Lecture 5

Digital Filtering Idea

$$x[n] \longrightarrow \boxed{H(z)} \longrightarrow y[n] = (h * x)[n]$$

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k] \quad \Big(= \sum_{k=0}^{N} h[k]\, X[n-k] \Big)$$

$=$ if causal & finite-duration $h[k]$

- Filter Order N
- Impulse Response $N+1$ samples long

$$\updownarrow$$

$$Y(z) = H(z)\, X(z) \qquad (z\text{-transform})$$

DTFT (when $z = e^{j2\pi f t_0}$, t_0 : sample duration, $f_0 = 1/t_0$: sample rate)

$$Y(e^{j2\pi f t_0}) = H(e^{j2\pi f t_0})\, X(e^{j2\pi f t_0})$$

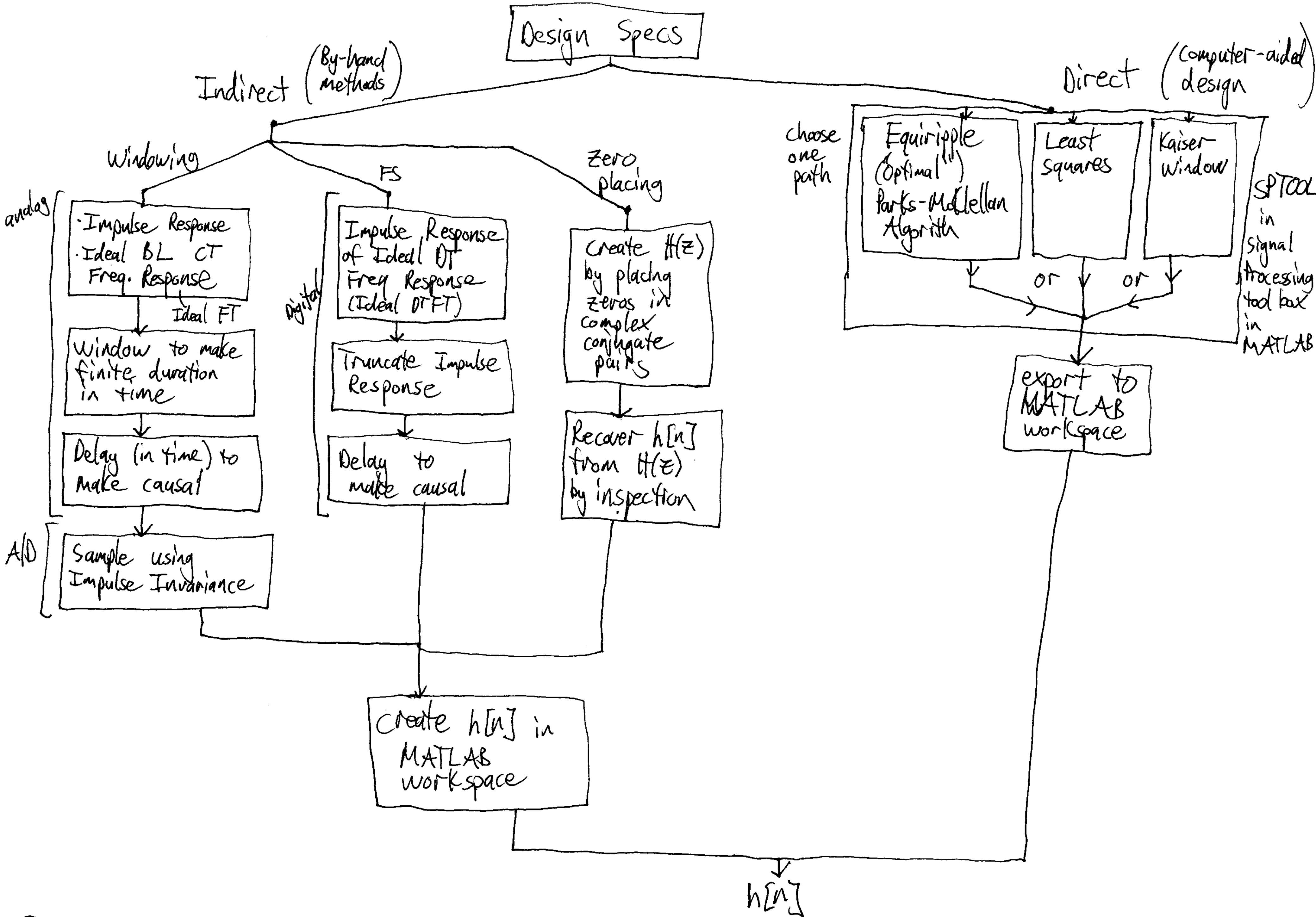
