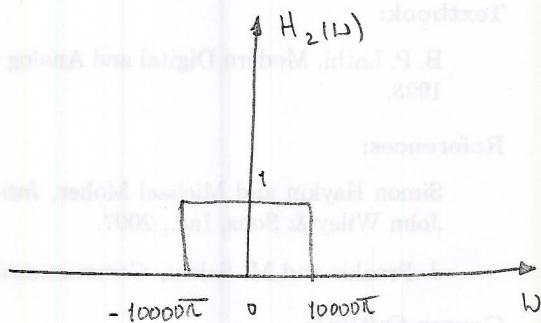
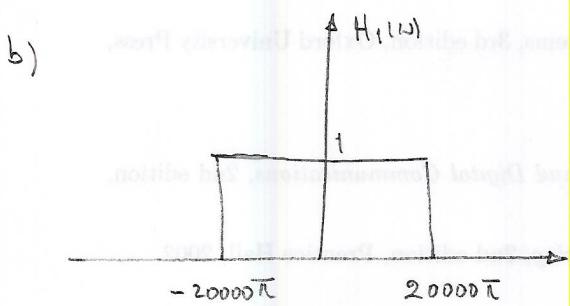
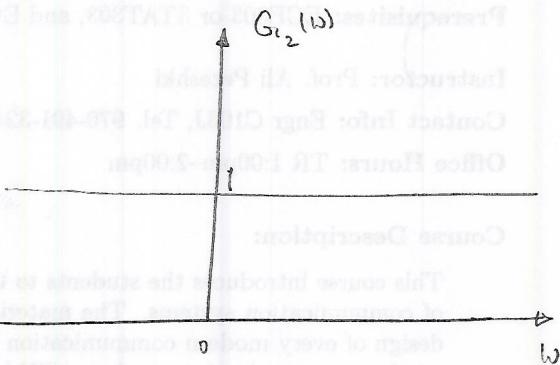
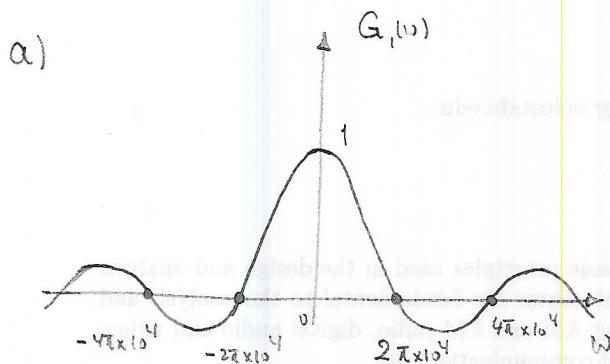
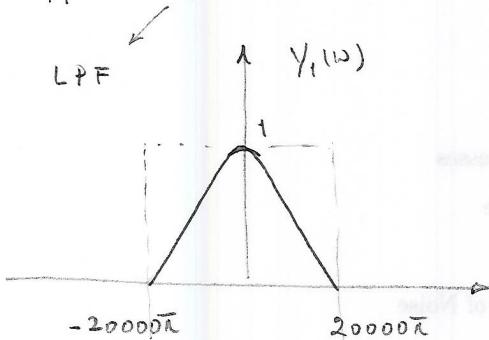


$$g_1(t) = 10^4 \operatorname{rect}(10^4 t) \longleftrightarrow G_1(\omega) = \operatorname{Sinc}\left(\frac{\omega}{2 \times 10^4}\right)$$

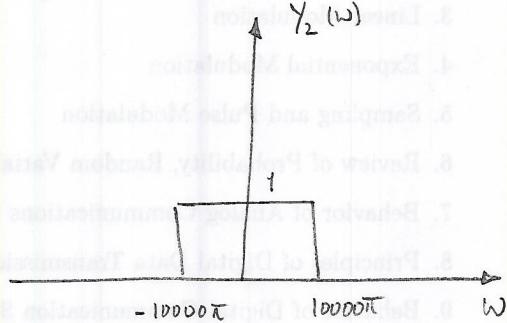
$$g_2(t) = S(t) \longleftrightarrow G_2(\omega) = 1$$



