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Some insights for the prediction of near-wall turbulence

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In this paper, we revisit the eddy viscosity formulation to highlight a number of important issues that have direct implications for the prediction of near-wall turbulence. For steady wall-bounded turbulent flows, we make the equilibrium assumption between rates of production (P) and dissipation (ϵ) of turbulent kinetic energy (k) in the near-wall region to propose that the eddy viscosity should be given by $\nu_t \approx \epsilon/S^2$, where S is the mean shear rate. We then argue that the appropriate velocity scale is given by $(ST_L)^{-1/2}k^{1/2}$ where $T_L = k/\epsilon$ is the turbulence (decay) time scale. The difference between this velocity scale and the commonly assumed velocity scale of $k^{1/2}$ is subtle but the consequences are significant for near-wall effects. We then extend our discussion to show that the fundamental length and time scales that capture the near-wall behaviour in wall-bounded shear flows are the shear mixing length scale $L_S = (\epsilon/S^3)^{1/2}$ and the mean shear time scale $1/S$, respectively. With these appropriate length and time scales (or equivalently velocity and time scales), the eddy viscosity can be rewritten in the familiar form of the k–ϵ model as $\nu_t = (1/ST_L)^2 k^2/\epsilon$. We use the direct numerical simulation (DNS) data of turbulent channel flow of Hoyas & Jiménez (*Phys. Fluids*, vol. 18, 2006, 011702) and the turbulent boundary layer flow of Jiménez *et al.* (*J. Fluid Mech.* vol. 657, 2010, pp. 335–360) to perform *'a priori'* tests to check the validity of the revised eddy viscosity formulation. The comparisons with the exact computations from the DNS data are remarkable and highlight how well the equilibrium assumption holds in the near-wall region. These findings could prove to be useful in near-wall modelling of turbulent flows.

Key words: turbulent boundary layers, turbulent mixing, turbulence modelling

1. Introduction

Wall-bounded turbulent flows are prevalent in many engineered and natural flows such as turbulent flow in channels, pipelines and rivers. The presence of the solid wall has profound effects on the transport of momentum, mass and heat. As such, it is not surprising that the subject of near-wall modelling has received much attention in the last few decades. However, modelling the near-wall effects is not trivial due to the highly inhomogeneous and anisotropic nature of the flow in the 'near-wall' region, which can be considered to be the most volatile region of the turbulent boundary layer where most of the turbulence is produced. Turbulence closure schemes such

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as the k–ϵ model (Launder & Spalding 1972) are often used in Reynolds-averaged Navier–Stokes (RANS) numerical simulations to model turbulence in wall-bounded flows. Such models use the eddy viscosity hypothesis to link the turbulent momentum flux (Reynolds stresses) with the mean shear rate S through a turbulent eddy viscosity ν_t. For example, in a uni-directional shear flow (such as in a turbulent channel flow) with a streamwise mean velocity $\overline{U}$, and taking y as the wall-normal coordinate, the turbulent momentum flux is given by

$$\overline{u'v'} = -\nu_t \frac{\mathrm{d}\overline{U}}{\mathrm{d}y} = -\nu_t S, \tag{1.1}$$

where the mean shear rate $S = \mathrm{d}\overline{U}/\mathrm{d}y$. There are a number of different closure schemes that have been developed to model ν_t, but two-equation models such as the k–ϵ model, have emerged to be the most widely used complete closure schemes (Pope 2000; Durbin & Pettersson Reif 2011).