Sometimes we use the notation

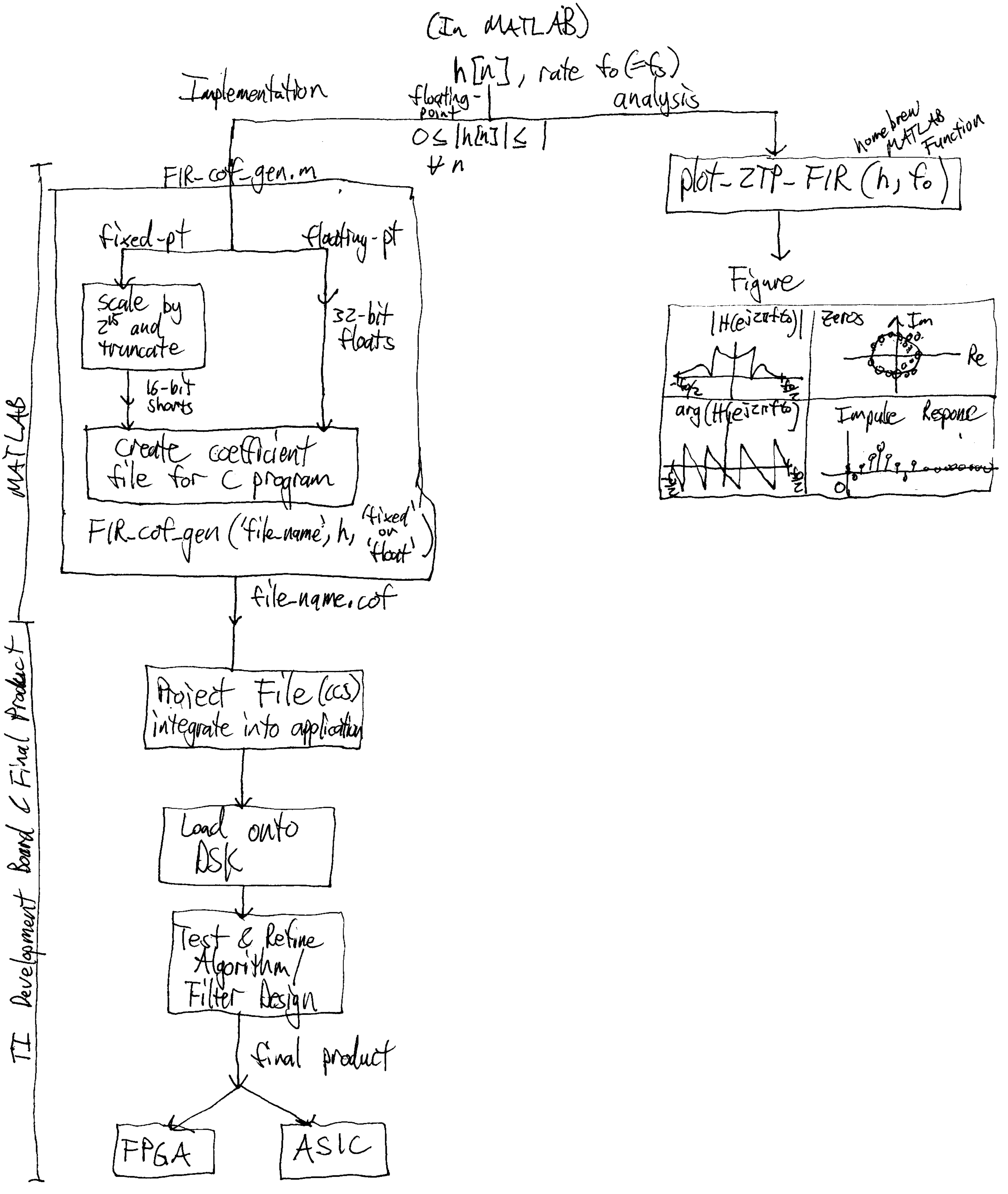
$$Y(e^{j2\pi f t_0}) = Y(e^{j\theta}) \quad \text{where } \theta = 2\pi f t_0 = 2\pi \frac{f}{f_0} \quad (= \omega t_0)$$

Now, passband and stopband frequencies are relative to the rate of the system. (i.e. $f = \frac{\theta}{2\pi t_0} = \frac{\theta f_0}{2\pi}$)

NB: In multi-rate systems f_0 need not be equal to sample rate of the codec (see the digital interpolation step in D/A conversion). However, in this lab $f_0 = f_s$ where $f_s = 8KHz$ for the on-board codec and $f_s = 24KHz$ on the D.C. codec.