c) $y_1(\omega) = H_1(\omega) G_1(\omega)$



$$y_2(\omega) = H_2(\omega) G_2(\omega)$$



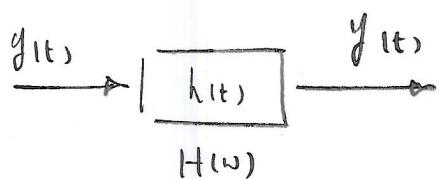
d) $BW(y_1(t)) = \frac{20000\pi}{2\pi} = 10 \text{ KHz}$

$$BW(y_2(t)) = \frac{10000\pi}{2\pi} = 5 \text{ KHz}$$

$$y(t) = y_1(t) y_2(t) \longleftrightarrow Y(\omega) = \frac{1}{2\pi} Y_1(\omega) * Y_2(\omega)$$

$$\Rightarrow BW(y(t)) = BW(y_1(t)) + BW(y_2(t)) = 15 \text{ KHz}$$

3.6-1



$$y(t) = h(t) * g(t)$$

$$Y(\omega) = H(\omega) G(\omega)$$

$$|H(\omega)| = 1$$

$$\theta_h(\omega) = -\omega t_0 - k \sin \omega T; \quad k \ll 1$$

$$\begin{aligned} H(\omega) &= |H(\omega)| e^{j\theta_h(\omega)} \\ &= e^{-j(\omega t_0 + k \sin \omega T)}; \quad k \ll 1 \end{aligned}$$

$$(a) \quad H(\omega) = e^{-j\omega t_0} e^{-jK \sin \omega T}, \quad k \ll 1$$

Taylor series for $e^{-jK \sin \omega T}$:

$$e^{-jK \sin \omega T} = 1 - jK \sin \omega T + \frac{1}{2!} (jK \sin \omega T)^2 + \dots$$

Very small when

$$\text{Thus } e^{-jK \sin \omega T} \approx 1 - jK \sin \omega T \quad \text{and} \quad k \ll 1$$

$$H(\omega) \approx e^{-j\omega t_0} (1 - jK \sin \omega T) = e^{-j\omega t_0} \left(1 - K \frac{e^{-j\omega T} - e^{j\omega T}}{2} \right)$$

$$= e^{-j\omega t_0} - \frac{K}{2} e^{-j\omega(t_0 - T)} + \frac{K}{2} e^{-j\omega(t_0 + T)}$$

$$\rightarrow Y(\omega) = e^{-j\omega t_0} G(\omega) - \frac{K}{2} e^{-j\omega(t_0 - T)} G(\omega) + \frac{K}{2} e^{-j\omega(t_0 + T)} G(\omega)$$

$$\rightarrow y(t) = g(t - t_0) - \frac{K}{2} g(t - t_0 + T) + \frac{K}{2} g(t - t_0 - T)$$

3.6-1 (b)

$$Y(\omega) = \begin{bmatrix} e^{-j\omega t_0} & e^{-j\omega(t_0-T)} \\ e^{-j\omega t_0} & -\frac{k}{2}e^{-j\omega(t_0-T)} + \frac{k}{2}e^{-j\omega(t_0+T)} \end{bmatrix} G(\omega)$$

$$\text{BW}(y) = \text{BW}(G) \quad \text{no increase in bandwidth}$$

→ no interference between adjacent channels in FDM.

$$y(t) = g(t-t_0) - \frac{k}{2}g(t-t_0+T) + \frac{k}{2}g(t-t_0-T)$$

spreads the signal in time by T & $-T$.

This time spread causes interference in TDM systems.

3.7-1

$$E_g = \int_{-\infty}^{\infty} |g(t)|^2 dt = \int_{-\infty}^{\infty} \frac{1}{\sigma^2 2\pi} e^{-\frac{t^2}{\sigma^2}} dt$$

let $x = \frac{t}{\sigma}$ then $dx = \frac{1}{\sigma} dt$ and

$$E_g = \int_{-\infty}^{\infty} \frac{\sigma}{\sigma^2 2\pi} e^{-x^2} dx = \frac{1}{\sigma 2\pi} \sqrt{\pi} = \frac{1}{2\sigma \sqrt{\pi}}$$