Wall-bounded flow (specifically channel flow) can be categorized into two main regions: an inner region and an outer region. The inner region can be further subdivided into three different layers namely: the viscous sublayer for $y^+ < 5$, buffer layer for $5 < y^+ < 30$ and the log-law region (constant-stress region that overlaps the inner and outer regions) for $y^+ > 30$, where y^+ is the wall unit defined as $y^+ = u_\tau y/\nu$ with u_τ, y and ν defined as the friction velocity, distance from the wall and the molecular (kinematic) viscosity, respectively (Pope 2000). The viscous sublayer and buffer layer together are classified as the near-wall (or viscous wall) region since viscosity is important and the energetic and dissipative scales overlap (Jiménez & Moser 2007). There have been numerous discussions on the appropriate velocity scale for both the inner and outer flow regions. The velocity scale that is assumed to be common to both regions of the flow is the friction velocity u_τ. However, the length scales are different; in the inner region, it is the viscous length scale ν/u_τ that is relevant while in the outer region, the boundary layer thickness is considered to be the appropriate length scale. Such arguments (in the limit of infinite Reynolds number) give rise to the logarithmic law for the mean velocity profile and are now commonly referred to as the 'classical' scaling (Jones, Nickels & Marusic 2008). Papers by Gad-el-Hak & Bandyopadhyay (1994), George (2007), Marusic *et al.* (2010) and Smits, McKeon & Marusic (2011) provide some comprehensive reviews on wall-bounded turbulent flows that highlight the issue of inner and outer scales of the flow and their universality.

In this paper, the focus is on the near-wall predictability of turbulence. Although the thickness of the near-wall region is two or more orders of magnitude smaller than the total flow depth, its effect extends throughout the whole flow as almost 50 % of the total flow velocity from the wall to the free surface occurs in this thin region (Hanjalić & Launder 1976). The remaining velocity differences occur mainly in the logarithmic layer. Therefore understanding the flow behaviour in the near-wall region is essential for modelling the turbulence. To this end, direct numerical simulations (DNS) have been used extensively to study the kinematics and dynamics of wall-bounded flows (for a recent in-depth review, see Jiménez 2012). Near the wall, the mean shear rate is very high and the local Reynolds number is low due to viscous effects. However, Durbin (1991) argues that the low-Reynolds-number effect is not as important as the wall blocking effect which results from the impermeability condition at the wall (i.e. zero normal velocity). All of the above conditions make this thin layer very interesting

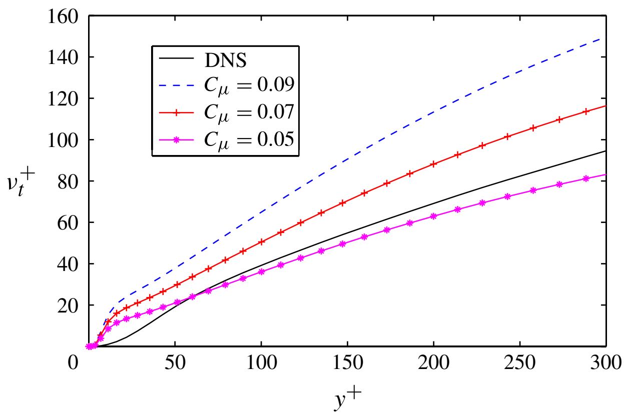

FIGURE 1. (Colour online) Comparison of the exact eddy viscosity and the k–ϵ prediction in a turbulent channel flow at $Re_\tau = 2003$, computed from the DNS data of Hoyas & Jiménez (2006).

to study and highlight the complexity involved in modelling the flow in the near-wall region compared to free shear flows.

The difficult and costly problem of resolving the very thin near-wall layer at high Reynolds number can be avoided by using wall functions which require the existence of a log-law region where production and dissipation of turbulent kinetic energy are nearly in balance. For example, in the standard k–ϵ model, ν_t is then calculated as

$$\nu_t = C_\mu \frac{k^2}{\epsilon}, \tag{1.2}$$

where k is the turbulent kinetic energy, ϵ is the dissipation rate of the turbulent kinetic energy, and $C_\mu = (|\overline{u'v'}|/k)^2$ is the eddy viscosity constant and usually assumed to be 0.09 in the constant-stress (log-law) region. This result for the eddy viscosity can be derived in a number of ways but it can be simply inferred from dimensional analysis by assuming that the characteristic velocity scale is $k^{1/2}$ and the characteristic time scale is $T_L = k/\epsilon$. The eddy viscosity formulation with a constant C_μ works well above the near-wall region, but as discussed by Durbin (1991) and indicated in figure 1, it overpredicts the eddy viscosity in the near-wall region even when C_μ is almost half of the commonly assumed value of 0.09. The results shown in figure 1 are computed from the channel flow DNS data of Hoyas & Jiménez (2006) at a friction Reynolds number $Re_\tau = 2003$. The eddy viscosities have been computed using both the exact definition obtained from (1.1) and the k–ϵ formulation given in (1.2) with the exact k and ϵ values from the DNS. Durbin & Pettersson Reif (2011) denote such comparisons as '*a priori*' tests.

