

## **SIGNALS AND SYSTEMS LABORATORY 12:**

### **Linear Modulation and Demodulation**

#### **INTRODUCTION**

When the Federal Communications Commission issues a license to operate a radio transmitter, it requires the operator to meet specifications on power and frequency. For example, the radio station KOA in Denver is required to limit its total transmitter power to 50 kilowatts, and can transmit only in the frequency band from 845 kHz to 855 kHz. This band has a center frequency of 850 kHz, which is the transmitter carrier frequency, and has a 5kHz range on both sides. That 5 kHz number is the bandwidth of the audio baseband signal that is being communicated. The bandwidth of the *radio frequency* (RF) signal is 10 kHz. FM radio stations have similar constraints, although their allowed bandwidth is much greater (200 kHz). Television stations are given a 6 MHz bandwidth and future High Definition TV may be given even more.

The analog signals to be transmitted, coming from microphones, TV cameras, digital data sources and so on are referred to as *baseband* signals. These typically have an intrinsic data rate or bandwidth. Voice quality requires 3 kHz. High fidelity audio requires 20 kHz. Video requires a few MHz. The means of transforming one or more of these signals so that they occupy a portion of the RF spectrum is called, in general, *modulation* and may take several forms. Acronyms abound, such as AM, DSB, SSB, FM, PM, PAM, PSK, FSK, QAM, QPSK. Some applications require more than one form. Color television, for example involves fitting two high quality audio signals, and three video signals into one 6 MHz band, and involves both AM and FM.

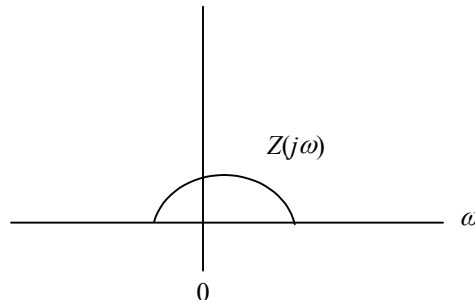
In this laboratory simulation exercise, we shall study a central issue, which is the characterization of signals which occupy an RF band. We will also consider a few common types of modulation.

#### **COMPLEX BASEBAND REPRESENTATIONS FOR REAL BANDPASS SIGNALS**

Let us begin with an arbitrary *complex baseband* signal, which we write in its polar and Cartesian forms:

$$(1) \quad \underset{\text{(polar)}}{z(t) = r(t)e^{j\phi(t)}} = \underset{\text{(cartesian)}}{x(t) + jy(t)}$$

Note that the polar representation is a generalized phasor representation, wherein magnitude and phase are time-varying. We say  $z(t)$  is baseband because its spectrum  $z(t) \leftrightarrow Z(j\omega)$  is bandlimited around dc ( $\omega = 0$ ). This is shown in Figure One below. This Spectrum is not generally Hermitian Symmetric because  $z(t)$  is complex.



**Figure One** The Complex Baseband Signal  $Z(j\omega)$

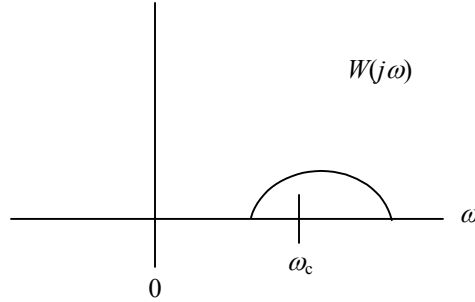
Perhaps from the *complex baseband signal*  $z(t)$  we can build a *real passband signal* that could be radiated from a mobile phone or transmitted through a coaxial cable. Let us consider the complex modulated signal

$$\begin{aligned}
(2) \quad w(t) &= z(t)e^{j\omega_c t}; \quad \omega_c : \text{carrier frequency} \\
&= r(t)e^{j\phi(t)}e^{j\omega_c t} \\
&= [x(t) + jy(t)][\cos(\omega_c t) + j\sin(\omega_c t)]
\end{aligned}$$

The complex spectrum of  $w(t)$  is

$$(3) \quad w(t) \longleftrightarrow W(j\omega) = Z(j(\omega - \omega_c))$$

The spectrum  $W(j\omega)$  is now bandlimited around the carrier frequency  $\omega_c$  as shown below in Figure Two.



