REAL-TIME DSP LABORATORY 7:
Analog Communications using DSP Techniques on the C6713 DSK

Contents

1 Introduction

In traditional analog communication systems, a message modulates a sinusoid (oscillating at the carrier frequency) that is transmitted (radiated from an antenna) across a channel. Modulation reduces the size of the antenna and allows multiple users to use the same channel. In this lab, we will be concerned with signals that are bandlimited to the audio range (less than 20kHz), which means that we will use low frequency sinusoids, voice, and music as our message sources.

In this lab, you will study

- quadrature modulation,
- coherent and non-coherent demodulation, and
- AM, DSB-SC, QAM, SSB, and FM implementations using DSP techniques.

2 Quadrature Modulation

The basic idea in an analog communication system is to transform a message $m(t)$ into another signal $s(t)$ that can be transmitted across a channel [1],[2]. This transformation, or modulation, must be done in such a way that $m(t)$ may be recovered via demodulation from $s(t)$. The transmitted signal $s(t)$ may be called a *modulated waveform*, since it is generated by using the message signal $m(t)$ to modulate a sinusoidal carrier waveform. The demodulator recovers the message from the modulated waveform.

We will call our message signal $m(t)$ a *baseband* signal, since it will be bandlimited around DC ($\omega = 0$), and we will call our transmit signal $s(t)$ a *passband* signal, since it will be bandlimited around the carrier frequency ($\omega = \omega_c$). In practice, the carrier frequency will be in the *radio frequency* (or RF) range (30kHz to 3GHz), so $s(t)$ may also be referred to as an RF signal. However, the codecs available for the DSK do not have the sampling rate required to process RF signals, so we will use frequencies in the audio range[1]. We will agree to overlook this fact and still refer to the transmit signal as an RF signal for the role that it would play in the communication system for which it was intended.

Sinusoidal carriers will be used to transmit a modulated waveform over a channel. At a given frequency, there are really two orthogonal sinusoids (90^o out of phase) that may be used as information bearing waveforms. To see this, consider a sinusoidal with frequency ω. Integrating this sinusoid with a 90^o phase shifted version of the same sinusoid over one period will yield

$$\int_{<T>} \cos(\omega t)\cos(\omega t - \tfrac{\pi}{2})dt = \int_{<T>} \cos(\omega t)\sin(\omega t)dt = 0, \qquad \omega = \frac{2\pi}{T}, \tag{1}$$

which means that $\cos(\omega t)$ and $\sin(\omega t)$ are orthogonal signals. Using this fact, we realize that we may transmit two channels worth of information (independently) over the same bandwidth. We will develop this idea next when we construct a real (one channel) passband signal from a complex (two channel) baseband signal.

2.1 Complex Baseband Representations For Real Passband Signals

The idea behind quadrature modulation is that a real baseband message signal $m(t)$ may be used to form a complex baseband signal $z(t)$ that complex modulates the time-varying phasor $e^{j\omega_c t}$, rotating at the carrier frequency. The real part of this complex modulated signal (real passband signal), namely $s(t)$, is then transmitted across a channel. Let's consider the arbitrary complex baseband signal

$$z(t) = A(t)e^{j\phi(t)} = x(t) + jy(t). \tag{2}$$

[1]There exist high-end DSP-based systems that sample well into the RF range ($f_s = 2.208$MHz). By using an oscillator and a mixer (a non-linear circuit that multiplies two signals together), an RF signal that is bandlimited around a carrier may be shifted to a frequency that is more suitable for DSP (< 1.1MHz). This process of frequency shifting is known as heterodyning.

Here, the polar representation is a generalized phasor representation, wherein magnitude and phase are time-varying. We say $z(t)$ is baseband because its spectrum $z(t) \longleftrightarrow Z(j\omega)$ is bandlimited around DC ($\omega = 0$). This is shown in Figure 1. This spectrum is not generally Hermitian symmetric because $z(t)$ is complex.

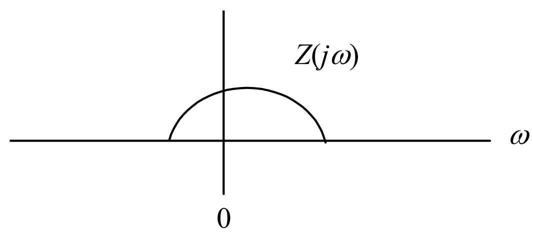

Figure 1: The complex baseband spectrum $Z(j\omega)$. Taken from [3].

From this complex baseband signal $z(t)$ we want to build a real passband signal that may be radiated from a mobile phone or transmitted through a coaxial cable. Consider the complex modulated signal

$$\begin{aligned} w(t) &= z(t)e^{j\omega_c t}; \qquad \omega_c\text{: carrier frequency} \\ &= A(t)e^{j\phi t}e^{j\omega_c t} \\ &= [x(t) + jy(t)][\cos(\omega_c t) + j\sin(\omega_c t)]. \end{aligned} \tag{3}$$

The complex spectrum of $w(t)$ is $w(t) \longleftrightarrow W(j\omega) = Z(j(\omega - \omega_c))$. The spectrum $W(j\omega)$ is now bandlimited around the carrier frequency ω_c as shown in Figure 2. This signal is often referred to as a *complex analytic signal*, which means that it contains no "negative frequencies".

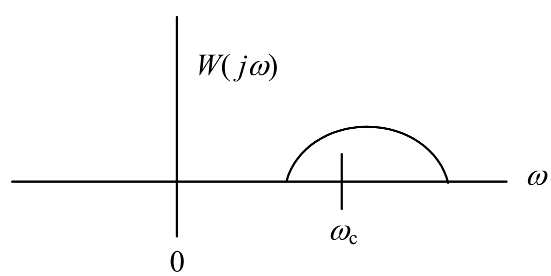

Figure 2: The complex bandpass spectrum $W(j\omega)$. Taken from [3].

Note that the spectrum in Figure 2 is still not Hermitian symmetric, since the signal $w(t)$ is complex. In order to build a real signal that can be radiated or transmitted, let's construct the real signal

$$
\begin{aligned}
s(t) &= Re\{w(t)\} = \frac{w(t)}{2} + \frac{\overline{w(t)}}{2} && (4)\\
&= Re\left\{z(t)e^{j\omega_c t}\right\} = Re\left\{A(t)e^{j\phi t}e^{j\omega_c t}\right\} = A(t)\cos(\omega_c t + \phi(t)) && (5)\\
&= Re\left\{[x(t) + jy(t)][\cos(\omega_c t) + j\sin(\omega_c t)]\right\} = x(t)\cos(\omega_c t) - y(t)\sin(\omega_c t). && (6)
\end{aligned}
$$

Each of these representations for the real signal $s(t)$ brings its own insights. For example, eqn (5) shows the bandpass signal to be an amplitude and phase modulation of a cosinusoidal carrier, and eqn (6) shows it to be a quadrature modulation of sinusoidal and cosinusoidal carriers. Equation (4) shows the spectrum of the bandpass signal to be

$$
s(t) = \frac{w(t)}{2} + \frac{\overline{w(t)}}{2} \longleftrightarrow S(j\omega) = \frac{1}{2}Z(j(\omega - \omega_c)) + \frac{1}{2}\overline{Z(j(-\omega - \omega_c))}. \quad (7)
$$

This is illustrated in Figure 3.