This severe shortcoming has made it essential to make modifications to the k–ϵ model to correctly capture the near-wall behaviour. Attempts to model the near-wall effects date back to van Driest (1956) who used a damping function to reduce the eddy viscosity near the wall. A damping function is the ratio of the exact eddy viscosity to the eddy viscosity predicted by the turbulence model, given by

$$f_\mu = \frac{\nu_t}{C_\mu \dfrac{k^2}{\epsilon}}. \tag{1.3}$$

Since then numerous proposals have been made to reduce the eddy viscosity in the near-wall region. Some of the commonly cited formulations include those by Jones & Launder (1973), Lam & Bremhorst (1981) and Patel, Rodi & Scheuerer (1985), and Rodi & Mansour (1993). Most of these formulations use damping functions (sometimes loosely referred to as low-Reynolds-number models) that are based on y^+ and/or u_τ, and tend to be generally ineffective. Durbin (1991) rejected the notion of using arbitrary damping functions to overcome the deficiency of the k–ϵ model. He pointed out that the k–ϵ formulation is isotropic while the near-wall turbulence is anisotropic, and argued that it is the wall-normal turbulent velocity which is responsible for transport. He proposed the so-called k–ϵ–$\overline{v^2}$ model which is essentially an elliptic relaxation model that allows for a representation of the wall blocking effect. It involves the solution of a fourth-order (i.e. a four-equation model) coupled system of differential equations in order to calculate the eddy viscosity as

$$\nu_t = c'_\mu \overline{v'^2} \frac{k}{\epsilon}, \tag{1.4}$$

where c'_μ is a constant with a suggested value of 0.20 (Durbin 1991). It is important to note that in the k–ϵ–$\overline{v^2}$ model, the velocity scale is chosen as $\overline{v^2}$ (a model for the variance of the wall-normal component of turbulent velocity $\overline{v'^2}$), while the time scale is still chosen to be T_L, like in the standard k–ϵ formulations given in (1.2) or (1.3) with a lower bound set by the Kolmogorov time scale $(\nu/\epsilon)^{1/2}$. This model agrees well with the DNS data, especially in the near-wall region, but is sensitive to the choice of c'_μ away from the near-wall region as shown in figure 2. It must be noted that this model has been shown to successfully predict different complex flows (for more details, see Pope 2000; Durbin & Pettersson Reif 2011). A lot of the more recent works have focused on developing wall conditions for large-eddy simulations (LES) (see e.g. Kawai & Larsson 2012). In the RANS context, some recent work includes that of Kalitzin *et al.* (2005) where implications for the development of wall functions are discussed. Other recent RANS turbulence modelling efforts include near-wall corrections to account for low-Reynolds-number effects near the wall (Rahman & Siikonen 2005) and eddy-viscosity formulations proposed for the atmospheric boundary layer by Wilson (2012).

In this paper, our main goal is to highlight some insights that may be useful for modelling near-wall turbulence in closure schemes. We do this by revisiting the turbulent kinetic energy equation for a turbulent channel flow in order to propose a revised formulation for the eddy viscosity, and hence derive more appropriate velocity, length and time scales. In § 2, we present the evolution equation of the turbulent kinetic energy, followed by a proposal for the eddy viscosity by extending the equilibrium assumption to the near-wall region. This is followed by a discussion on relevant velocity, length and time scales. In § 3, '*a priori*' tests using DNS data are presented to highlight the validity of the revised formulation. Conclusions are given in § 4.