"Production Line FIR Filter Design, Analysis, and Implementation"




(In MATLAB)
h[n], rate fo (=fs)
Implementation
floating-point
0 ≤ |h[n]| ≤ 1 ∀ n
analysis
homebrew MATLAB Function
plot_ZTP_FIR (h, fo)
Figure
|H(ej2πf/fo)|
Zeros
Im
Re
arg(H(ej2πf/fo))
Impulse Response
0
MATLAB
FIR_cof_gen.m
fixed-pt
floating-pt
scale by 2^15 and truncate
16-bit shorts
32-bit floats
create coefficient file for C program
FIR_cof_gen ('file_name', h, 'fixed' or 'float')
file_name.cof
TI Development Board & Final Product
Project File (ccs) integrate into application
Load onto DSK
Test & Refine Algorithm/Filter Design
final product
FPGA
ASIC

Lowpass Filter Design Example Using Windowing

Design Specs

$f_0 = f_s = 8\,kHz$ (On-board Codec)

pass band 0 - 1400 Hz

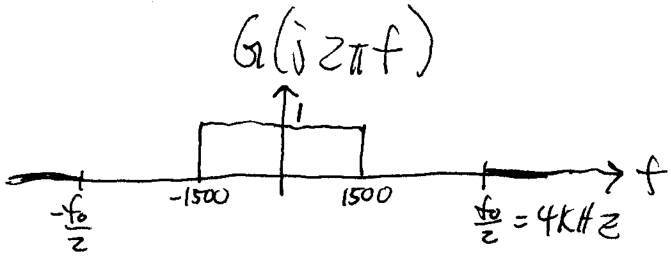
stop band 1600 - 4000 Hz

Filter Order $N \leq 70$

Minimize Aliasing $\longleftrightarrow$ Hanning Window

Design Process

1.) Choose Ideal Bandlimited Frequency Response



$$g(t) = \int_{-\infty}^{\infty} G(j2\pi f) e^{j2\pi f t}\, df$$

$$= \int_{-1500}^{1500} 1\, e^{j2\pi f t} df = \frac{1}{j2\pi t} e^{j2\pi f t}\Big|_{-1500}^{1500}$$

$$= \frac{1}{j2\pi t}\, e^{j2\pi(1500)t} - e^{-j2\pi(1500)t}$$

$$= \frac{1}{\pi t}\left(\frac{e^{j(\cdot)} - e^{j(\cdot)}}{2j}\right)$$

$$= \frac{1}{\pi t}\sin(2\pi(1500)t)\;\frac{2(1500)}{2(1500)}$$

$$= 2(1500)\,\frac{\sin(2\pi(1500)t)}{2\pi(1500)t}$$

$$= 2(1500)\,\mathrm{sinc}(2\pi(1500)t)$$

Note

$$g(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G(j\omega)e^{j\omega t}\,d\omega$$

let $\omega = 2\pi f$

$d\omega = 2\pi\, df$

$$g(t) = \cancel{\frac{1}{2\pi}}\int_{-\infty}^{\infty} G(j2\pi f)e^{j2\pi f t}\,\cancel{2\pi}\,df$$

$$= \int_{-\infty}^{\infty} G(j2\pi f)e^{j2\pi f t}\,df$$

In lab write-up, see table 1, LPF.

