

### Sequential (Gray) Encoding

- represent states with 2 state bits  $[Q_1, Q_0]$

Encoding  $HG = 00 = \bar{Q}_1 \cdot \bar{Q}_0$ ;  $HY = \bar{Q}_1 \cdot Q_0$ ;  $FG = Q_1 \cdot Q_0$ ;  $FY = Q_1 \cdot \bar{Q}_0$

$$\begin{aligned} Q_1^+ &= TS \cdot HY + C \cdot \bar{TL} \cdot FG + (\bar{C} + TL) \cdot FG + \bar{TS} \cdot FY \\ &= TS \cdot HY + FG + \bar{TS} \cdot FY \end{aligned}$$

$$\begin{aligned} Q_0^+ &= C \cdot TL \cdot HG + \bar{TS} \cdot HY + TS \cdot HY + C \cdot \bar{TL} \cdot FG \\ &= C \cdot TL \cdot HG + HY + C \cdot \bar{TL} \cdot FG \end{aligned}$$

$$Output ST = C \cdot TL \cdot HG + TS \cdot HY + (\bar{C} + TL) \cdot FG + TS \cdot FY$$

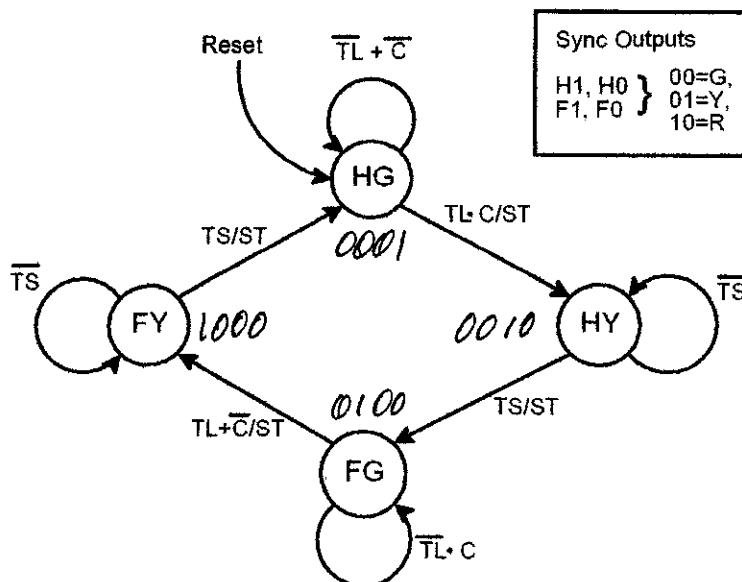
In terms of  $[Q_1, Q_0]$

$$Q_1^+ = TS \cdot \bar{Q}_1 \cdot \bar{Q}_0 + Q_1 \cdot Q_0 + \bar{TS} \cdot Q_1 \cdot \bar{Q}_0$$

$$Q_0^+ = C \cdot TL \cdot \bar{Q}_1 \cdot \bar{Q}_0 + \bar{Q}_1 \cdot Q_0 + C \cdot \bar{TL} \cdot Q_1 \cdot Q_0$$

$$\begin{aligned} ST = & C \cdot TL \cdot \bar{Q}_1 \cdot \bar{Q}_0 + \bar{Q}_1 \cdot Q_0 + C \cdot Q_1 \cdot Q_0 + \bar{C} \cdot Q_1 \cdot Q_0 + TL \cdot Q_1 \cdot Q_0 \\ & + TS \cdot Q_1 \cdot \bar{Q}_0 \end{aligned}$$

Worst case (ST) 15 literals, 5 terms,  
4-input gates



4 State bits  
 $[Q_3, Q_2, Q_1, Q_0]$

$$HG = 0001 = Q_0; HY = Q_1; FG = Q_2; FY = Q_3$$

## One Hot Encoding

$$Q_3^+ = \overline{TS} \cdot FY + (\bar{C} + TL) \cdot FG; \quad Q_2^+ = C \cdot \overline{TL} \cdot FG + TS \cdot HY$$

$$Q_1^+ = \overline{TS} \cdot HY + C \cdot TL \cdot HG; \quad Q_0^+ = (\bar{C} + \bar{TL}) \cdot HG + TS \cdot FY$$

$$ST = C \cdot TL \cdot HG + TS \cdot HY + (\bar{C} + TL) \cdot FG + TS \cdot FY$$