2. Parameterization of the eddy viscosity

2.1. *Turbulent kinetic energy equation*

The evolution equation for the turbulent kinetic energy (k) for an inhomogeneous constant-density shear flow can be written as (using the Einstein summation

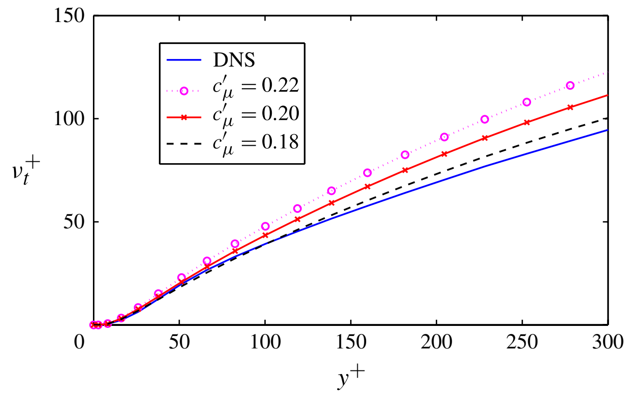

FIGURE 2. (Colour online) Comparison of the exact eddy viscosity and the k–ϵ–$\overline{v^2}$ model prediction in a turbulent channel flow at $Re_\tau = 2003$, computed from the DNS data of Hoyas & Jiménez (2006). Curves for different c'_μ values are shown.

convention)

$$\frac{\partial k}{\partial t} + \overline{U_j}\frac{\partial k}{\partial x_j} = P - \epsilon + D_\nu + T + \Pi, \tag{2.1}$$

where $P = -\overline{u'_i u'_j}\partial \overline{U_i}/\partial x_j$ is the production rate of k, $\epsilon = \nu\overline{(\partial u'_i/\partial x_j)(\partial u'_i/\partial x_j)}$ is the dissipation rate of k, $D_\nu = \nu\partial^2 k/(\partial x_j \partial x_j)$, is the viscous transport of the turbulent kinetic energy, $T = -(1/2)\partial\overline{(u'_j u'_i u'_i)}/\partial x_j$ is the turbulent velocity transport of k and $\Pi = -(1/\rho)\partial\overline{(p' u'_j)}/\partial x_j$ is the pressure transport of k, respectively. It is worth noting that all the three transport terms arise due to the inhomogeneity in the flow.

2.2. *Eddy viscosity and appropriate velocity scale*

For steady fully developed turbulent channel flow, (2.1) simplifies to

$$-\overline{u'v'}\frac{\mathrm{d}\overline{U}}{\mathrm{d}y} = \epsilon - D_\nu - T - \Pi, \tag{2.2}$$

where now the transport terms are also simpler (for details see e.g. Pope 2000). Equation (2.2) implies that the production of k is balanced by the dissipation and transport of k in the flow. Using the eddy viscosity hypothesis, P can be replaced with $\nu_t S^2$ and rearranging gives an expression for the eddy viscosity as

$$\nu_t = (\epsilon - D_\nu - T - \Pi)/S^2. \tag{2.3}$$

Evidence from DNS data (dating back to the seminal DNS of channel flow by Kim, Moin & Moser 1987) shows that in the near-wall region (especially in the buffer region), the dominant terms are P and ϵ, while the transport of k is substantially impeded. If we make the assumption to neglect all the transport terms in the near-wall region (i.e. assume equilibrium in the near-wall region), then (2.3) simplifies to

$$\nu_t \approx \epsilon/S^2. \tag{2.4}$$

The eddy viscosity given by (2.4) can be defined as an irreversible momentum diffusivity since it is based on ϵ, which is an irreversible quantity in the turbulent kinetic energy budget (Venayagamoorthy & Stretch 2010). Equation (2.4) also directly follows from the eddy viscosity hypothesis (1.1), once the equilibrium assumption is made. However, let us also take a somewhat lengthy path using the eddy viscosity formulation from a dimensional point of view, to illustrate how (2.4) is also equivalent to (1.2). Dimensional reasoning suggests that the eddy viscosity should be given by

$$\nu_t = u^* l^* = u^{*2} T^* = l^{*2}/T^*, \tag{2.5}$$

where u^* is a characteristic velocity scale, l^* is a characteristic mixing length scale (which will be discussed later), and T^* is a characteristic time scale. Pope (2000) suggests that a favourably disposed specification for the velocity scale is

$$u^* = |\overline{u'v'}\,|^{1/2}. \tag{2.6}$$

In the context of two-equation models, a good choice is to base the velocity scale on k as

$$u^* = ck^{1/2}, \tag{2.7}$$

where c is usually assumed to be a constant. However, it is easy (and important) to note from (2.6) and (2.7) that c is given as the square root of the stress-intensity ratio