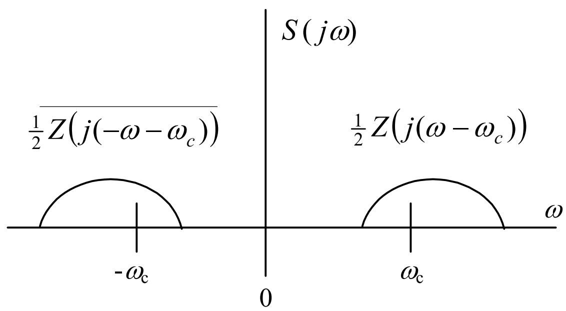

Figure 3: The complex spectrum of the real bandpass signal $S(j\omega)$. Note the Hermitian symmetry. Taken from [3].

2.2 Recovering the Complex Baseband Signal from the Real Passband Signal

Once we have transmitted data across a channel, we will want to demodulate the received signal to recover the in-phase and quadrature components of our transmitted signal. Generally, there will be noise present in the channel, so the demodulated signals will be an estimate of what was actually transmitted. To begin let's assume that the received signal, $r(t)$, is identical to the sent signal (i.e. $r(t) = s(t)$). Recall that our complex baseband representation of $s(t)$ is bandlimited around the positive frequency ω_c and its complex conjugate is bandlimited around the negative frequency $-\omega_c$, each scaled by $\frac{1}{2}$. Therefore, what we ultimately want is the positive frequencies of $S(j\omega)$ scaled by two and complex demodulated to baseband. To do this, the negative frequencies of $S(j\omega)$ are eliminated using a phase splitter, which is shown in Figure 4.

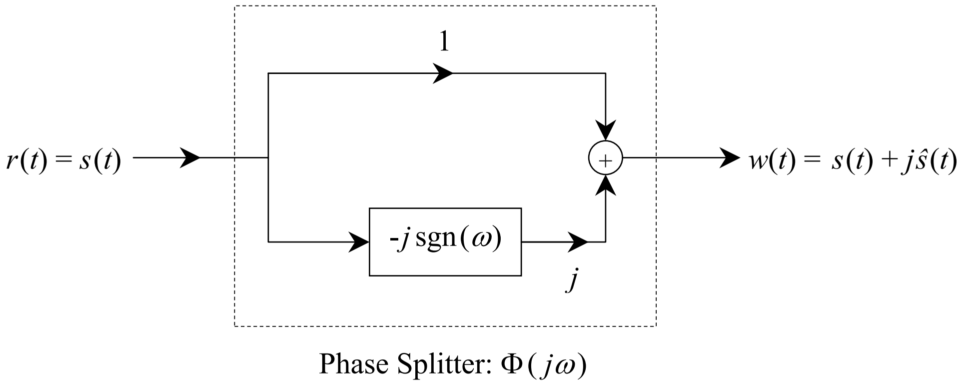

Figure 4: Phase splitter with Fourier transform $\Phi(j\omega)$

Recall that the Fourier transform of a Hilbert transformer (see lab 4) is $H(j\omega) = -j\text{sgn}(\omega)$, where $\text{sgn}(\cdot)$ is the signum function[2]. The resulting Fourier transform of a phase splitter (using linearity) is then

$$\Phi(j\omega) = 1 + j(-j\text{sgn}(\omega)) = 1 + \text{sgn}(\omega) = 2u(\omega). \tag{8}$$

Here, $u(\omega)$ is the unit step function:

$$u(\omega) = \begin{cases} 1, & \omega \geq 0 \\ 0, & \omega < 0 \end{cases} \tag{9}$$

This will give us the desired complex analytic signal, $w(t)$. Next, this complex signal is down-demodulated to baseband by multiplying it by $e^{-j\omega_c t}$, which will result in

$$w(t)e^{-j\omega_c t} = \left(z(t)e^{j\omega_c t}\right) e^{-j\omega_c t} = z(t), \tag{10}$$

and we will have successfully recovered the complex baseband signal $z(t)$ from real passband signal $s(t)$.

2.3 The Quadrature Modulator

We want to build the real RF signal $s(t)$ from a bandlimited baseband message $m(t)$. From eqns (4) - (6), the transmit signal $s(t)$ has the polar and Cartesian representations

$$s(t) = Re\left\{z(t)e^{j\omega_c t}\right\} = A(t)\cos(\omega_c t + \phi(t)) = x(t)\cos(\omega_c t) - y(t)\sin(\omega_c t), \tag{11}$$

where $x(t)$ and $y(t)$ are the *in-phase* and *quadrature* components of the RF signal, respectively. Note that $x(t)$ and $y(t)$ are the real and imaginary parts of the complex baseband representation

[2]The signum function returns the sign (± 1) of the real variable passed to the function.

of $s(t)$ in eqn (2). The basic block diagram for implementing a quadrature modulator is shown in Figure 5.

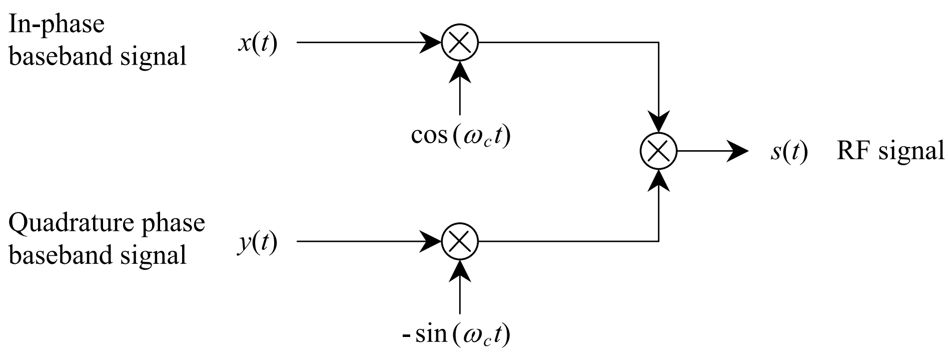

Figure 5: The quadrature modulator for creating the real bandpass signal $s(t)$. Taken from [3].

Here, the real bandpass signal $s(t)$ is created by modulating the two independent bandlimited signals $x(t)$ and $y(t)$. In the polar representation of $z(t) = A(t)e^{j\phi(t)}$, $A(t)$ is the time-varying magnitude, which is called the *envelope* of $s(t)$, and $\phi(t)$ is the time-varying *phase*, which will not be renamed. Note that $A(t)$ is called the envelope of $s(t)$ since it is the slow varying magnitude that rides on the peaks, or envelope, of the transmitted signal $s(t)$, and $z(t)$ is called the *complex envelope* of $s(t)$, since it contains the information about the magnitude of the envelope and the phase offset from the carrier frequency, namely ω_c, of $s(t)$[3].

The signal $s(t)$ is a *quadrature amplitude modulated* (or QAM) signal. From eqn (11), the QAM signal $s(t)$ may be interpreted in two ways. One, the signal $s(t)$ is a signal whose amplitude is modulated by $A(t)$ and whose phase is modulated by $\phi(t)$. Another way to look at this is to say that the in-phase component of $s(t)$ is modulated by $x(t)$ and the quadrature phase component is modulated by $y(t)$ (The second way being demonstrated in Figure 5). The game in analog communications (and digital, with a slight twist on our story) is to select $x(t)$ and $y(t)$ to get a certain type of modulation.

2.4 The Quadrature Demodulator

The general QAM demodulator is shown in Figure 6.