**Figure Two** The Complex Passband Spectrum  $W(j\omega)$

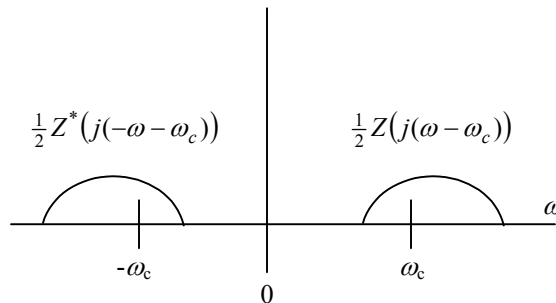
Note that the spectrum in Figure Two is still not Hermitian symmetric since our signal is still complex. In order to build a real signal that can be radiated or transmitted, let's constraint the real signal:

$$\begin{aligned}
(4) \quad s(t) &= \text{Re}[w(t)] = \frac{w(t)}{2} + \frac{w^*(t)}{2} & (a) \\
&= \text{Re}[z(t)e^{j\omega_c t}] = \text{Re}[r(t)e^{j\phi(t)}e^{j\omega_c t}] = r(t)\cos(\omega_c t + \phi(t)) & (b) \\
&= \text{Re}[x(t) + jy(t)][\cos(\omega_c t) + j\sin(\omega_c t)] = x(t)\cos(\omega_c t) - y(t)\sin(\omega_c t) & (c)
\end{aligned}$$

Each of these representations for the real signal  $s(t)$  brings its own insights. For example, equation (4b) shows the passband signal to be an amplitude and phase modulation of a cosinusoidal carrier, and equation (4c) shows it to be a quadrature modulation of sinusoidal and cosinusoidal carriers. Equation (4a) shows the spectrum of the passband signal to be

$$(5) \quad s(t) = \frac{w(t)}{2} + \frac{w^*(t)}{2} \longleftrightarrow S(j\omega) = \frac{1}{2}Z(j(\omega - \omega_c)) + \frac{1}{2}Z^*(j(-\omega - \omega_c))$$

This is illustrated in Figure Three.



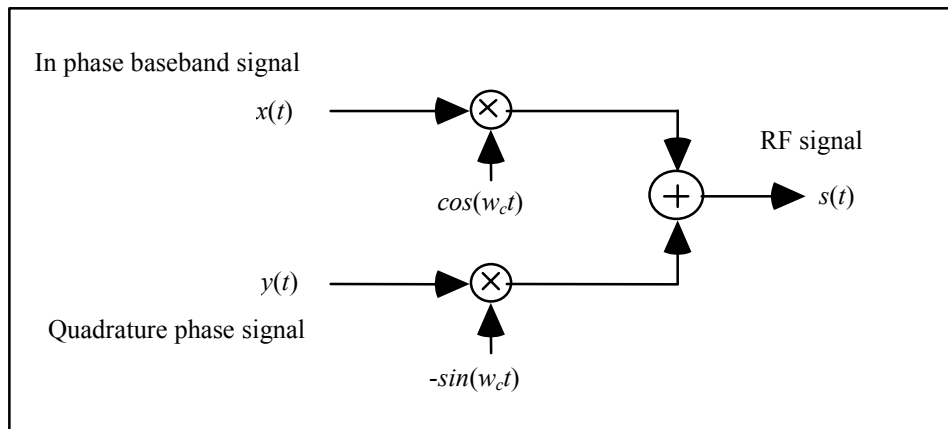
**Figure Three** Complex Spectrum of Real Passband Signal  $S(j\omega)$ . Note complex symmetry.

## IMPLEMENTATION

The signal  $s(t)$  is produced in Figure Four by a transmitter multiplying the output of an oscillator of frequency  $\omega_c$  with a baseband signal centered around dc. But bandpass signals don't have to be produced in this way. One can also get them by passing a broadband signal like white noise through a bandpass filter. The important thing to know about such signals is that they really involve not one but *two* baseband signals, which are called the *inphase* (or I) signal, and the *quadrature* phase (or Q) signal, both of which are centered around dc. These are like the real and imaginary parts of a phasor. Above, we used the notation  $x(t)$  for the I signal,  $y(t)$  for the Q signal, and  $s(t)$  for the RF signal. We have shown that every bandpass signal has the representation

$$(6) \quad s(t) = x(t) \cos(\omega_c t) - y(t) \sin(\omega_c t) = r(t) \cos(\omega_c t + \phi(t))$$

A block diagram for this equation looks like a simplified scheme of a transmitter as shown in Figure Four. There is really only one oscillator which produces both the sine and the cosine, but the two outputs must differ by a quarter wave. This representation also goes by the name of *Quadrature Amplitude Modulation*, or QAM. But no matter how the RF signal is actually produced, there will be exactly two independent baseband signal components, and they can be recovered from  $s(t)$ , assuming that a *local oscillator* (at the receiver) can be exactly synchronized in both frequency and phase, to the original. We shall examine this problem of QAM synchronous demodulation later in the lab.



