in six distinct ways, as shown in Fig. 21.22. It follows that the output voltage E_{KA} is composed of segments of the ac line voltages. That is why we draw line voltages in Fig. 21.21 instead of line-to-neutral voltages.

Each dotted connection in Fig. 21.22 represents a diode that is conducting. The successive 60-degree intervals correspond to those in Fig. 21.20. For

60 segment @ 6ϕ vs 3pϕ 120° segment @ 3ϕ

1.414 / 1.225 √2 E rms VS 70% E rms for 3pϕ @ 3ϕ



123 cyclic

Figure 21.21
Another way of showing E_{KA} using line voltage potentials. Note also the position of E_{2N} with respect to the line voltages.

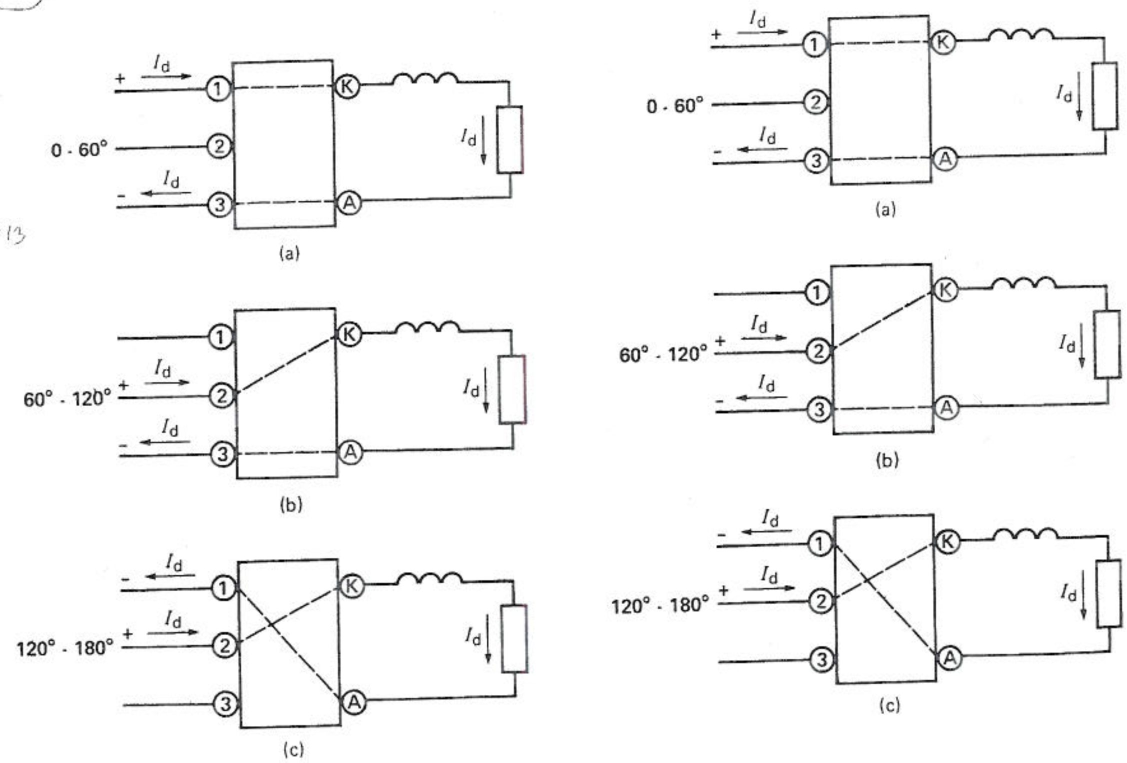


Figure 21.22
Successive diode connections between the 3-phase input and dc output terminals of a 3-phase, 6-pulse rectifier.

OK P = P what about Q?

example, from 300° to 360°, because i_1 and i_5 are flowing, diodes D1 and D5 are conducting. It follows from Fig. 21.19 that K is effectively connected to line 1 while A is effectively connected to line 2.

Because the diode voltage drop is small, we can assume that each dotted line represents a loss-free connection. The dc power absorbed by the load must therefore be equal to the active power drawn from the 3-phase source.