For a QAM demodulator to work properly, the local oscillator that generates the function $e^{j\omega_c t} = \cos(\omega_c t) + j\sin(\omega_c t)$ must be exactly in phase with the oscillator that generated the $\cos(\omega_c t)$ and $\sin(\omega_c t)$ functions in the transmitter. This type of synchronous demodulation requires either a phase-locked loop or that a pilot tone be sent with transmit signal. In the case of phase-locked loop, an algorithm must track the current phase of the carrier frequency. These types of demodulators will be used in the DSB-SC and FM cases. The other method for synchronous demodulation is to include a pilot tone in the transmit signal. In this lab, the

[3]In some texts, $s(t)$ is the labelled the sent signal, which is how we have defined it, $s_L(t)$ is the complex lowpass representation of $s(t)$ (i.e. $s_L(t) = z(t)$), and $s_+(t)$ is the complex analytic representation of $s(t)$ (i.e. $s_+(t) = w(t)$).

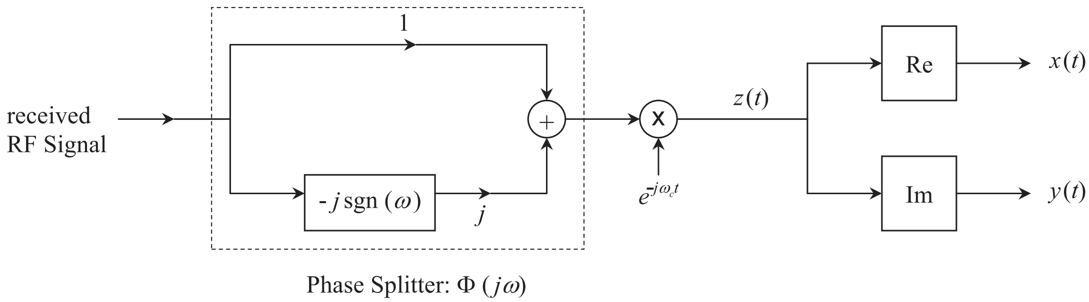

Figure 6: Complex Implementation of a Synchronous Demodulator or Quadrature Receiver for QAM.

message will have no DC ($\omega = 0$) components. This means that a small DC offset may be added to the baseband message signal that may be removed at the receiver end without affecting the message signal. When this DC offset is modulated up to the passband by a local oscillator, it will carry enough information about the phase of the carrier sinusoid to do synchronous demodulation. This type of demodulation will be used in the coherent implementation of QAM and SSB. In this lab, various coherent and non-coherent methods of demodulating will be studied.

2.5 Linear Modulation Schemes

All of the modulation schemes in this lab (with the exception of FM) are linear modulation schemes. Linear modulation means that the in-phase component is the message signal and the quadrature component is a filtered version of the message signal (or zero). In these systems, the Fourier transform of the transmitted signal may be expressed in terms of the message signal. Practical linear demodulation may be done by using the method described above. However, this requires synchronous demodulation, which means that a pilot tone must be embedded in the transmitted signal, or a carrier recover scheme must be used. This will done in the QAM, DSB-SC, and SSB schemes. In the AM and FM schemes, specialized non-linear systems will used to demodulate message signal. These non-linear systems raise some concerns, namely inefficient use of transmit power in the AM case and an inability to specify the bandwidth of the transmitted signal in the FM case.

Table 1 describes how to construct the in-phase and quadrature components of $s(t)$ from the baseband message signal $m(t)$ for various modulation schemes.

Table 1 bears the following comments:

- QAM: This is the general case. In practice, there will be a small carrier component called a *pilot* that the local oscillator will lock onto. This will be addressed when the quadrature demodulator is implemented.
- AM: The carrier amplitude A_c must be greater than the signal amplitude. AM signals may be demodulated via a simple envelope detector or a square law detector, but will require more power to transmit than the other schemes.

QAM Type	I signal $x(t)$	Q signal $y(t)$	Envelope $A(t)$	Phase $\phi(t)$
AM	$A_c + m(t)$	0	$A_c + m(t)$	0
DSB-SC	$m(t)$	0	$\|m(t)\|$	0 or π
SSB	$m(t)$	$\hat{m}(t)$		
FM	$\cos(\phi(t))$	$\sin(\phi(t))$	1	$k_\omega \int_0^t m(\tau)d\tau$

Table 1: Various Modulation Schemes for a QAM Modulator.

- DSB-SC: Double sideband suppressed carrier signals require synchronous demodulation, which will be implemented using a Costas receiver. This scheme will require less power to transmit than AM signals mentioned above, but is bandwidth inefficient.
- SSB: Single sideband needs only half the RF bandwidth of AM or DSB-SC and will require synchronous demodulation. The signal $\hat{m}(t)$ is the Hilbert transform of the message signal $m(t)$.
- FM: The baseband signal is encoded in the phase of the RF signal. Demodulation is done by non-linear devices like a phase-locked loop or an FM discriminator.

As stated earlier, all transmitted RF signals carry two channels worth of information (i.e. the in-phase and quadrature channels). In the cases where the quadrature component is not used (i.e. AM and DSB-SB), the transmit signal contains twice the bandwidth of its baseband messgae signal, which means that bandwidth is being wasted. This can be remedied by choosing a modulation scheme that uses the symmetry property of real signals to reduce the bandwidth, such as SSB. In general, we will want to pick a modulation scheme that conserves bandwidth and power and has a high tolerance to noise.

In each of these schemes, $m(t)$ is assumed to be a single channel or mono source. When we want to transmit a two channel or stereo source (e.g. the left and right channels of from a CD), we will want to use a scheme like QAM. In this case, we could modulate the left and right channels of our stereo signal onto the in-phase and quadrature carriers of QAM signal, respectively. We will address this again when we implement the various modulation schemes.

3 Amplitude Modulation (AM)

Amplitude modulation, or AM is the oldest modulation scheme used in commercial radio. From Table 1 and eqn (11), it may be seen that AM signals have no quadrature component. This means that the transmit signal $s(t)$ is

$$s(t) = [A_c + m(t)]\cos(\omega_c t) = A_c \cos(\omega_c t) + m(t)\cos(\omega_c t). \tag{12}$$

When this is implemented, $A_c > |m(t)| \ \forall \ t$. This is required so that the message signal $m(t)$ rides on the peaks, or envelope, of the carrier sinusoid $\cos(\omega_c t)$. From the righthand side of eqn (12), we can see that the transmit signal contains a carrier sinusoid modulated by the message signal plus a scaled sinusoid that carries no information about the transmit signal. This extra signal contains most of the power of the transmit signal and is generally considered a waste of power. However, it does allow for demodulation via a simple envelope detector[4]. In DSP hardware, we will use a square law detector as describe in [2].

3.1 Square Law Detector

Consider squaring the sent signal $s(t)$. The result is

$$s^2(t) = [A_c + m(t)]^2 \cos^2(\omega_c t) = \frac{1}{2}[A_c + m(t)]^2 + \frac{1}{2}[A_c + m(t)]^2 \cos(2\omega_c t). \tag{13}$$

From the righthand side of eqn (13), we can see that the squaring operation created the original baseband signal, namely $x(t) = A_c + m(t)$, squared and scaled by two, and a high frequency component that is the original baseband signal squared and scaled by two that is modulating $\cos(2\omega_c t)$ (a sinusoid oscillating twice as fast as the original carrier frequency ω_c). Under a bandlimited assumption, the higher frequency term may be filtered out using a lowpass filter. The result of lowpass filter would be

$$\frac{1}{2}x^2(t) = \frac{1}{2}[A_c + m(t)]^2. \tag{14}$$

Now, taking the square root of eqn (14) gives

$$\sqrt{\frac{1}{2}x^2(t)} = \frac{1}{4}x(t) = \frac{1}{4}[A_c + m(t)]. \tag{15}$$

The last step to recovering the message signal is to remove the DC component A_c. Most of the message signals that we will use in this lab will have no DC components, so the A_c term may be removed by either using a highpass filter or subtracting off the gain. However, the codec on the DSK is capacitively coupled so that DC signals are not transmitted. Therefore, the highpass filter is not necessary, but it is good practice to put one in. The basic hardware diagram for this square law demodulator is given in Figure 7.