**Figure Four** The quadrature modulator for creating a real passband signal  $s(t)$

## A REPRISE: ENVELOPE, PHASE, AND THE COMPLEX BASEBAND SIGNAL

If we think of the I and Q signals  $x(t)$  and  $y(t)$  as being two components of a phasor, then we are inevitably lead to considering the magnitude and phase, obtained by a rectangular to polar conversion. The magnitude  $r(t)$  is called the *envelope* of the RF wave because you can see it as the signal which rides the peaks of the sinusoid. It is the time-varying amplitude of the RF signal. The time-varying phase  $\phi(t)$  is not renamed. It is simply called the *phase* of the RF signal. The usual phasor relation between the Cartesian in-phase and quadrature signals  $x(t)$ ,  $y(t)$  and the envelope and phase  $r(t)$  and  $\phi(t)$  is

$$(7) \quad r(t)e^{j\phi(t)} = x(t) + jy(t)$$

or

$$(8) \quad \begin{aligned} x(t) &= r(t) \cos(\phi(t)), & \text{and} & & y(t) &= r(t) \sin(\phi(t)). \\ r(t) &= \sqrt{x^2(t) + y^2(t)}, & \text{and} & & \phi(t) &= \arctan(y(t)/x(t)) \end{aligned}$$

The complex time-varying phasor which can be built from these is called the *complex baseband signal*, or sometimes, the *analytic signal*:

$$(9) \quad z(t) = x(t) + j y(t) = r(t) e^{j\phi(t)}.$$

Using the complex baseband signal  $z(t)$ , we can write a very simple formula for the RF signal in its usual phasor form:

$$(10) \quad s(t) = \text{Re}[z(t)e^{j\omega_c t}]$$

It can be inferred from equations (6) and (10) that  $s(t)$  is a generalized rotating phasor whose slowly varying phasor is  $r(t)e^{j\phi(t)}$ . The signal  $s(t)$  is also a QAM signal. This can be interpreted in two ways. One, the signal  $s(t)$  is a signal whose amplitude is modulated by  $r(t)$  and whose phase is modulated by  $\phi(t)$ . Another way to look at this is to say that the inphase component of  $s(t)$  is modulated by  $x(t)$  and the quadrature phase component is modulated by  $y(t)$  (The second way being demonstrated in Figure Four). The game in analog communications (and digital, with a slight twist on our story) is to select  $x(t)$  and  $y(t)$  to get a certain type of modulation. The following table describes the components for some common methods of modulation. The signal  $m(t)$  is a baseband message signal. Can you fill in the two blank boxes? Why is there no magnitude  $|A+m(t)|$  in row 2, column 4 of the table?

Type	I signal	Q signal	Envelope	Phase	Remarks
QAM	$x(t) = a+m(t)$	$y(t)$	$r(t)$	$\phi(t)$	This is the general case. Generally there is a small carrier component $a$ for the local oscillator to lock onto. This is called a <i>pilot</i> .
AM	$A+m(t)$	0	$A+m(t)$	0	The carrier amplitude $A$ must be greater than the signal amplitude. AM signals can be demodulated with an envelope detector.
DSB-SC	$m(t)$	0	$ m(t) $	0 or $\pi$	Double sideband suppressed carrier signals require synchronous demodulation.
SSB	$m(t)$	Hilbert transform of $m(t)$			Single sideband needs only half the RF bandwidth. Requires synchronous demodulation. There may be a carrier pilot.
FM	$\cos(\phi(t))$	$\sin(\phi(t))$	1	$2\pi k_f \int_0^t m(\tau) d\tau$	The baseband signal is encoded in the phase of the RF signal. Demodulation via either a phase lock loop, or an FM discriminator.