Example 21-4

A 3-phase bridge rectifier has to supply power to a 360 kW, 240 V dc load. If a 600 V, 3-phase, 60 Hz feeder is available, calculate the following:

a. Voltage rating of the 3-phase transformer
b. DC current per diode
c. PIV across each diode
d. Peak-to-peak ripple in the output voltage and its frequency

Solution

a. Secondary line voltage is $[V_{sec}(ac)]_{RMS}$

$$E = E_d/1.35 = 240/1.35$$
$$= 177 \text{ V}$$

No preference

Thus, a 3-phase transformer having a line voltage ratio of 600 V/177 V would be satisfactory. The primary and secondary windings may be connected either in wye or in delta.

b. dc load current I_d = 360 kW/240 = 1500 A

dc current per diode = 1500/3 = 500 A — 3 pairs $I_{DC}(eff)$

peak current in each diode = 1500 A — $I_{DC} = I_{pk}$

c. PIV across each diode

$$= \sqrt{2}\,E = 1.414 \times 177$$
$$= 250 \text{ V}$$

ONLY V_{sec}(rms)

d. The output voltage E_{KA} fluctuates between 1.225 E and 1.414 E (Fig. 21.21). In other words, the voltage fluctuates between

$$E_{min} = 1.225 \times 177 = 217 \text{ V and}$$
$$E_{max} = 1.414 \times 177 = 250 \text{ V}$$

The peak-to-peak ripple is, therefore,

$$E_{peak\text{-}to\text{-}peak} = 250 - 217 = 33 \text{ V}$$

Fundamental ripple frequency

$$= 6 \times 60 \text{ Hz} = 360 \text{ Hz}$$

$\frac{\Delta I}{I} = .05$

Example 21-5

a. Calculate the inductance of the smoothing choke required in Example 21-4, if the peak-to-peak ripple in the current is not to exceed 5 percent.
b. Does the presence of the choke modify the peak-to-peak ripple in the output voltage E_{KA}?

Solution

a. Using Eq. 21.5, we have

$$\text{ripple} = \frac{0.17\,P}{f\,W_L}$$
$$5 = \frac{0.17 \times 360\,000}{60 \times W_L}$$
$$W_L = 204 \text{ J}$$

I_{DC}

Consequently, the inductor must store 204 J in its magnetic field. The inductance is found from

$$W_L = \frac{1}{2} L I_d^2$$
$$204 = \frac{1}{2} L\,(1500)^2$$
$$L = 1.8 \times 10^{-4}$$
$$= 0.18 \text{ mH}$$

K–A same but ΔV_R

b. The presence of the choke does not affect the voltage ripple between K and A. It remains at 33 V peak-to-peak.

Key

21.11 Effective line current, fundamental line current

Line

We saw in Fig. 21.20 that the ac line currents consist of 120-degree rectangular waves having an amplitude I_d, where I_d is the dc current flowing in the load. Let us direct our attention to the current I_b flowing in line 2 and to the corresponding line-to-neutral voltage E_{2N}. They are shown in Fig. 21.23 and it can be seen that the rectangular current wave is symmetrically located with respect to the sinusoidal voltage maximum. In other words, the center of the positive current pulse coincides with the peak of the positive voltage wave. Thus, I_b can be considered to be "in phase" with E_{2N}.

Naturally via mains commutation

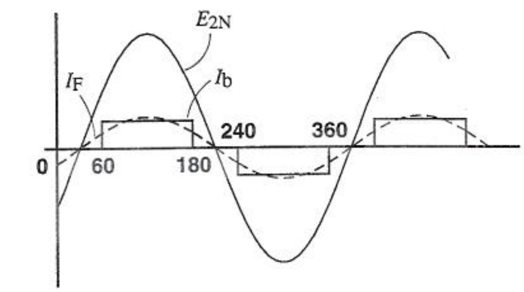


Figure 21.23
Line-to-neutral voltage and line current in phase 2 of Fig. 21.20.

The effective value I of the rectangular line current can be deduced from the relationship

$$I^2 \times 180° = I_d^{\,2} \times 120°$$

therefore

$$I = \sqrt{120/180}\; I_d$$
$$= 0.816\, I_d \tag{21.6}$$

This effective current is composed of a *fundamental* rms component I_F plus all the harmonic components. As we have seen, I_F is in phase with the line-to-neutral voltage.

What is the value of I_F? To calculate it we reason as follows:

The dc power to the load is

$$P_d = E_d I_d$$

The active ac power supplied to the rectifier (and its load) is

$$P_{ac} = \sqrt{3}\, E I_F \tag{8.9}$$

Because no power is lost or stored in our ideal rectifier, it follows that $P_{ac} = P_d$. We can therefore write

$$P_{ac} = P_d$$
$$\sqrt{3}\, E I_F = E_d I_d$$
$$= 1.35\, E I_d$$

and so

$$I_F = 0.78\, I_d \tag{21.7}$$

Combining Eqs. 21.6 and 21.7 we find

$$I_F = 0.955\, I \tag{21.8}$$

Thus, owing to the presence of harmonics,* the fundamental component I_F is slightly less than the effective value of the line current I.

