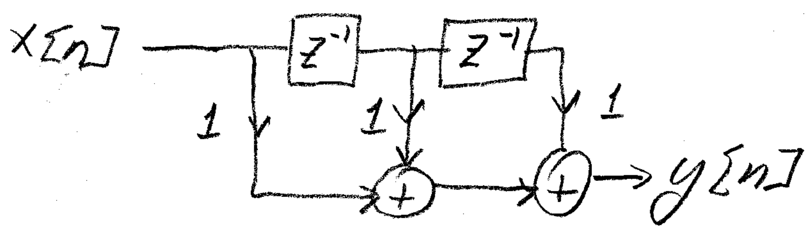
FIR Filter Implemented As Recursive Structure.

We think of FIR filters normally as feed-forward structures, but an FIR can also be implemented with a recursive part (and still be a finite impulse response filter).

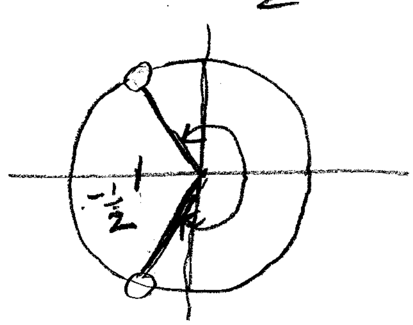
Simple example: moving average filter with equal tap weights.

(1) $y[n] = x[n] + x[n-1] + x[n-2]$. The usual feedforward implementation is



For this, $H(z) = 1 + z^{-1} + z^{-2}$. The zeros are $z_{1,2} = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$

zero plot



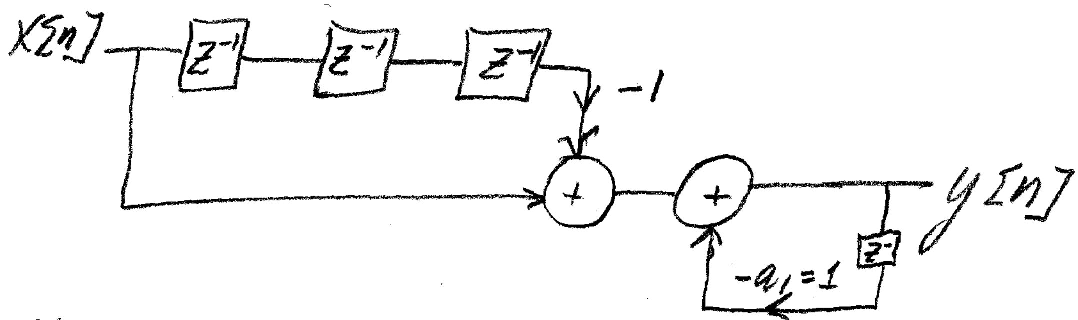
We can also write

$$y[n-1] = x[n-1] + x[n-2] + x[n-3]$$

So can rewrite (1) as

(2) $$y[n] = x[n] - x[3] + y[n-1]$$

The corresponding structure has both feedforward and feedback



For clarity, two adders are shown, but they could be combined

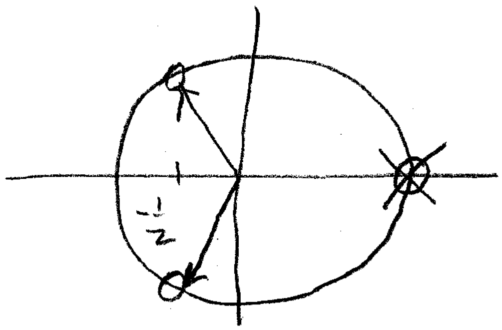
For this structure,

$$H(z) = (1+z^{-3})\left(\frac{1}{1-z^{-1}}\right)$$

This has zeros at $z_i = e^{+j \cdot \frac{2\pi}{3} n}$ $n=0,1,2$

There is a single pole at $p = 1$

The pole-zero plot is



So the pole and zero at $z=1$ cancel (with ∞ arithmetic precision), and the two remaining zeros are at the same location as for (1). Therefore, the two implementations have the same behavior, and feedback doesn't necessarily mean Infinite Impulse response.

- Although this was shown only for one simple example, it is generally true: any FIR structure can be converted into a (usually more compact) structure with a feedback section.