## AN INITIALIZATION SCRIPT FILE

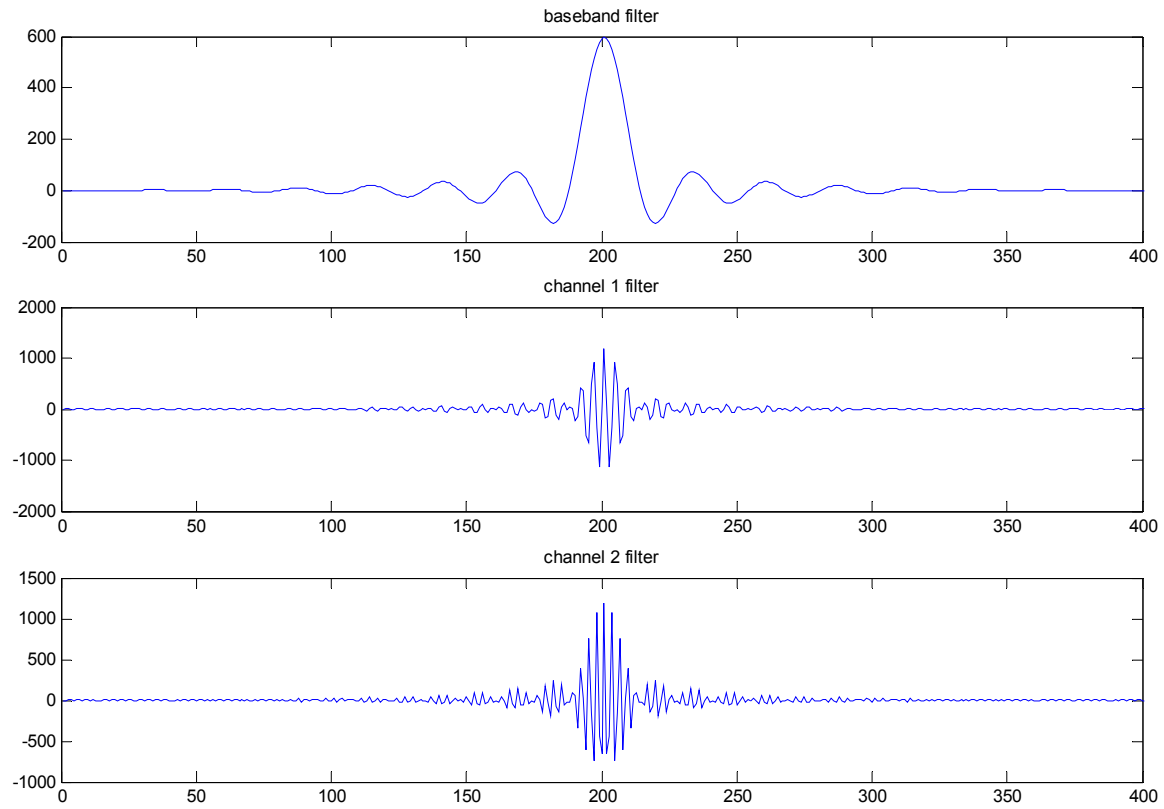
For the purpose of building a standard set of filters, baseband signals, carrier oscillators, and other parameters, the following script m-file is written. Recall that in MATLAB, script (as opposed to function) m-files leave all variables in the workspace. None are hidden. It is simply a way of getting a list of commands together, and is executed as if they were typed individually while in command mode. Study this file to see what parameters are produced. There is a time base vector, four baseband signals limited to  $f_0$  Hz, two RF channels, each with a cosine and a sine oscillator signal, a baseband filter and two RF bandpass filters. Locate 'script12.m' on the webpage and run it. The output is shown below in Figure Five.

```
% script12.m
% setup global data
```

```

L=2048; % data size
fs=8000; % sampling frequency
ts=1/fs; % sampling period
T=L*ts; % duration of signals
f0=300; % bandwidth of base-band
f1=1855; % center frequency of channel 1
f2=2755; % center frequency of channel 2
t=ts*[0:L-1]; % vector of sampling times
M=201; % offset of FIR filters in samples
k=[L/2:L/2+511]; % snapshot index set
tk=ts*k; % a time base for the snapshot
tkk=(ts/4)*[2*L:2*L+2047];
v=2*pi*f0*ts*[1:M-1];v=sin(v)./v; % construct a
bbf=2*f0*[flipud(v)',1,v]; % linear phase
bbf=bbf.*hamming(length(bbf))'; % FIR baseband filter
c1=cos(2*pi*f1*t); % inphase carrier, channel one
s1=sin(2*pi*f1*t); % quadrature carrier, channel one
c2=cos(2*pi*f2*t); % inphase carrier, channel two
s2=sin(2*pi*f2*t); % quadrature carrier, channel two
[b0,a0]=butter(4,.6*f0/(fs/2));
x1=filter(b0,a0,randn(1,L)); % channel 1 baseband I signal
x1=x1/max(abs(x1)); % set amplitude to 1
x2=filter(b0,a0,randn(1,L)); % channel 2 baseband I signal
x2=x2/max(abs(x2));
y1=filter(b0,a0,randn(1,L)); % channel 1 baseband Q signal
y1=y1/max(abs(y1));
y2=filter(b0,a0,randn(1,L)); % channel 2 baseband Q signal
y2=y2/max(abs(y2));
bp1=modulate([1,v],fs,f1); %build filter:
bp1=4*f0*[flipud(bp1(2:M))',bp1]; % channel 1 bandpass
bp1=bp1.*hamming(length(bp1))';
bp2=modulate([1,v],fs,f2); % build filter:
bp2=4*f0*[flipud(bp2(2:M))',bp2]; % channel 2 bandpass
bp2=bp2.*hamming(length(bp2))';
figure
subplot(3,1,1),plot(bbf),title('baseband filter')
subplot(3,1,2),plot(bp1),title('channel 1 filter')
subplot(3,1,3),plot(bp2),title('channel 2 filter')