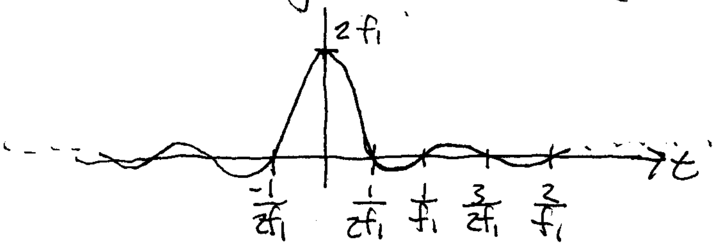
$g(t) = 2f_1\,\mathrm{sinc}(2\pi f_1 t)$

where $f_1 = 1500$ Hz is the cut-off frequency

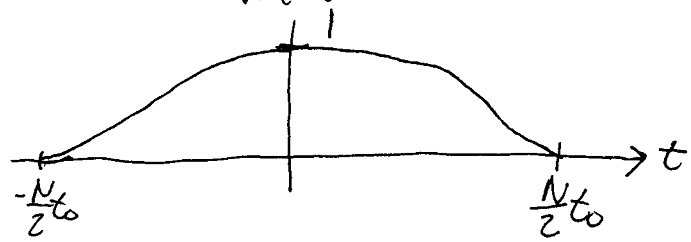
$-\frac{N}{2}t_0 < t < \frac{N}{2}t_0$

2.) Apply Window (Hanning Window, $w(t) = \begin{cases} \frac{1}{2} + \frac{1}{2}\cos\left(\frac{2\pi}{Nt_0} t\right) \\ 0 & \text{o.w.} \end{cases}$)

$g(t) = 2f_1 \,\mathrm{sinc}(2\pi f_1 t)$



$w(t)$

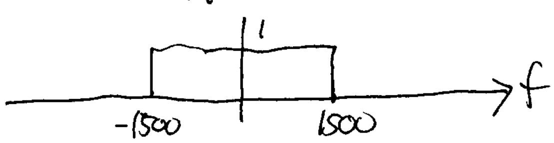


$f(t) = g(t)\,w(t)$ (multiplication in time)
see handout

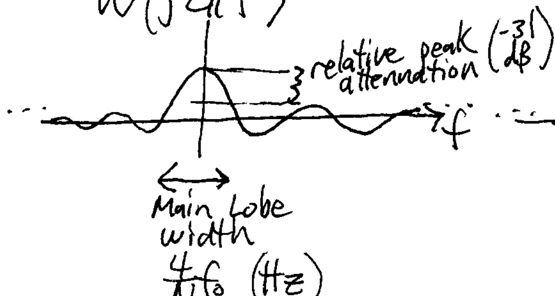
$\updownarrow$

$F(j2\pi f) = G(j2\pi f) * W(j2\pi f)$ (convolution in freq.)
(see Lab Notes Figure 5)

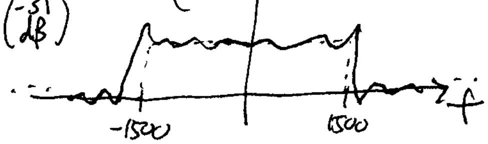
$G(j2\pi f)$



$W(j2\pi f)$

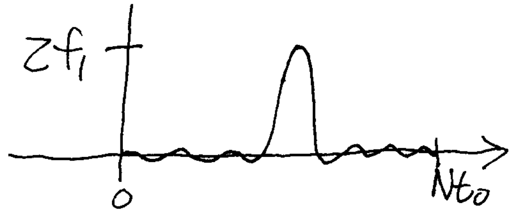


$F(j2\pi f)$

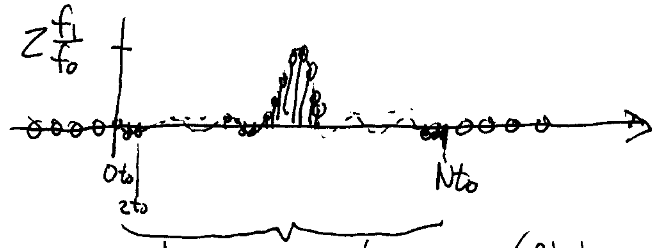


3.) Define $h(t) = f\left(t - \frac{N}{2}t_0\right)$

$h(t)$



4.) Define $h[n] = t_0 h(nt_0)$ (sample Using Impulse Invariance)



$N+1$ samples (finite-duration)