From Task 3.1 #22

$$g(t) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{t^2}{2\sigma^2}} \longleftrightarrow e^{-\sigma^2 \omega^2 / 2} = G(\omega)$$

$$E_g = \frac{1}{2\pi} \int_{-\infty}^{\infty} |G(\omega)|^2 d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-\sigma^2 \omega^2} d\omega$$

let $x = \sigma\omega$ then $dx = \sigma d\omega$ and

$$E_g = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-x^2} \frac{dx}{\sigma} = \frac{1}{2\pi \sigma} \sqrt{\pi} = \frac{1}{2\sigma \sqrt{\pi}}$$

3.7-3

$$\begin{aligned} \int_{-\infty}^{\infty} g_1(t) g_2(t) dt &= \int_{-\infty}^{\infty} \left(\frac{1}{2\pi} \int_{-\infty}^{\infty} G_1(\omega) e^{j\omega t} d\omega \right) g_2(t) dt \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} G_1(\omega) e^{j\omega t} d\omega \right) g_2(t) dt \\ &\quad \text{---} \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \left(- \int_{\infty}^{-\infty} G_1(-\omega) e^{-j\omega t} d\omega \right) g_2(t) dt \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} G_1(-\omega) e^{-j\omega t} d\omega \right) g_2(t) dt \\ &\quad \text{---} \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} G_1(-\omega) \underbrace{\left(\int_{-\infty}^{\infty} g_2(t) e^{-j\omega t} dt \right)}_{G_2(\omega)} d\omega \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} G_1(-\omega) G_2(\omega) d\omega \end{aligned}$$

The second identity may be derived in a similar way by replacing $g_2(t)$ with $\frac{1}{2\pi} \int_{-\infty}^{\infty} G_2(\omega) e^{j\omega t} d\omega$.

3.7-5

$$f(t) = \frac{2a}{t^2 + a^2}$$

Table 3.1 #3

$$e^{-at} \longleftrightarrow \frac{2a}{a^2 + \omega^2}, \quad a > 0$$

Table 3.2 #3 (symmetry) or eq. 3.24

$$G(t) \longleftrightarrow 2\pi g(-\omega)$$

Thus

$$f(t) = \frac{2a}{t^2 + a^2} \longleftrightarrow 2\pi e^{-a|\omega|} = 2\pi e^{-a|\omega|} = G(\omega)$$

$$|G(\omega)| = 2\pi e^{-a|\omega|}$$

$$\begin{aligned} E_g &= \frac{1}{2\pi} \int_{-\infty}^{\infty} |G(\omega)|^2 d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} (2\pi e^{-2a|\omega|})^2 d\omega \\ &= 2\pi \left(\int_{-\infty}^0 e^{2aw} dw + \int_0^{\infty} e^{-2aw} dw \right) = 4\pi \int_0^{\infty} e^{-2aw} dw \\ &= \frac{4\pi}{-2a} e^{-2aw} \Big|_0^{\infty} = \frac{2\pi}{a} \end{aligned}$$

3.7-5 Cont.

Energy between $-W$ to W $\frac{\text{rad}}{\text{sec.}}$

$$E_g[-W, W] = \frac{1}{2\pi} \int_{-W}^W |G(\omega)|^2 d\omega = \int_{-W}^W \frac{-2\alpha|\omega|}{2\pi e^{-2\alpha|\omega|}} d\omega$$

$$= 4\pi \int_0^W e^{-2\alpha\omega} d\omega = \left[\frac{4\pi}{-2\alpha} e^{-2\alpha\omega} \right]_0^W$$

$$= \frac{-2\pi}{\alpha} (e^{-2\alpha W} - 1) = \frac{2\pi}{\alpha} (1 - e^{-2\alpha W})$$

99% band:

$$\frac{E_g[-W, W]}{E_g} = \frac{0.99}{1}$$

$$\frac{\cancel{2\pi} \cancel{\frac{1}{\alpha}} (1 - e^{-2\alpha W})}{\cancel{2\pi} \cancel{\frac{1}{\alpha}}} = 0.99 \rightarrow e^{-2\alpha W} = 0.01$$

$$\rightarrow W = \frac{2.3026}{\alpha}$$

$$\rightarrow B = \frac{W}{2\pi} = \underline{\underline{\frac{0.3665}{\alpha}}}$$