[4] An envelope detector an analog circuit that may be build using a diode in series with a parallel combination of a resistor and a capacitor.

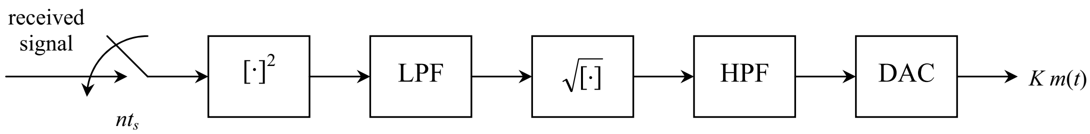

Figure 7: DSP Implementation of a Square Law AM Demodulator. Adapted from [2].

In Figure 7, notice that output is the original message signal scaled by the constant K. From the previous discussion, $K = 4$. In practice, you will want to add a gain to the output. This gain may be greater than 4, but it must be small enough to not overdrive the codec.

In practice, this implementation will require floating-point arithmetic to calculate the square root of a function.

3.2 Other AM Demodulators

Another possible AM demodulator is given in [2]. In this implementation, the envelope detector is implemented using a Hilbert transformer. For more information on this, refer to [2].

4 Quadrature Amplitude Modulation (QAM)

The most general coherent demodulator is the QAM demodulator, which is shown in Figure 6. In order to implement this demodulator, the receiver has to be able to lock onto the phase of the $\cos(\cdot)$ and $\sin(\cdot)$ waves that the in-phase and quadrature components modulated. In the general case, this is done by including a pilot tone that is outside the passband of the transmit signal. This pilot tone must carry information about the phase of the transmit signal. A specialized circuit is then used to extract this phase information and use it for coherent demodulation.

In the case where voice and music are being transmitted, the baseband message signal will contain no frequency components around DC. This means that we can add a small DC component to our in-phase signal (i.e. replace $x(t)$ with $p + x(t)$, where p is small). Now, the transmit signal from eqn (11) becomes

$$s(t) = x(t)\cos(\omega_c t) - y(t)\sin(\omega_c t) + p\cos(\omega_c t), \tag{16}$$

where $p\cos(\omega_c t)$ is the pilot tone that the demodulator will be able to lock onto. Since the message signal will contain no frequency components around DC, the pilot tone $p\cos(\omega_c t)$ will be the only frequency component around the frequency ω_c in the sent signal. This pilot tone and its quadrature counterpart, namely $p\sin(\omega_c t)$, may be extracted from $s(t)$ using finely tuned IIR comb filters. The transfer functions of the filters that do this are [2]

$$H_{in}(z) = \frac{(1-r)(1-r\cos(\omega_c t_o)z^{-1})}{1-2r\cos(\omega_c t_o)z^{-1}+r^2z^{-2}}, \tag{17}$$

and

$$H_{quad}(z) = \frac{(1-r)r\sin(\omega_c t_o)z^{-1}}{1-2r\cos(\omega_c t_o)z^{-1}+r^2 z^{-2}}. \tag{18}$$

Here, $H_{in}(z)$ is used to recover $p\cos(\omega_c t)$, and $H_{quad}(z)$ is used recover $p\sin(\omega_c t)$. In practice, the value of r in eqns (17) and (18) will be slightly less than one. If the DTFT's of eqns (17) and (18) are labelled $H_{in}(e^{j\omega t_o})$ and $H_{quad}(e^{j\omega t_o})$, respectively, and they are each evaluated at $\omega = \omega_c$, then the following relationships hold:

$$\lim_{r\to 1} H_{in}(e^{j\omega_c t_o}) = .5 \tag{19}$$

and

$$\lim_{r\to 1} H_{quad}(e^{j\omega_c t_o}) = -.5j, \tag{20}$$

which means that the in-phase and quadrature components will be recovered exactly. However, as r approaches 1, the transfer functions $H_{in}(z)$ and $H_{quad}(z)$ are approaching zero in the numerator and the poles (roots in the denominator polynomial) are approaching the unit circle. When implementing these filters using digital (quantized) values, they become unstable as r gets arbitrarily close to one (NB: theoretically, $r < 1$ will guarantee a stable implementation, however due to rounding errors in the digital hardware, it is not restrictive enough to guarantee stablity). According to [2], the optimal value of r should be chosen so that $H_{in}(z)$ and $H_{quad}(z)$ have a 50Hz 3dB bandwidth centered at ω_c[5]. These filters will allow for demodulation of all QAM signals. In particular, they will be useful for demodulating DSB-SC and SSB signals.

Caveat: These filters only extract the information about the in-phase and quadrature components of the received signal, but they do not remove it. To remove this component, either subtract it before demodulating or use a highpass filter at the output to remove it.

The general hardware diagram that will be used to implement a QAM demodulator is given in Figure 8.

In Figure 8, the solid lines represent the real channel and the dotted lines represent the imaginary channel. The multiplication operation is a complex multiplication (i.e. it will require four real multiplications, two real additions, and will produce a two channel output). After the complex demodulation, the output must be passed through a lowpass filter to remove other users on the channel. It must also filter out the DC component that was used to carry the pilot tone. These two operations may be combined into one by designing a bandpass filter whose passband extends from 50Hz to the bandwidth of the channel. A simple FIR filter or the reconstruction filter on the codec will work well for this lab.

NB: Be sure to account for the filter latency of the Hilbert transformer.

[5] By a 50Hz 3dB bandwidth, we mean that if $|H(e^{j\omega_c t_o})| = 1$, then $|H(e^{j(\omega_c+\Delta\omega)t_o})| \approx |H(e^{j(\omega_c-\Delta\omega)t_o})| \approx .5$, where $\Delta\omega = 2\pi \times 25$ rad/sec.