Notes $f_1 = \frac{f_0}{2} \equiv$ all pass filter (i.e. $h[n] = \delta[n]$

In general, $f_1 < \frac{f_0}{2} \Rightarrow 2\frac{f_1}{f_0} < 1$

From Sampling Theorem

$$H(e^{j2\pi f t_0}) = \frac{1}{t_0} \sum_{K=-\infty}^{\infty} H(j2\pi(f - Kf_0))$$

$$\updownarrow$$

$$h[n] = h(nt_0)$$

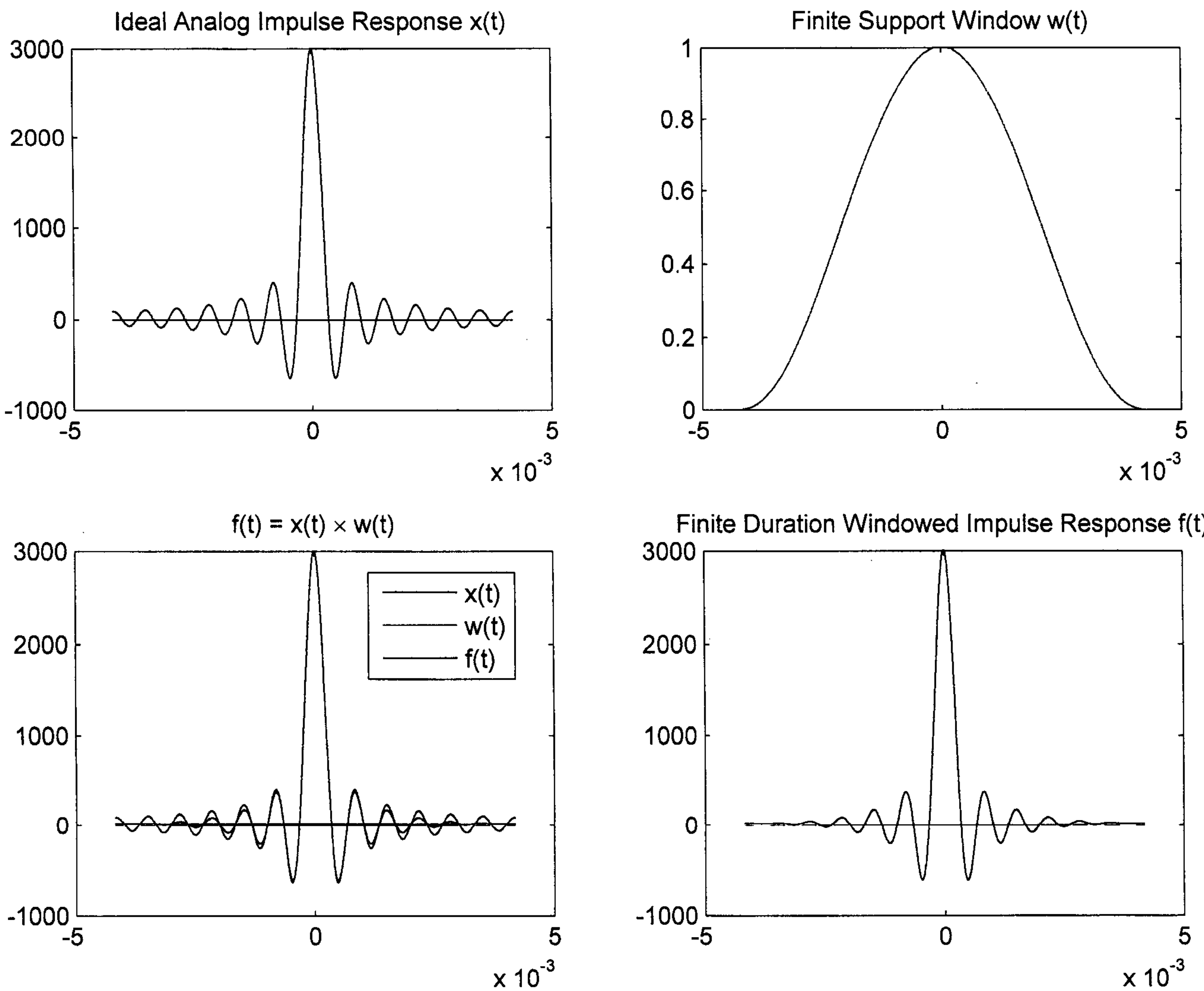
However, we want the DTFT of $h[n]$ to equal the FT of $h(t)$ for $-\frac{f_0}{2} < f < \frac{f_0}{2}$.