3.8-1

$$g(t) = C \cos(\omega_0 t + \theta_0)$$

$$R_g(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} g(t) g(t+\tau) dt$$

$$= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} C^2 \cos(\omega_0 t + \theta_0) \cos(\omega_0(t+\tau) + \theta_0) dt$$

$$= \frac{C^2}{2} \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \left(\cos 2(\omega_0 t + \frac{\tau}{2}) + \cos \omega_0 \tau \right) dt$$

$$= \frac{C^2}{2} \left[\lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \cos 2(\omega_0 t + \frac{\tau}{2}) dt + \cos \omega_0 \tau \right]$$

At most equal to the
area under a half-cycle
of a cosine and hence $< \infty$

$$\lim_{T \rightarrow \infty} \frac{1}{T} (\text{limited value}) = 0.$$

$$\rightarrow R_g(\tau) = \frac{C^2}{2} \cos \omega_0 \tau$$

3.8-1 cont.

$$R_g(\tau) \longleftrightarrow S_g(\omega) : \text{PSD}$$

$$R_g(\tau) = \frac{C^2}{2} \cos \omega_0 \tau \longleftrightarrow \frac{C^2 \pi}{2} \left[\delta(\omega - \omega_0) + \delta(\omega + \omega_0) \right]$$

$$y(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n)$$

Auto correlation for $y_1(t) + y_2(t)$:

$$\begin{aligned} R_{y_1+y_2}(\tau) &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} (y_1(t) + y_2(t)) (y_1(t+\tau) + y_2(t+\tau)) dt \\ &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} [y_1(t)y_1(t+\tau) + y_2(t)y_2(t+\tau) + y_1(t)y_2(t+\tau) \\ &\quad + y_2(t)y_1(t+\tau)] dt \\ &= R_{y_1}(\tau) + R_{y_2}(\tau) + R_{y_1 y_2}(\tau) + R_{y_2 y_1}(\tau) \end{aligned}$$

$$\text{Now let } y_1(t) = C_m \cos(m\omega_0 t + \theta_m) \quad \&$$

$$y_2(t) = C_l \cos(l\omega_0 t + \theta_l)$$

3.8-1 Cont.

$$\begin{aligned} R_{g_1 g_2}(\tau) &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} c_n c_l \cos(n\omega_0 t + \theta_n) \cos(l\omega_0 t + \tau + \theta_l) dt \\ &= \frac{c_n c_l}{2} \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \cos(n\omega_0 t + l\omega_0 t + \tau + \theta_n + \theta_l) dt \\ &\quad + \frac{c_n c_l}{2} \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \cos(n\omega_0(lt + \tau - nt) + \theta_l - \theta_n) dt \end{aligned}$$

At most equal to the area under a half-cycle of the integrand, and thus limited value.

$$\lim_{T \rightarrow \infty} \frac{1}{T} (\text{limited value}) = 0.$$

Similarly the first term is zero.

$$\rightarrow \underline{R_{g_1 g_2}(\tau) = 0}$$

Similarly $\underline{R_{g_2 g_1}(\tau) = 0}$.

3.8-1 Cont.

In computing $R_y(\tau)$ there are four groups of terms of the forms

$$1 - R_{g_1 g_1}(\tau) = \frac{C_0^2}{2} \cos m \omega_0 \tau \quad \text{from p. 15}$$

$$2 - R_{g_1 g_2}(\tau) = 0$$

$$3 - R_{g_2 g_1}(\tau) = 0 \quad \text{for the same reason as 2.}$$

$$4 - R_{c_0 c_0}(\tau) = C_0^2$$

Therefore

$$R_y(\tau) = C_0^2 + \frac{1}{2} \sum_{n=1}^{\infty} C_n^2 \cos n \omega_0 \tau$$

$$\text{PSD} = S_y^{(0)} = F\{R_y(\tau)\} = 2\pi C_0^2 \delta(\omega) + \frac{\pi}{2} \sum_{n=1}^{\infty} C_n^2 [\delta(\omega - n\omega_0) + \delta(\omega + n\omega_0)]$$