```

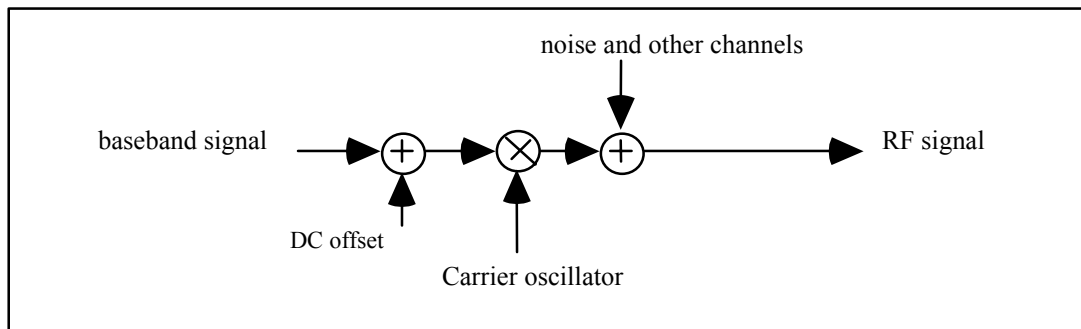


**Figure Five** Models of a baseband filter and two RF filters generated by `'script12.m'`

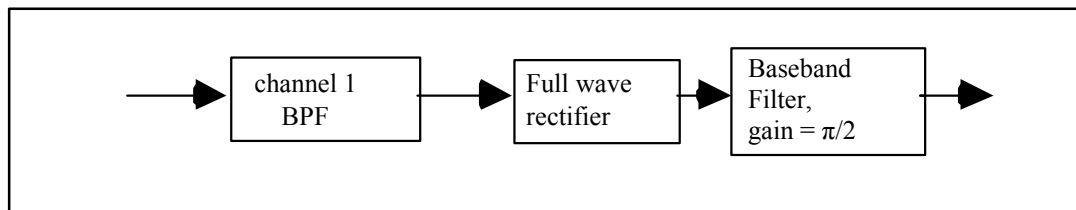
The I and Q signals for channels 1 and 2 are denoted  $x_1$ ,  $x_2$  and  $y_1$ ,  $y_2$ . They will be used in the assignment. Can you describe what kinds of signals they are?

## AN EXAMPLE: AMPLITUDE MODULATION

The following example, which illustrates Amplitude Modulation with carrier, is provided as a means of describing how the modulation and filtering is done using the variables produced by 'script12.m'. In commercial AM, the baseband signal is the envelope of the RF signal. This makes it especially easy to demodulate. Children once made their own crystal radio sets which contained only a coil (with wiper), a capacitor (which was called a *condenser*), a crystal with a "cat's whisker" (which amounted to a diode), a long piece of wire acting as an antenna, and a pair of primitive headphones. The diode served as an envelope detector.



AM transmitter and Channel



AM Receiver

**Figure Six** Simplified AM transmitter and Receiver

The following script m-file, 'demo12\_am.m', builds two AM RF signals, mimics the blocks for the receiver found in Figure Six, and plots the results. There is considerable overhead in the plotting. The three blocks in the receiver require only one line each. Download the files: 'script12.m', 'modulate.m', 'demo12\_am.m', 'spectrm.m' and 'upsample.m' from the webpage and run 'demo12\_am.m'.

```

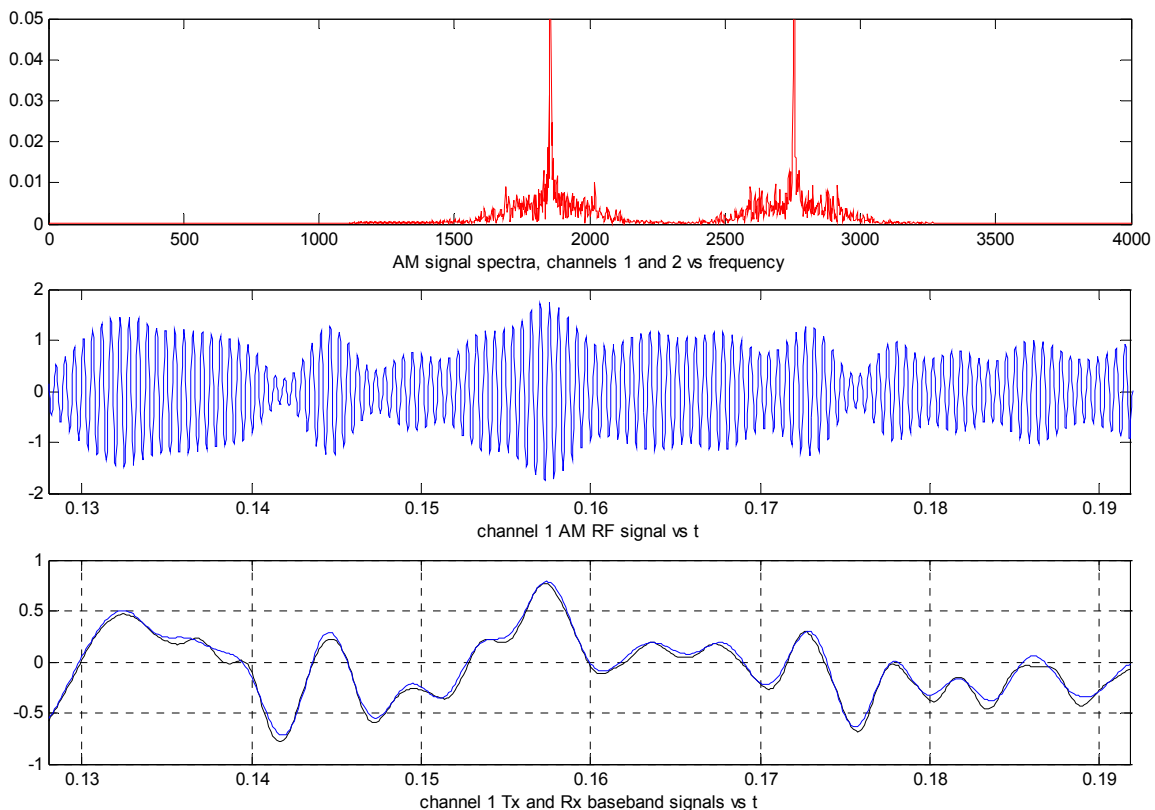
% demo12_am.m
% This script m-file requires variables that have been generated by script12.m
r1=(x1+1).*c1; % channel 1 AM signal
r2=(x2+1).*c2; % channel 2 AM signal
sound(r1+r2,fs)
subplot(3,1,1)
spectrm(r1+r2,fs)
axis([0,fs/2,0,.05])
xlabel('AM signal spectra, channels 1 and 2 vs frequency')
subplot(3,1,2)
w=ts*conv(bp1,r1+r2); % channel one bandpass filter
plot(tkk,upsample(w(k+M-1)));
axis([min(tk),max(tk),-2,2])
xlabel('channel 1 AM RF signal vs t')
w=abs(w); % full wave rectify
w=(pi*ts/2)*conv(bbf,w); % lowpass filter with gain of pi/2
  