Note that $\frac{1}{t_0} = 8000$ is a rather large scaling in the frequency response. To compensate, we scale the impulse response by t_0. Since the DTFT pair $h[n] \longleftrightarrow H(e^{j2\pi f t_0})$ is a linear relationship, scaling one will scale the other.

This completes the design stage. For analysis, code the impulse response and pass it to plotZTP_FIR.m

The MATLAB code used to create the windowed continuous-time impulse response is shown below.

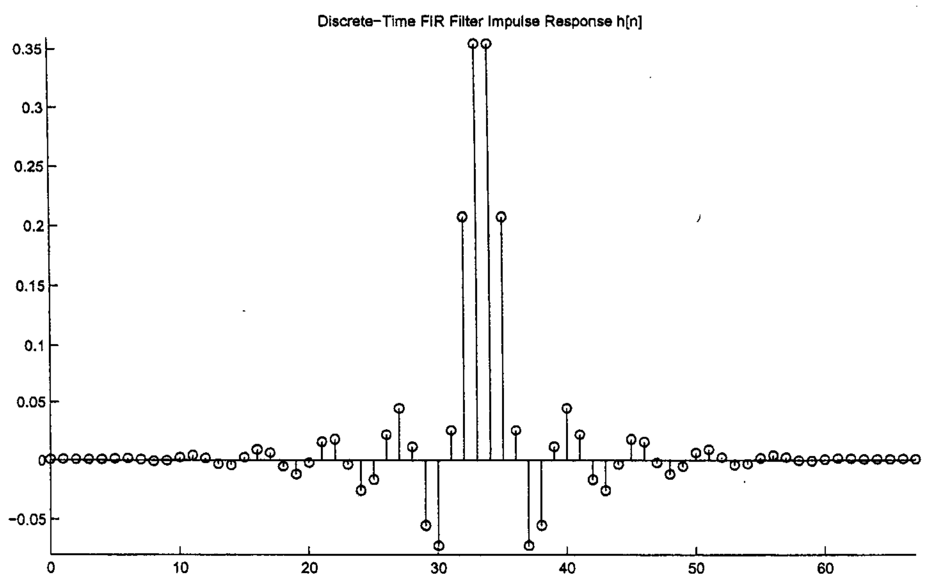
```
f0 = 8000;
t0 = 1/f0;

f1 = 1500;
N = 67;

t = (-N*t0/2):0.00001:(N*t0/2);   % Continuous time approx.
xt = 2*f1*sinc(2*f1*t);
wt = 0.5 + 0.5*cos((2*pi)/(N*t0)*t);

ft = xt .* wt;

% Plot Results
endpt = N*t0/2;

subplot(221)
plot(t,xt)
hold on
plot([-endpt endpt],[0 0],'k')
title('Ideal Analog Impulse Response x(t)')

subplot(222)
plot(t,wt,'g')
hold on
plot([-endpt endpt],[0 0],'k')
title('Finite Support Window w(t)')

subplot(223)
plot([-endpt endpt],[0 0],'k')
hold on
tmp = plot(t,xt,'b',t,wt,'g',t,ft,'r');
legend(tmp, 'x(t)','w(t)','f(t)')
title('f(t) = x(t) \times w(t)')

subplot(224)
plot([-endpt endpt],[0 0],'k')
hold on
plot(t,ft,'r')
title('Finite Duration Windowed Impulse Response f(t)')
```