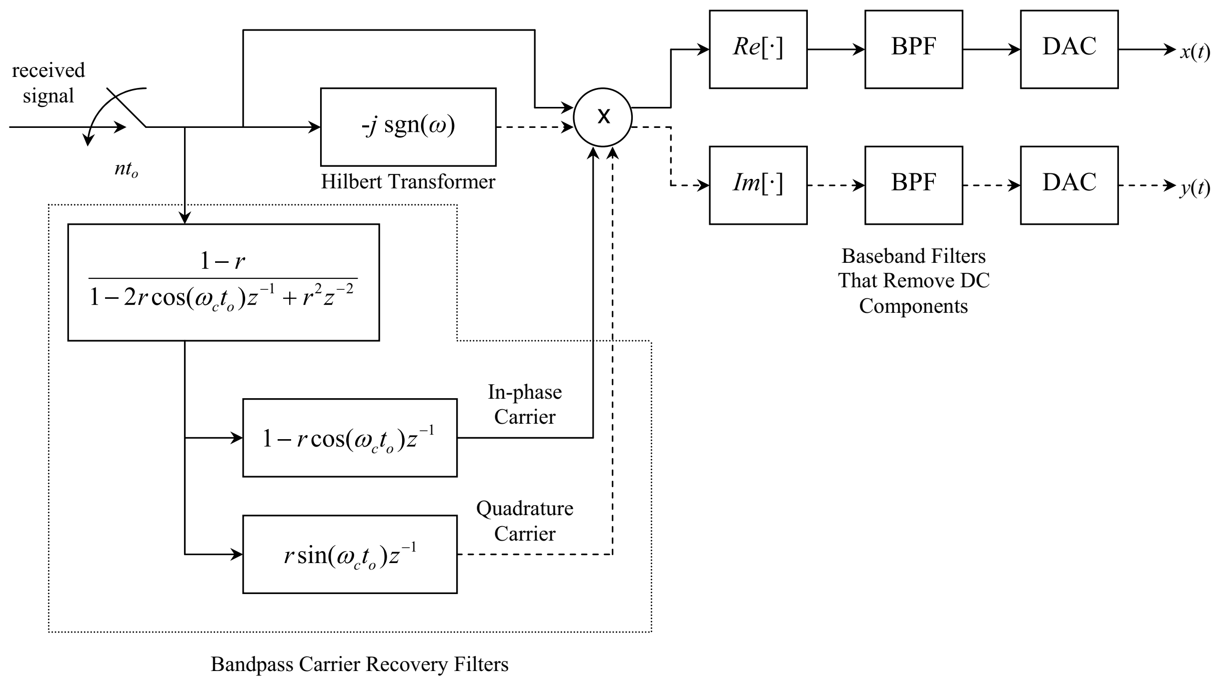

Figure 8: Hardware Diagram of a QAM Demodulator. Adapted from [2].

5 Single-Sideband (SSB) Modulation

Single-sideband modulation is the most efficient way to use bandwidth. This method takes full advantage of complex baseband signal by creating a complex (two channel) signal where each channel has half the bandwidth of the real (one channel) message signal. This operation of creating a two channel signal with half the bandwidth of a one channel signal does not reduce the bandwidth in the baseband signal, but it will reduce the amount of bandwidth in the passband (or transmit) signal (centered around a carrier frequency). In the AM and DSB-SC methods, the passband signal has twice the bandwidth as the message signal. In SSB, the message signal and the passband signal have the same bandwidth[6]. This is the most bandwidth efficient way to transmit information across a channel. This works because of the Hermitian symmetry of real signals. If you want to communicate using less bandwidth, then you will have to not be able to transmit the entire spectrum of the message signal. This is a physical limitation that can not be circumvented.

The basic SSB modulator may be implemented by creating the Hilbert transform of the message signal, accounting for filter delays, and then creating the transmit signal as defined in eqn (16), where $x(t)$ is the delayed message signal, $y(t)$ is the Hilbert transform of the message signal (aligned in time with the unfilter message signal), and p is chosen in an appropriate manner. Choosing p will need to be done by trial and error.

The basic SSB demodulator is the same as the QAM demodulator shown in Figure 8. The

[6]In these comparisons, the bandwidth of the message signal is the bandwidth centered around DC and the bandwidth of the transmit signal is with respect to the bandwidth centered around a carrier frequency. To clarify what is meant by bandwidth, the transmit signal is labelled a passband signal.

only consideration is that the message signal will be on the in-phase channel. This means that filtering the demodulated imaginary channel is unnecessary, since it will be of no use at the output.

6 Double-Sideband Suppressed Carrier (DSB-SC) Modulation

Double-sideband suppressed carrier modulation is the same as AM, except there is no carrier component. The basic transmit signal for DSB-SC is

$$s(t) = m(t)\cos(\omega_c t). \tag{21}$$

This signal may be demodulated using a Costas receiver, which is a modified phase-locked loop[7]. A well suited DSP implementation of a Costas receiver is shown in Figure 9, which is taken from [2].

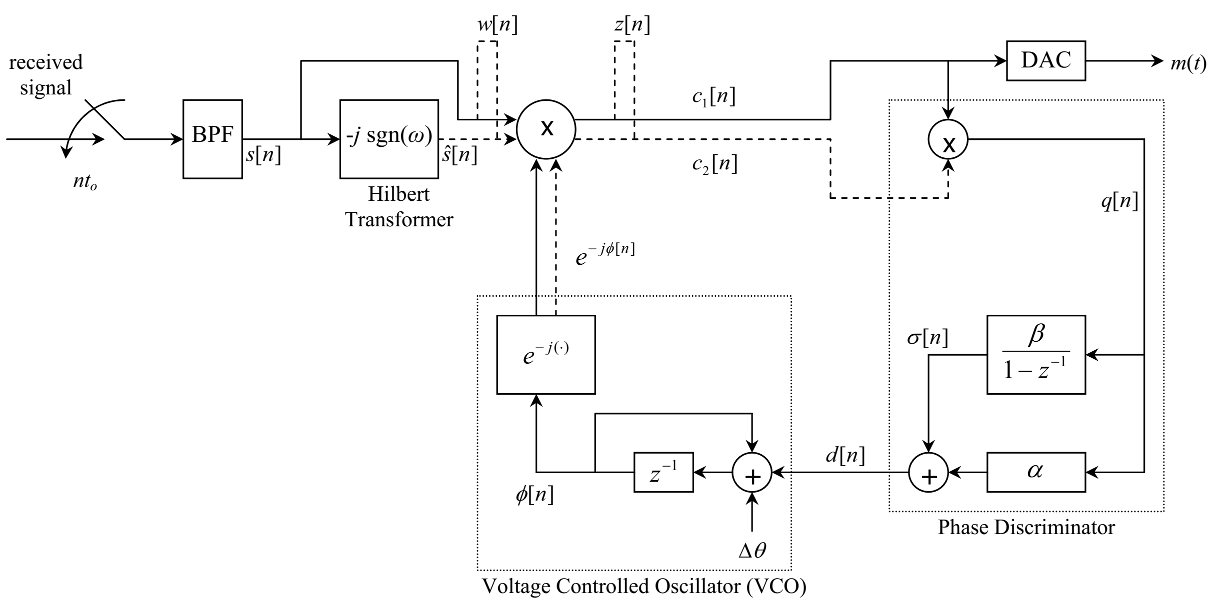

Figure 9: Second-Order Costas Loop For Demodulating DSB-SC Signals. Adapted from [2].

The basic idea of a Costas receiver is to track the phase of the modulated waveform $\cos(\omega_c t)$. The DSP implementation given in Figure 9 is taken from [2], which will be explained shortly. For more information on this implementation and Costas loops in general, see [1] and [2].

The first task of the demodulator shown in Figure 9 is to bandpass filter the received signal to get the transmitted signal

[7] If the transmit DSB-SC signal has a carrier component added to it, then it may be demodulated using a basic QAM demodulator. In this case, the transmit signal would need to be defined as $s(t) = m(t)\cos(\omega_c t) + a\cos(\omega_c t)$. While this will work, it is not the preferred way to do DSB-SC modulation.

$$s[n] = m[n]\cos[\Delta\theta n + \theta_1]. \tag{22}$$

Here, $\theta_1[n]$ is the phase error at the receiver and $\Delta\theta = \omega_c t_o$ is the change in angle of the carrier sinusoid between samples. This error will consist of a constant that is the nominal phase error and may have a slowly changing phase due to a doppler shift. As a result, this phase error will be expressed as

$$\theta_1[n] = \Delta\omega n t_o + \gamma, \tag{23}$$

where $\Delta\omega$ is the Doppler shift and γ is the nominal phase error.