```

```

w=w-sum(w)/L; % subtract DC offset
subplot(3,1,3)
plot(tk,x1(k),'k',tk,w(k+2*M-2),'b')
axis([min(tk),max(tk),-1,1])
xlabel('channel 1 Tx and Rx baseband signals vs t')
grid

```

The output of this program is in Figure Seven. Some comments: The RF signal was not plotted directly since it is grossly undersampled. The `'upsample.m'` tool and the more closely spaced time base vector `'tkk'` was used to provide accurate representations of the peaks in the RF signal. Make sure you understand how this works. These peaks tell you where the envelope is. Also, in order to compare Rx (receiver) and Tx (transmitter) baseband signals, a time shift was used to accommodate the inherent filter delay. More specifics are given in the text below. The two baseband signals in the third panel are not quite the same. There is an error due to imperfect filters, and the imperfect envelope detection. It is easy to see that the RF envelope is the original baseband signal with a DC offset  $x(t)=1+m(t)$ . The Q signal is zero and the phasor always has zero phase. AM with carrier is profligate in its use of power. As can be seen in the RF spectrum in the top panel, a great deal of transmitter power ends up in the carrier. This is the price that one must pay to make the receiver structure (envelope detection) easy.



**Figure Seven** AM signals produced by `'demo12_am.m'`



### Initialization and AM demodulation

Download the file '[script12.m](#)' from the webpage. While in MATLAB command mode, type

```
»script12
»whos
```

This will run the script m-file to compute global variables. The command 'whos' will produce a list of these variables and their sizes.

#### Assignment:

1. Use '[plotFT.m](#)' to look at the baseband filter 'bbf', and make a copy of the plots. Annotate it with the legend "baseband filter". Notice the linear phase characteristic of the baseband filter which results from the symmetry of the impulse response. The linear phase gives us a precise control over the delay inherent in the filter.

```
»figure;plotFT(bbf,fs,M*ts,2*f0)
```

2. Look at baseband, Channel One, and Channel Two, by plotting the spectrum of the sum 'bbf+bp1+bp2'. Make a copy of the results and annotate it.

```
»figure; spectrm(bbf+bp1+bp2,fs)
```

#### Three RF waveforms

The MATLAB function '[plotRF.m](#)' is designed to show the RF signal  $s(t)$ , the I and Q signals  $x(t)$  and  $y(t)$ , the envelope  $r(t)$  and the phase  $\phi(t)$  given data vectors  $x$ ,  $y$ , a carrier frequency and a data vector sampling frequency. Download it from the webpage and examine it using the MATLAB 'help' and 'type' commands.

3. Construct two baseband signals  $b1=\sin(2\pi f_1 t)$ , and  $b2=\sin(2\pi f_2 t)$ , where  $f_1 = 250$  Hz, and  $f_2 = 500$  Hz. Use a sampling frequency  $f_s = 50$  kHz, and a data vector length of  $L=512$ . (It is important to have the tool '[upsample.m](#)' working correctly) Then construct the I and Q signals for each of the following cases, and use the homebrew MATLAB tool '[plotRF.m](#)' to display the results. Hand annotate each plot, including analytic formulas for the I and Q signals  $x(t)$  and  $y(t)$ . When the envelope, phase, I, or Q signals are trivialized in some sense (for example when they are constant), be sure to make note of it and offer an explanation. Note here that both  $x$  and  $y$  must be the same length for '[plotRF.m](#)' to work correctly.