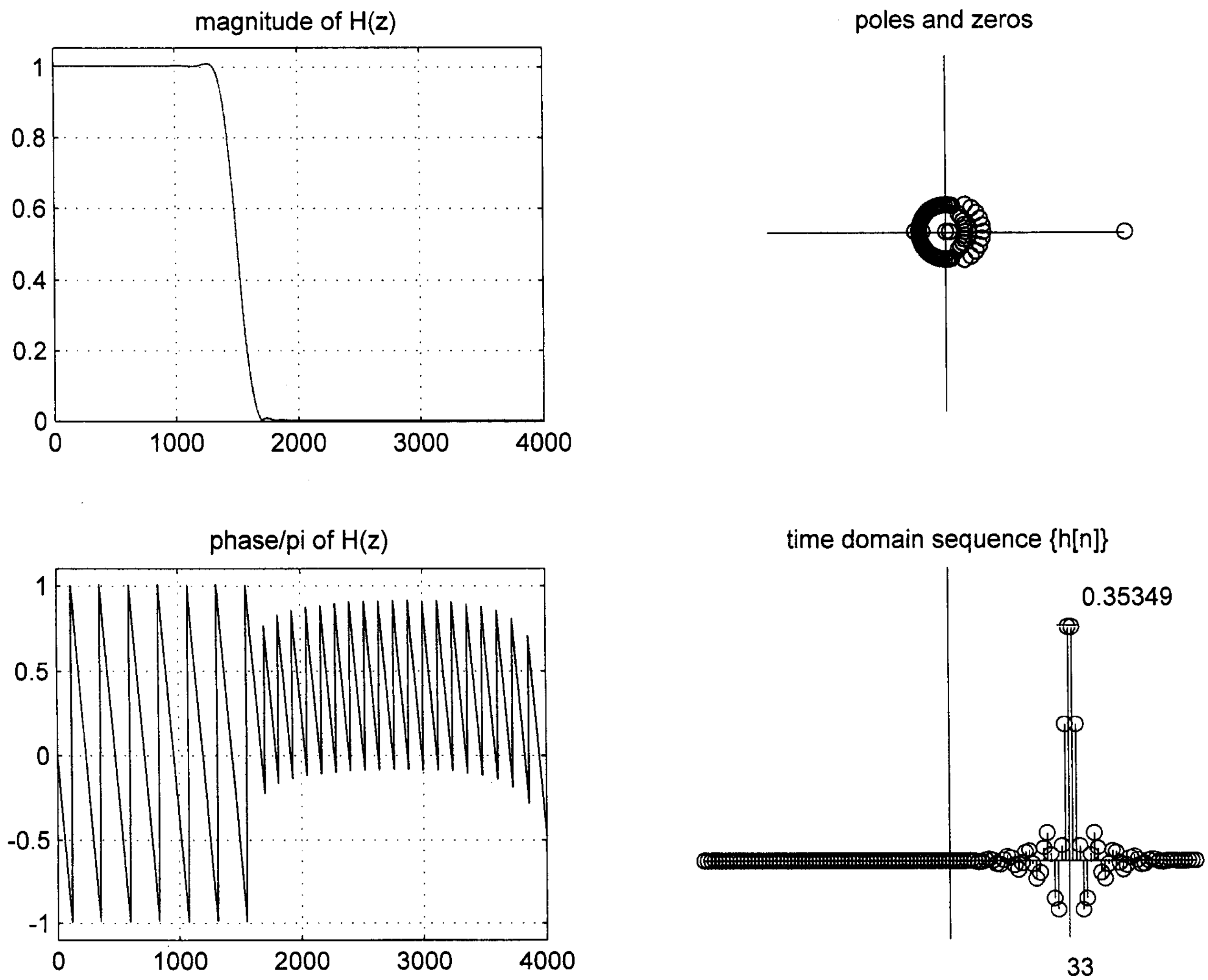


Ideal Analog Impulse Response x(t)
3000
2000
1000
0
-1000
-5
0
5
x 10^-3
Finite Support Window w(t)
1
0.8
0.6
0.4
0.2
0
-5
0
5
x 10^-3
f(t) = x(t) × w(t)
x(t)
w(t)
f(t)
3000
2000
1000
0
-1000
-5
0
5
x 10^-3
Finite Duration Windowed Impulse Response f(t)
3000
2000
1000
0
-1000
-5
0
5
x 10^-3

The MATLAB code used to calculate the causal impulse response $h[n]$ is shown below. The coefficients are stored in the workspace variable h. The resulting filter is analyzed using the homebrew MATLAB function plotZTP_FIR.m.

```
f0 = 8000;
t0 = 1/f0;

f1 = 1500;
N = 67;

n = 0:N;  % DT index
delay = N/2*t0;
xd = 2*f1*sinc(2*f1*(n*t0 - delay));
wd = 0.5 + 0.5*cos((2*pi)/(N*t0)*(n*t0 - delay));

h = t0 * (xd .* wd);
stem(n,h)
axis([0 67 -0.08 0.36])
title('Discrete-Time FIR Filter Impulse Response h[n]')

figure
plotZTP_FIR(h,f0)
```




magnitude of H(z)
poles and zeros
phase/pi of H(z)
time domain sequence {h[n]}
0.35349
33
0
1000
2000
3000
4000
0.2
0.4
0.6
0.8
1
-0.5
-1
0.5