In terms of  $Q_3 \dots Q_0$

$$Q_3^+ = \overline{TS} \cdot Q_3 + \bar{C} \cdot Q_2 + TL \cdot Q_2$$

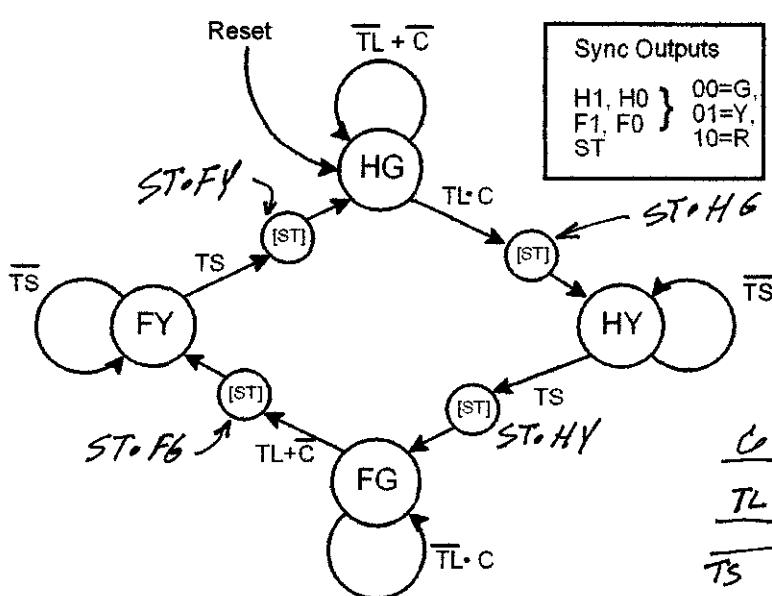
$$Q_2^+ = C \cdot \overline{TL} \cdot Q_2 + TS \cdot Q_1$$

$$Q_1^+ = \overline{TS} \cdot Q_1 + C \cdot TL \cdot Q_0$$

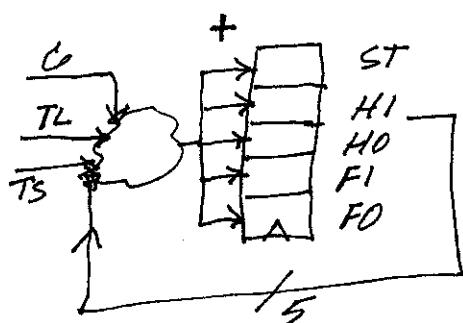
$$Q_0^+ = \bar{C} \cdot Q_0 + \bar{TL} \cdot Q_0 + TS \cdot Q_3$$

$$ST = C \cdot TL \cdot Q_0 + TS \cdot Q_1 + \bar{C} \cdot Q_2 + TL \cdot Q_2 + TS \cdot Q_3$$

Worst Case (ST) 11 literals, 5 terms, 3-input gates



State bits  
 $[ST, H_1, H_0, F_1, F_0]$



### Output oriented Encoding

$$HG^+ = ST \cdot FY + \bar{ST} \cdot HG$$

$$HY^+ = ST \cdot HG + \bar{ST} \cdot HY$$

$$F6^+ = ST \cdot HY + \bar{ST} \cdot F6$$

$$FY^+ = ST \cdot FG + \bar{ST} \cdot FY$$

$$ST^+ = C \cdot TL \cdot HG + TS \cdot HY + (\bar{C} + TL) \cdot F6 + TS \cdot FY$$