a) **AM:** Set  $x(t) = 1+0.8 b_1(t)$ ,  $y(t) = 0$ . Use  $f_c = 5$  kHz.  
(Hint: Code  $y(t)$  as  $y=zeros(1,length(x));$ )

b) **QAM:** Set  $x(t) = b_1(t)$ , and  $y(t) = b_2(t)$ . Let  $f_c = 10$  kHz.

c) **FM:** Compute the I and Q signals so that the envelope is  $r(t) = 1$ , and the phase is  $\phi(t) = 2\pi k_f \int_0^t b_1(\tau) d\tau$ . Use a carrier frequency of  $f_c = 5$  kHz, and a frequency sensitivity of  $k_f = 0.5 f_c$ . Note: If  $b1$  is the data vector for  $b_1(t)$ , then the data vector for the integral is  $ts*cumsum(b1)'$ .

## SIMULATION NOTES

Study the script m-file '[demo12\\_am.m](#)'. You will need some of the techniques used there to do the QAM demodulation in the next section. Here are some important points to keep in mind.

- Modulation can be done by multiplying the signal vector, element by element with the appropriate sinusoid. For example, to modulate against sine at the frequency for channel one, the operation is `y.*s1`, and the *dot* is important. The '[modulation.m](#)' tool does something similar, but it will assume a phase. It's best to use the vectors `c1` and `s1` produced by the initialization program.
- Filtering can be done using convolution. Suppose that we pass the signal `u` through the Channel One bandpass filter to get an output `v`. The filter simulation is

```
»v=ts*conv(bp1,u);
```

but be warned, you might have to fix things up. First, the vector `v` will be longer than the vector `u` by `length(bp1)-1`. If you want it to come out the same length, you must trim the excess. Second, the filter `bp1` has an inherent delay of `M-1` samples. If you want to compare input and output you will have to make an adjustment. For an example, consider the line in '[demo12\\_am.m](#)' that plots the receiver output against the transmitter input:

```
»plot(tk,x1(k),'g',tk,w(k+2*M-2),'b').
```

There is an adjustment here of `2*M-2` samples, because *two* filters were used!

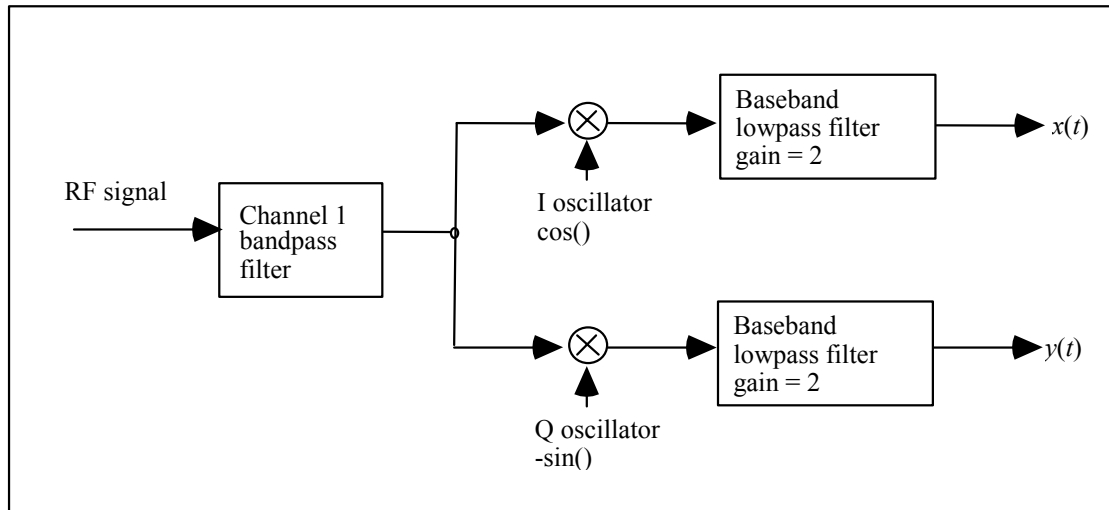
- When you plot an RF signal, use the time base `tkk` against the up-sampled version of the snapshot of the RF signal. This will somewhat faithfully reproduce the peaks of the RF wave, which support the envelope. Here is an example:

```
»plot(tkk,upsample(w(k+M-1)));
```

Note that the RF wave was also adjusted by `M-1` samples, so that proper alignment between plots is achieved.