Next, the complex analytic signal $w[n]$ is formed, where

$$\begin{aligned} w[n] &= s[n] + j\widehat{s}[n] \\ &= m[n]e^{j[\Delta\theta n + \theta_1[n]]}. \end{aligned} \tag{24}$$

In this notation, $\widehat{s}[n]$ is the Hilbert transform of $s[n]$.

In the feedback loop, the system must generate an estimate of the angle of the carrier $\cos(\omega_c t)$ in the received signal. This angle is determined by using a phase discriminator and a voltage controlled oscillator, which will be explained shortly. For now, the angle will be defined as

$$\phi[n] = \Delta\theta n + \theta_2[n], \tag{25}$$

This estimate of the angle is used to complex demodulate the signal to baseband, which yields

$$\begin{aligned} z[n] &= w[n]e^{-j\phi[n]} \\ &= m[n]e^{j[\theta_1[n] - \theta_2[n]]} \\ &= c_1[n] + jc_2[n]. \end{aligned} \tag{26}$$

Here, $c_1[n]$ and $c_2[n]$ are the estimates of the in-phase and quadrature components of the baseband signal $z[n]$. These estimates may be expressed as

$$\begin{aligned} c_1[n] &= s[n]\cos[\phi[n]] + \widehat{s}[n]\sin[\phi[n]] \\ &= m[n]\cos[\theta_1[n] - \theta_2[n]] \end{aligned} \tag{27}$$

and

$$\begin{aligned} c_2[n] &= -s[n]\sin[\phi[n]] + \widehat{s}[n]\cos[\phi[n]] \\ &= m[n]\sin[\theta_1[n] - \theta_2[n]] \end{aligned} \tag{28}$$

When $\theta_1[n] \approx \theta_2[n]$, the system is *in lock* and $c_1[n] \approx m[n]$ and $c_2[n] \approx 0$. In this case, the in-phase channel carries a good estimate of the original message signal and quadrature channel is approximately zero. Provided that we can generate $\theta_2[n] \approx \theta_1[n]$, this method of demodulation will allow for coherent detection without the use of a pilot tone.

6.1 Phase Discriminator

The next step is to implement a *phase discriminator*, which will generate an estimate of of the phase error. To begin this process, the real and imaginary components of the demodulated signal are multiplied together to get

$$\begin{aligned} q[n] = c_1[n]c_2[n] &= m^2[n]\cos[\theta_1[n] - \theta_2[n]]\sin[\theta_1[n] - \theta_2[n]] \\ &= .5m^2[n]\sin[2(\theta_1[n] - \theta_2[n])]. \end{aligned} \tag{29}$$

If $\theta_1[n]$ and $\theta_2[n]$ differ by less than 90^o, then $q[n]$ has the same sign as the phase error $\theta_1[n]-\theta_2[n]$, so it indicates which direction the phase estimate $\theta_2[n]$ should be changed to reduce the phase error to zero. When the loop is in lock, the small angle approximation $\sin(x) \approx x$ may be used to accurately approximate $q[n]$ as

$$q[n] = m^2[n](\theta_1[n] - \theta_2[n]) + \sigma[n-1] \quad \text{for} \quad |\theta_1[n] - \theta_2[n]| \ll 1. \tag{30}$$

In this case $q[n]$ is approximately zero and the phase $\theta_1[n]$ is being tracked almost perfectly by $\theta_2[n]$.

The next task of the phase discriminator is to generate a DC signal that will correct for the phase error. This is done by passing $q[n]$ through a lowpass filter to generate

$$\sigma[n] = \beta q[n] + \sigma[n-1], \tag{31}$$

and adding it to $\alpha q[n]$ to get an adjustment to the error estimate for the next angle sample, namely

$$\begin{aligned} d[n] &= \sigma[n] + \alpha q[n] \\ &= (\alpha + \beta)q[n] + \sigma[n-1]. \end{aligned} \tag{32}$$

In practice, α and β will be small positive constants with $\beta < \frac{\alpha}{50}$. When $\theta_1[n] \approx \theta_2[n]$, $q[n] \approx 0$, which means that $\sigma[n] \approx 0$, and the error adjustment $d[n]$ will not change much from sample-to-sample. This approximation of the phase error is then passed to a voltage controlled oscillator that uses the approximated phase error to correct the phase of the next angle sample.

6.2 Voltage Controlled Oscillator (VCO)

The voltage-controlled oscillator (VCO) generates the phase corrected angle for the in-phase and quadrature demodulator signal generator, namely $e^{-j(\cdot)}$ in Figure 9. This is done by calculating the next angle to be passed to $e^{-j(\cdot)}$, which is

$$\phi[n+1] = \phi[n] + \Delta\theta + d[n]. \tag{33}$$

The VCO recursively calculates the phase corrected angle for the next demodulated sample based on the current angle and phase, namely $\phi[n]+\theta_2[n]$, plus the error adjustment for current demodulated complex baseband sample. When this is implemented, the current angle estimate will need to be used, not the next estimate. This is a basic discrete-time VCO that we will use in this lab.

6.3 Comments on the Second-Order Costas Loop

It is well known in control theory that an integrator (a transfer function with a pole at $\omega = 0$) will track constant inputs with a zero steady-state error, and that double integrators (a transfer function with a double pole at $\omega = 0$) will track first-order input polynomials with a zero steady-state error. From eqn (23), we can see that $\theta_1[n]$ is a first-order polynomial that may be tracked by a double integrator. In discrete-time systems, an integrator is the transfer function, $U(z)$, whose impulse response is the unit step response $u[n]$. Using the well known DTFT pair $u[n] \longleftrightarrow U(z) = \frac{1}{1-z^{-1}}$, we can see that $U(z)$ has a simple pole at $\omega = 0$. To see this, consider $z = e^{j\omega t_o}$. When $\omega = 0$, $z = 1$, which is pole of $U(z)$, as we would expect. A double integrator may be formed by cascading two integrators together. In a second-order Costas loop, one integrator appears in phase discriminator, namely $\frac{\beta}{1-z^{-1}}$, and one is formed by the feedback loop in the VCO. Therefore, this second-order (a cascade of two first order integrators) will track linear phase errors, such as $\theta_1[n]$, with zero steady-state error.

7 Frequency Modulation (FM)

Frequency modulation (FM) is non-linear modulation scheme that encodes the message signal into the phase of the transmit signal. The basic FM transmit signal is

$$\begin{aligned} s(t) &= \cos(\omega_c t + \phi(t)) && (34) \\ &= \cos(\omega_c t)\cos(\phi(t)) - \sin(\omega_c t)\sin(\phi(t)), && (35) \end{aligned}$$

where

$$\phi(t) = k_\omega \int_0^t m(\tau)d\tau. \qquad (36)$$

In this notation, k_ω is a positive constant called the *frequency sensitivity*. In general, k_ω will be a fraction of the carrier frequency (e.g. $.5\omega_c$).

To implement a discrete-time FM modulator as described in eqn (34), a VCO may be used. A basic FM modulator is given in Figure 10.

In Figure 10, $\beta = k_\omega t_o$. Typically, this value will be less than 2π since $k_\omega < \omega_c < \frac{\omega_0}{2}$. In the lab, FM signals may either be generated using the FM modulator in Figure 10 or by using the signal generator to generate FM signals.

