In this experiment we will examine the step response of several RC and RLC circuits. We begin by finding the output waveform \( v(t) \) from RC circuits designed to operate as “analog computers”, where the derivative with respect to time (or the integral) of an input waveform \( v_i(t) \) is taken continuously.

**DIFFERENTIATOR AND INTEGRATOR**

A simple first order RC circuit can approximate an integrator or a differentiator, which, respectively, are circuits which have as an output the time integral or time differential of an input signal.

1. Consider the circuit:

![Circuit Diagram](image)

If we assume there is no load on this circuit, the current through the capacitor and the resistor will be equal,

\[
\frac{v(t)}{R} = C \frac{d}{dt} [v_i(t) - v(t)]
\]

If the output voltage \( v(t) \) is much smaller than the input voltage \( v_i(t) \), we can write

\[
\frac{v(t)}{R} \approx C \frac{dv_i(t)}{dt}
\]

so that

\[
v(t) \approx RC \frac{dv_i(t)}{dt}
\]

In other words: The output of this circuit, \( v(t) \), is approximately proportional to the time rate of change (i.e., the derivative) of the input voltage.

The assumption that \( v(t) < v_i(t) \) requires \( RC \) to be small. How small depends on the rate at which \( v_i(t) \) is changing. The larger \( dv_i(t)/dt \), the smaller must be the \( RC \).
2. Now consider the integrator circuit:

\[
\text{If we assume there is no load on this circuit, the current through the capacitor and the resistor will be equal,}
\]

\[
\frac{v_i(t) - v(t)}{R} = C \frac{dv(t)}{dt}
\]

\[\text{(1)}\]

\[
\text{If the output voltage } v(t) \text{ is much smaller than the input voltage } v_i(t), \text{ we can write (assuming that } v(t)=0, \ t<0) \]

\[
v(t) \approx \frac{1}{RC} \int_0^t v_i(t') dt'
\]

\[\text{(2)}\]

Show steps that get us from (1) to (2)

**In other words:** The output of this circuit \(v(t)\) is approximately proportional to the integral of the input voltage.

The assumption that \(v(t) < v_i(t)\) requires \(RC\) to be large. How large depends on the value of integral of \(v_i(t)\). The larger is the integral, the larger must be the \(RC\).

Using graphing paper, carefully sketch the output signal \(v(t)\) you would expect to obtain from the preceding differentiator and integrator circuits when driven with a square wave voltage, for the two cases below. Include a plot of \(v_i(t)\) for reference. Attach your plots.

a) When the assumptions about \(RC\) are satisfied.

b) When the assumptions about \(RC\) are not satisfied.
3. In the first order RC circuit below, the following values are given: $V_o = 5 \text{ V}$, $R_3 = 4.7 \text{ k}\Omega$, and $C = 17.4 \text{ nF}$.

What must $R_1$ and $R_2$ be to obtain the output voltage $v(t) = 3.15 \left(1 - e^{-8476t}\right) \text{ V}$

Show your work:

\[ R_1 = \underline{\text{___________}} \]

\[ R_2 = \underline{\text{___________}} \]
4. Choose one of the 2\textsuperscript{nd} order circuits given below and derive its characteristic equation. Determine conditions for which your circuit would have each of the three responses: overdamped, critically damped or underdamped. Your answer will be in most general form, in terms of variables $R$, $L$ and $C$. Do not forget to turn off the independent sources before deriving characteristic equation.

\textbf{Note:} Set the switch in such a way that source 1 sets the initial value and source 2 “drives” circuit to steady state ($t \to \infty$).