21.12 Distortion power factor

We have just seen that the fundamental component I_F is in phase with the corresponding line-to-neutral voltage (Fig. 21.23). Consequently, we would be inclined to say that the power factor of the 3-phase, 6-pulse rectifier is 100 percent. However, by definition, power factor is given by the expression

$$\text{power factor} = \frac{\text{active power}}{\text{apparent power}}$$

$$= \frac{\text{active power}}{\text{effective voltage} \times \text{effective current} \times \sqrt{3}}$$

$$= \frac{P_{ac}}{EI\sqrt{3}} = \frac{EI_F\sqrt{3}}{EI\sqrt{3}} = \frac{I_F}{I}$$

But according to Eq. 21.8, $I_F = 0.955\, I$. As a result,

$$\text{power factor} = 0.955$$

Thus, the actual power factor is not 100% but only 95.5%. The reason is that the line current is rectangular and not sinusoidal. Thus, the power factor of 95.5% is due to distortion in the current.

Although the power factor of our rectifier is less than 100%, the fundamental component of current is nevertheless in phase with the line-to-neutral voltage. Consequently, this ideal rectifier absorbs no reactive power from the line.

21.13 Displacement power factor, total power factor

In Fig. 21.23, the fundamental component of current is in phase with the line-to-neutral voltage. However, in later circuits we will discover that the rectangular current wave can shift so that it lags behind the line-to-neutral voltage. This causes the fundamental component I_F to shift along with it.

* Harmonics are discussed in detail in Chapter 30.

Key — I has 60° lag but I_F is in phase with E neutral / E phase ⇒ $\cos\phi = 1$

This angular shift of the fundamental component of current with respect to the line-to-neutral voltage is called *displacement,* and the cosine of the angle is called *displacement power factor.* The displacement power factor in Fig. 21.23 is unity. The *total power factor* of a load or electrical installation is given by the expression:

$$\text{Total power factor} = \frac{P}{EI_L} \qquad (21.9a)$$

I total rms

The displacement power factor is given by:

Fund

$$\text{Displacement power factor} = \frac{P}{EI_F} \qquad (21.9b)$$

rms

In these equations,

P = active power per phase [W]

E = effective value of voltage per phase [V]

I_L = effective value of line current including the fundamental and harmonics [A]

I_F = effective value of fundamental component of line current [A]

Key

21.14 Harmonic content and THD

The rectangular current wave of Fig. 21.23 occurs very frequently in power electronics. It is therefore worthwhile to examine it more closely, particularly as regards its harmonic content. First, any periodic current in a line can be expressed by the equation

$$I^2 = I_F^2 + I_H^2 \qquad (21.10)$$

in which

I = rms value of the line current

I_F = rms value of the fundamental component of line current

I_H = rms value of all the harmonic components combined

It can also be shown that the total harmonic content I_H^2 is equal to the sum of the squares of the individual harmonics. Thus, we can write

$$I_H^2 = I_{HA}^2 + I_{HB}^2 + I_{HC}^2 + I_{HD}^2 + \ldots \qquad (21.11a)$$

in which I_{HA}, I_{HB}, I_{HC}, etc., are the rms values of the harmonic components in the line current.

The rectangular wave in Fig. 21.23 contains the 5th, 7th, 11th, 13th, ~~17th, harmonics, and so forth; in other words,~~ all odd harmonics that are not multiples of 3. The remarkable feature of these harmonic components is that their respective amplitudes are equal to the amplitude of the fundamental I_F divided by the order of the harmonic. For example, if the fundamental component has an rms value of 1500 A, the 17th harmonic has an rms value of 1500/17 = 88 A.

Why triplen gone? Simple I_F/N

The degree of distortion of an ac voltage or current is defined as the ratio of the rms value of all the harmonics divided by the rms value of the fundamental component. This *total harmonic distortion* (THD) is given by the formula

$$\text{THD} = \frac{I_H}{I_F} \qquad (21.11b)$$

rms

where I_F and I_H are defined as before.

For more information on harmonics, the reader should refer to Chapter 30.

V or I with corresponding I_{AC} or V_{AC} sine

Example 21-6

The 3-phase, 6-pulse rectifier in Fig. 21.19 furnishes a dc current of 400 A to the load. Estimate, for line 1:

a. The effective value of the line current measured by an rms hook-on ammeter
b. The effective value of the fundamental component of line current
c. The peak value of the 7th harmonic
d. The rms value of the 7th and 11th harmonics combined

I_F

I_{DC} (load) 400 I_{RMS}

Solution

a. The effective or rms value of the line current is

$$I = 0.816\, I_d = 0.816 \times 400 = 326 \text{ A} \qquad (21.6)$$

b. The rms value of the fundamental is

$$I_F = 0.955\, I = 0.955 \times 326 = 311 \text{ A}$$

$I_F \approx I$ but not fully

c. The rms value of the 7th harmonic is

$$I_{H7} = I_F/7 = 311/7 = 44 \text{ A}$$

The peak value of the $I_{H7} = 44\sqrt{2} = 62$ A