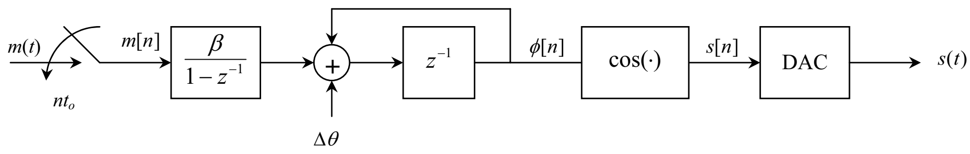

Figure 10: VCO FM Modulator.

The two most common ways to demodulate FM signals is to use an FM discriminator or to use a phase-locked loop. Both methods may be implemented on the DSP hardware, but a phase-locked loop will give better performance. Therefore, only the phase-locked loop will be covered here. For information about a DSP implementation of an FM discriminator, see [2].

7.1 FM Demodulation Using A Phase-Locked Loop

The block diagram for a discrete-time phase-locked loop FM demodulator is given in Figure 11.

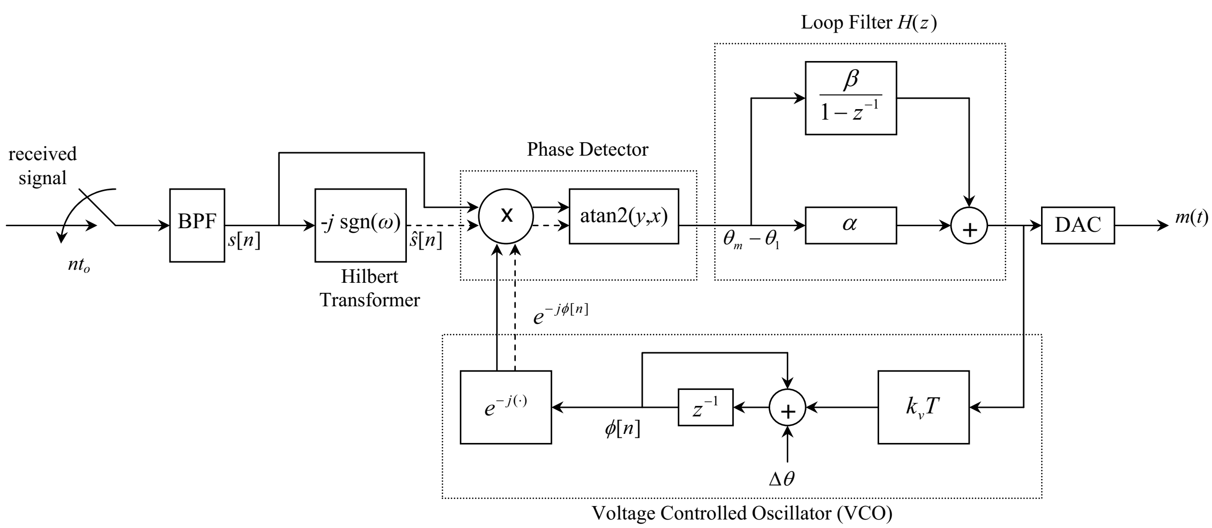

Figure 11: A Discrete-Time Phase-Locked Loop FM Demodulator. Adapted From [2].

The FM demodulator in Figure 11 is very similar to the the discrete-time Costas receiver shown in Figure 9. If the received signal is assumed to be noiseless, then it may be expressed as

$$s[n] = \cos[\omega_c n t_o + \theta_m[n]]. \tag{37}$$

This signal is then passed through a phase splitter to form the complex analytic signal

$$w[n] = s[n] + j\hat{s}[n] = e^{j[\omega_c n t_o + \theta_m[n]]}. \tag{38}$$

Next, the analytic signal is demodulated to baseband as follows

$$z[n] = w[n]e^{-j\phi[n]} = w[n]e^{-j[\omega_c n t_o + \theta_1[n]]} = e^{j[\theta_m[n]-\theta_1[n]]}. \tag{39}$$

The phase angle is $\phi[n]$ is calculated recursively by the VCO. Starting at $n = 0$ and iterating the equation, it follows that

$$\begin{aligned} \phi[n+1] &= \phi[n] + \omega_c t_o + k_v t_o y[n] & (40) \\ &= \phi[n] + \theta_1[n], & (41) \end{aligned}$$

where

$$\theta_1[n] = \theta[0] + k_v t_o \sum_{k=0}^{n-1} y[n]. \tag{42}$$

The phase $\theta_1[n]$ is tracked by the loop filter $H(z)$ and is the phase of received signal one sample ago. Since $\theta_m[n]$ is the phase of the received signal at the current sample, the difference $\theta_m[n] - \theta_1[n]$ is an estimate of the derivative of $\theta_m[n]$ at low frequencies. Recall that the message signal was used to form the instantaneous frequency of the transmit signal $s(t)$. Therefore, by estimating the derivative of the phase of the received signal, we have that $y[n] \approx \frac{k_f}{k_v} m[n]$ for all n, and we have demodulated an FM signal.

Recovering the phase difference $\theta_m[n] - \theta_1[n]$ is done by calculating the phase of complex baseband signal $z[n]$. It is well known that this phase may be calculated as

$$\theta_m[n] - \theta_1[n] = \arg z[n] = \tan^{-1}\left(\frac{\Im m z[n]}{\Re e z[n]}\right) = \tan^{-1}\left(\frac{s[n]}{\widehat{s}[n]}\right) = \mathrm{atan2}(\widehat{s}[n], s[n]), \tag{43}$$

where `atan2(y,x)` is a built-in C function that may be used when the header file `math.h` is included in a C program. The function `atan2(y,x)` is a four-quadrant inverse tangent (arctangent) that returns angles between $-\pi$ and π.

Between the loop filter and the VCO (second-order pole at $\omega = 0$), this phase-locked loop will track the linear instantaneous phase of the received signal with zero steady-state error.

This implantation is very similar to the Costas loop in the DSB-SC case, except that the FM demodulator will require more bandwidth than the Costas loop. In the DSB-SC demodulator, the loop only had to track a single sinusoid at the carrier frequency, so it only required a small amount of bandwidth. In the FM demodulator the phase-locked loop must be able to track signals over the bandwidth of the message signal, which will require more bandwidth. This is accounted for in design by using larger values of α and β in the loop filter. In a typical FM demodulator, α might be equal to one and $\beta < \frac{\alpha}{100}$.

8 Practical Considerations for this Lab

8.1 Hardware Limitations

There will be three main hardware limitations in this lab. These include overdriving the codec, filter latency (timing), and code speed. In all of these implementations, the output will have to be integers between $\pm 2^{15}$ (type `short`) with maps to 3Vpp output voltage. For example, in an AM communication system the message will have to have a relative peak-to-peak voltage of less than 1.5Vpp. A point that will be obvious when AM is implemented. For filter latency, if the real channel of a complex signal is the unfiltered message and the imaginary channel is the message filtered by a 21-tap FIR filter ($N = 20$), then the imaginary channel will have a 10 sample delay (filter order divided by 2). This delay must then be accounted for in the real channel. This is done so that the real and imaginary components of the complex baseband signal align. Finally, in some systems, there will be many operations that need to be done. Therefore, it is necessary to code algorithms as efficiently as possible. An example of this would be to use the optimized assembly coded FIR filter for all FIR filters (see lab 4). These types of design issues will play a constant theme throughout this lab.