In terms of  $H_1, H_0, F_1, F_0$ :

$$H1 = FG + FY; \quad H1^+ = FG^+ + FY^+ = ST \cdot (HY + F6) + \bar{ST} \cdot (FY + F6)$$

$$H1^+ = ST \cdot H1 \cdot H0 + \bar{ST} \cdot F1 \cdot F0 + \bar{F1} \cdot F0$$

$$H1^+ = ST \cdot \bar{H1} \cdot H0 + \bar{ST} \cdot \bar{F1} \cdot F0 + \bar{F1} \cdot \bar{F0}$$

$$H0^+ = HY^+ = ST \cdot \bar{H1} \cdot \bar{H0} + \bar{ST} \cdot \bar{H1} \cdot H0$$

$$F1^+ = HG^+ + HY^+ = ST \cdot \bar{F1} \cdot F0 + \bar{ST} \cdot \bar{H1} \cdot H0 + \bar{H1} \cdot \bar{H0}$$

$$F0^+ = FY^+ = ST \cdot \bar{F1} \cdot \bar{F0} + \bar{ST} \cdot \bar{F1} \cdot F0$$

$$ST^+ = C \cdot TL \cdot \bar{H1} \cdot \bar{H0} + TS \cdot \bar{H1} \cdot H0 + \bar{C} \cdot \bar{F1} \cdot \bar{F0} + TL \cdot \bar{F1} \cdot \bar{F0}$$

$$+ TS \cdot \bar{F1} \cdot F0$$

Worst Case ( $ST^+$ ) 16 literals, 5 terms, 4-input gates

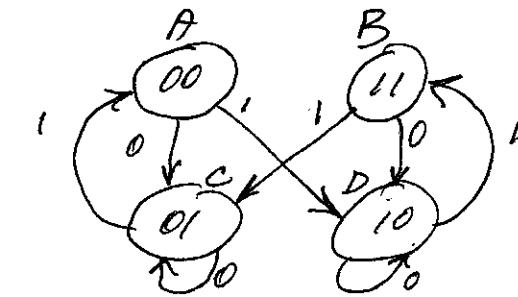
# Heuristic State Assignment

## Common Next State Association Example

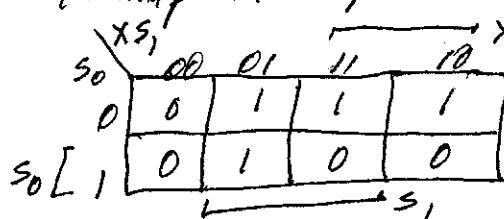
BAD Encoding

$$A=00, B=11$$

S	$S^+$	
	$x=0$	$x=1$
00	01	10
01	01	00
10	10	11
11	10	01



K-map for  $S_1^+$

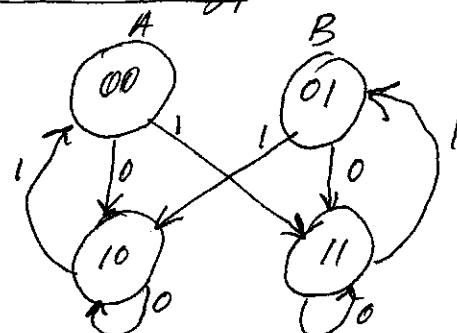


$$S_1^+ = x \cdot \bar{S}_0 + \bar{x} \cdot S_0$$

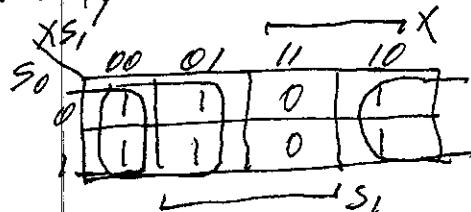
Encoding According to Rule (Good Encoding)

$$A=00, B=01$$

S	$S^+$	
	$x=0$	$x=1$
00	10	11
01	11	10
10	10	00
11	11	01



K-map for  $S_1^+$



$$S_1^+ = \bar{x} + \bar{S}_0$$

Comparison: Bad encoding - 4 literals, 2 prod terms, 2 input gates

Good encoding - 2 literals, 2 "prod terms", 1 input gate