Comparison: Convolution vs DFT for Filtering

Example: FIR filter w/ $h = \{1, 2\}$ $M=2$

Look at input $x = \{1, -2, 2\}$ $L=3$

Output y will have length $N = M+L-1 = 4$

Convolution Method

$$y[n] = \sum_{k=0}^{3} h[k]\, x[n-k]$$

Graphical Method: $\overrightarrow{h\text{-rev}} = \{2, 1\}$ *

$x = \{1, -2, 2\}$

$y = \{1, 0, -2, 4\}$ check if DFT same

DFT Method

$H(z) = 1 + 2z^{-1}$; DFT $H[k] = H(z)\Big|_{z=e^{j\left(\frac{2\pi k}{4}\right)}}$

Now $e^{j\frac{2\pi}{4}k} = j^k \Rightarrow H[k] = 1 + 2(j)^{-k} = 1 + 2(-j)^k$

$H[k] = \{3, 1-2j, -1, 1+2j\}$

$X(z) = 1 - 2z^{-1} + 2z^{-2}$

$X[k] = 1 - 2(j)^{-k} + 2(j^2)^{-k} = 1 - 2(-j)^k + 2(-1)^k$

$X[k] = \{1, -1+2j, 5, -1-2j\}$

multiply out DFTs

$Y[k] = H[k]X[k] = \{3,\ 3+4j,\ -5,\ 3-4j\}$

Now take IDFT

$$y[n] = \frac{1}{4}\sum_{k=0}^{3} Y[k]\left(e^{-j\frac{2\pi}{4}}\right)^{kn} = \frac{1}{4}\sum_{k=0}^{3} Y[k](-j)^{kn}$$

$$y[0] = \frac{1}{4}\{3+3+4j-5+3-4j\} = \frac{1}{4}\cdot 4 = 1 \checkmark$$

$$y[1] = \frac{1}{4}\{3,\ 3+4j,\ -5,\ 3-4j\} * \{1,\ j,\ -1,\ -j\} = \frac{1}{4}\{3+3j-4+5-3j-4\}$$
$$= 0 \checkmark$$

$$y[2] = \frac{1}{4}\{3,\ 3+4j,\ -5,\ 3-4j\} * \{1,\ -1,\ 1,\ -1\} = \frac{1}{4}\{3-3+4j-5-3+4j\}$$
$$= -2 \checkmark$$

$$y[3] = \frac{1}{4}\{3,\ 3+4j,\ -5,\ 3-4j\} * \{1,\ -j,\ -1,\ j\} = \frac{1}{4}\{3-3j+4+5+3j+4\}$$
$$= 4 \checkmark$$

Same $y[n] = \{1, 0, -2, 4\}$ via DFT/IDFT as conv sum.