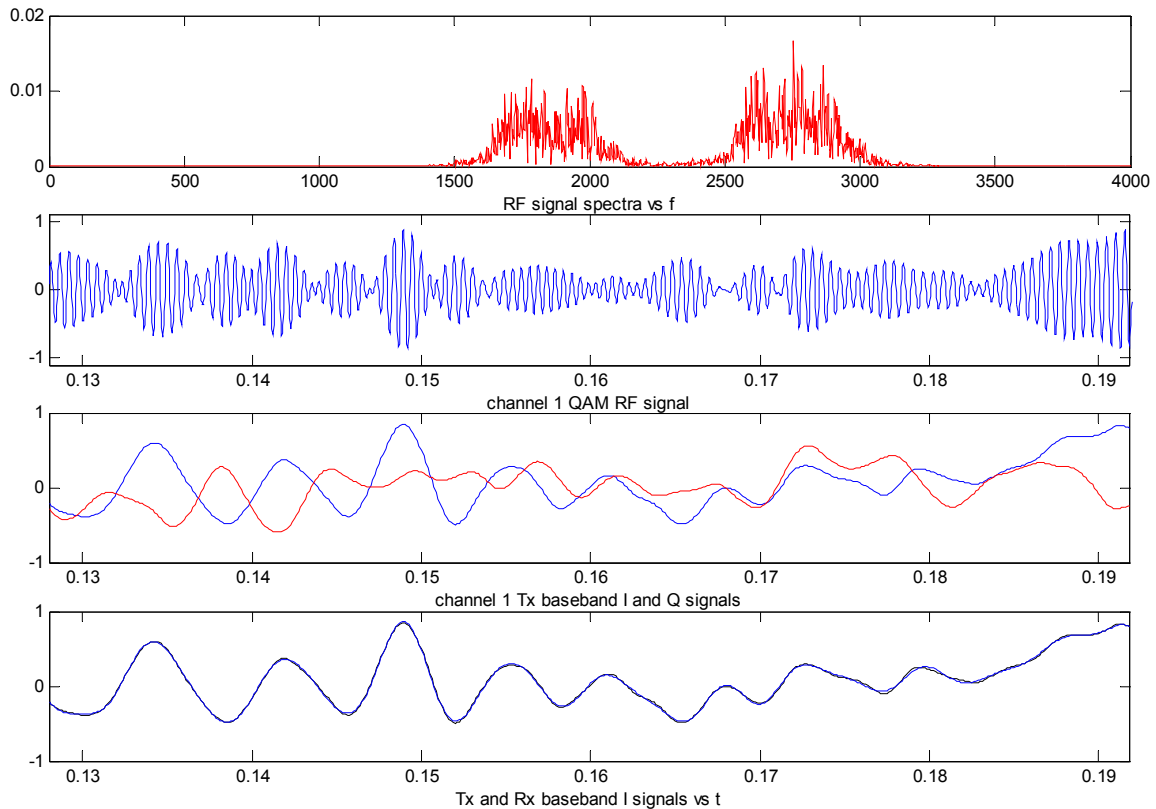
## QUADRATURE AMPLITUDE MODULATION (QAM)

Figure Four shows a QAM transmitter. The corresponding receiver structure, or demodulator, is shown in Figure Eight.



**Figure Eight.** Synchronous Demodulation receiver structure for QAM

Figure Nine shows what happens when the RF signal in Figure Eight contains two bandpass signals called channels one and two. The RF spectrum is in the first panel, and the two channels show up clearly. A time snapshot of the RF output of the channel One bandpass filter is shown in the second panel. The I and Q baseband components of this signal are shown in the third panel. Finally the transmitted ( $T_x$ ) and received ( $R_x$ ) I signals are compared in the fourth panel. The error is just enough to be noticeable.



**Figure Nine** QAM signals

**Assignment:**

5. Write a script m-file to simulate the QAM system, and reproduce Figure Nine. The input to your receiver should use both channels of RF:

```
»s=x1.*c1-y1.*s1+x2.*c2-y2.*s2; % two channel RF signal
```

The goal is to extract the I signal in Channel One and compare it to the original, which is the data vector **x1**. Be sure to accommodate filter delay, and to use the `'upsample.m'` tool to plot the RF wave. The shape of your baseband signal may differ, because it was generated randomly.

6. After you have constructed a receiver which successfully duplicates Figure Nine, you can study additional sources of error. First, consider adding white noise to the RF spectrum. This is present in any real system, and can be heard as the static between stations. Modify only the RF signal generation in your code. Add the line

```
»s=s+sigma*randn(size(s)); % Add noise
```

where **sigma** is an actual number. Start with **sigma = .2**, and vary it. Find the smallest value which produces a noticeable error in the fourth panel of Figure Nine. Also, generate a plot with considerable background noise. You should see the *white noise floor* in the RF spectrum in the first panel.

**Assignment:**

7. Now remove the noise statement. This time we will study the degradation caused by an error in local oscillator phase. In your receiver simulation, you must have had a line which demodulates the output of the bandpass filter. If it looked like this

```
»u=c1.*w; % demodulate;
```

then change the phase by an angle '**theta**' as follows:

```
»u=(cos(theta)*c1-sin(theta)*s1).*w; % demodulate with phase error.
```

Choose some values of theta, beginning near zero, and show what happens. You will see an increase in error in the fourth panel. Explain this analytically. Use the value

```
»theta=pi/2;
```

What will the output of the receiver I channel be with this phase error?