8.2 Implementation Using Two Boards

For the lab assignment, you will be expected to implement both the modulator and the demodulator (a full modem) for each modulation scheme. The most straightforward way of implementing this is to use one DSK as the modulator and another DSK for the demodulator (see the next section for an alternative method). This can be done if each group "pairs up" with another group in the sense that each group must be willing to share their DSK board with another group. However, each individual group will be expected to turn in their own lab report. Details of how to do this will be given in the recitation. An alternative is to borrow a board from the Instructor. Note that you should troubleshoot and assure that both the modulator and demodulator are working before you try to put them together.

8.3 Implementation Using Both Codec Channels Simultaneously

It is possible to use both channels of codec for this lab at the same time; however, it complicates the implementation process somewhat. For example, you could use the left channel input as the message signal input, loop the modulated output of the left channel into the right channel input, and use the right channel output to display your demodulated output. This should be straightforward from a programming point of view but will require some special cabling. Let the Instructor know beforehand if you would like to try this method.

NB: Both this method and the method mentioned above (using two boards) are acceptable for this lab.

9 Design and Implementation Tips

This section gives a list of tips that will aid in the design and implementation of the various modulation schemes in this lab.

- For many of the modulation schemes, it is possible to use two signal generators to create the modulator. Therefore, it is often wise to built and test the demodulator first. This is done for two reasons. First, the demodulator may be tested using a known working modulator (the signal generators). Second, the demodulators are generally much more difficult to implement than the modulators. Building and testing the demodulator first will reduce the sources of error in your modem implementation greatly.
- When designing a phase splitter, the input (straight wire) must be aligned in time with the output of the Hilbert transformer.
  On the class webpage, you will find 30, 60, and 90 order Hilbert transformer coefficient files. The fixed-point files are: `HT_30.cof`, `HT_60.cof`, and `HT_90.cof`. The floating-point files are `HT_30_float.cof`, `HT_60_float.cof`, and `HT_90_float.cof`.
  Note that there are two sources of latency in this implementation. The first comes from the way the filter was designed ($N/2$ samples) and the second comes from the way the filter is implemented on the DSK (a one sample delay). In your lab report, you will be expected to justify where the delay come from.
- For the Costas receiver (and the phase-locked loop), the arithmetic was done assuming that the received signal was normalized to ± 1. When you implement this, you have to convert the received signal (and its time aligned Hilbert transformer) to type `float`. Then, you must scale the values by 2^{-15} to normalize them, implement the receiver, and scale the output by 2^{15} before you output it to the codec. (*Caveat*: Do **not** try to use the bit shift operator to scale a floating point number. Use the standard floating-point multiplication to do the scaling)
- In the Costas receiver design, the parameters α and β determine the bandwidth of the carrier signal that you will be tracking. Sample values are $\alpha = 0.01$ and $\beta = 0.0002$. Note that $\beta = \frac{\alpha}{50}$.
- In the phase-locked loop receiver design, the bandwidth must be large enough to track the entire message signal. In this case, you will want $\alpha = 1$ and $\beta = 0.01$ (or less). Note that $\beta = \frac{\alpha}{100}$.

10 Assignments*

For this lab, you will have two options for implementation. One is to use the two wire codec program and use the DSK as both a modulator and a demodulator. The second way is to use two DSK's. This means that you will need to either borrow a DSK from a different group or set up a time to meet with the TA, who will provide another board. Each group is expected to do their own work (see "Practical Considerations for this Lab") .

Required Assignment

1. Consider the narrow baseband message signal magnitude spectrum $M(j\omega)$ shown in Figure 12. For each of the modulation schemes AM, DSB-SC, and SSB, sketch the complex baseband spectrum $Z(j\omega)$, the complex analytic spectrum $W(j\omega)$, and real passband spectrum $S(j\omega)$. Can you sketch the frequency spectrum for an FM scheme? Why?

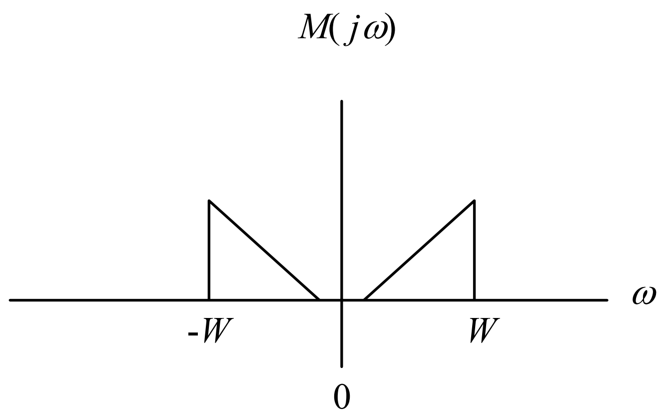

Figure 12: Message Signal Spectrum $M(j\omega)$

Required Assignment

2. Design and implement a SSB modem (modulator and demodulator) that does *not* use a pilot tone. Implement the QAM demodulator in Figure 6. How does the non-coherence affect the output signal? Use music/voice as an input and listen to the output.
3. Repeat the previous problem, except include a pilot tone. Use the carrier recovery circuit in Figure 8 to recover the in-phase and quadrature components of the carrier sinusoid. How does the phase coherence improve the quality of the demodulated signal?
4. Design and implement a DSB-SC modem. Use a Costas receiver in Figure 9 as your demodulator. Comment on all of the parameters used. Include an explain your code.

10.1 Extra Credit Assignments*

The following questions are *not* required, but they will be worth extra credit.

Extra Credit Assignment

5. Design and implement a FM modem. Use a phase-locked loop as your demodulator. Comment on all of the parameters used. Include an explanation of your code.

6. Design and implement an AM modem using a square-law detector as your demodulator. Comment on all of the parameters used. Include an explanation of your code.

   Caveat: When a square-law detector is implemented, the non-linear squaring operation will increase the bandwidth of the message signal. Note that in order to implement the AM modulator on the DSK, the frequency spectrum of the transmit signal must be must be bandlimited to $f_c + W < \frac{f_s}{2}$, where f_s is the rate of the codec and W is the bandwidth of the message signal. However, the (nonlinear) squaring operation in the demodulator will result in a signal that has bandwidth $2(f_c + W)$. Therefore, to avoid aliasing in the modem, the parameters f_c and W must be chosen so that they satisfy $2(f_c + W) < \frac{f_s}{2}$.

11 End Notes

The various modulation schemes implemented in this lab are used in everyday life. Many of these implementations are very suitable for DSP implementation, but as an engineering, you have to decide what method is the most practical for the application. For example, implementing a square law detector in place of simple envelope detector would probably not be a good decision. Also, doing high rate signal processing with a DSP chip may be very costly. Therefore, an analog circuits may be more suitable for heterodyning and then using a low rate DSP algorithm for modulation and demodulation.

11.1 Lab Suggestions for Course Project

- Implement a digital communication lab using the modulation schemes discussed here.
- Implement an OFDM system using an FFT algorithm

References

[1] S. Haykin, *Communication Systems*. John Wiley and Sons, Inc., New York, 2001.

[2] S. A. Tretter, *Communication Design Using DSP Algorithms: With Laboratory Experiments for the TMS320C6701 and TMS320C6711*. Kluwer Academic/Plenum Publishers, New York, 2003.

[3] Colorado State University, Fort Collins, CO, *Signals and Systems Laboratory 12: Linear Modulation and Demodulation*, 2001.