$$c = \left(\frac{|\overline{u'v'}|}{k}\right)^{1/2}. \tag{2.8}$$

In the constant-stress region (i.e. in the log-law region in wall-bounded flows), $c \approx 0.55$, based on empirical evidence that the stress intensity $-\overline{u'v'}/k \approx 0.3$ in this region. However, elsewhere it should hold as a dynamic 'constant'. Using the eddy viscosity hypothesis given in (1.1), c can be expressed as

$$c = \left(\frac{\nu_t S}{k}\right)^{1/2} = \left(\frac{P}{Sk}\right)^{1/2}, \tag{2.9}$$

where the production $P = -\overline{u'v'}S = \nu_t S^2$. The quantity $P/(Sk)$ is the ratio of the mean shear time scale $1/S$ to the turbulence production time scale k/P. Hence, for equilibrium flows (when $P \approx \epsilon$), this simplifies to

$$c \approx \left(\frac{\epsilon}{Sk}\right)^{1/2} = \frac{1}{(ST_L)^{1/2}}. \tag{2.10}$$

At this point, it follows that the eddy viscosity constant C_μ in (1.2) is given by

$$C_\mu = c^4 \approx \frac{1}{(ST_L)^2}. \tag{2.11}$$

We note that (1.2) is therefore equivalent to (2.4) if C_μ is given by (2.11). The assertion in (2.11) can be tested using DNS data. Figure 3 shows the behaviour of $C_\mu = (1/ST_L)^2$ (computed from the DNS data) and $C_\mu = 0.09$. The exact value $\nu_t\epsilon/k^2$ computed from the DNS data is also shown. First, it is remarkable to see the excellent agreement between the curves given by (2.11) and the exact computation, especially in the near-wall region, indicating that the assumption made in neglecting the transport terms to arrive at (2.4) seems to be valid. Second, as already shown in figure 1, it is not surprising that the exact quantity is very different from the usually assumed constant value of 0.09 for C_μ, especially close to the wall.

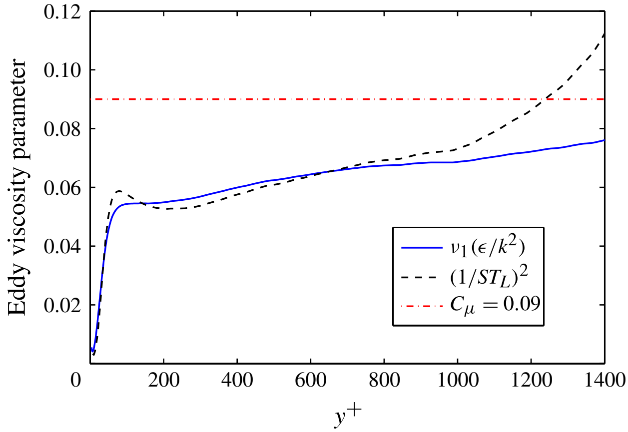

FIGURE 3. (Colour online) Profiles of $\nu_t\epsilon/k^2$ and $1/(ST_L)^2$ in a turbulent channel flow at $Re_\tau = 2003$, computed from the DNS data of Hoyas & Jiménez (2006). $C_\mu = 0.09$ is also shown for comparison.

Furthermore, substitution of the expression for c given by (2.10) in (2.7) indicates that the appropriate velocity scale should be prescribed as

$$u^* = U_S = \left(\frac{1}{ST_L}k\right)^{1/2} = \left(\frac{\epsilon}{S}\right)^{1/2}, \tag{2.12}$$

where ST_L is the ratio of the turbulence decay time scale T_L to mean shear time scale $1/S$. As an aside, it is worth noting that when $ST_L \to \infty$, the turbulence (fluctuations) in homogeneous shear flows can be described by rapid-distortion theory (see e.g. Pope 2000 for a detailed discussion), while for $ST_L \to 0$, turbulence production and eddy viscosity vanish. This limit is nicely discussed in Pope (2000) using the so-called return-to-isotropy models. The behaviour of ST_L obtained from channel flow DNS at $Re_\tau = 2003$ (Hoyas & Jiménez 2006) is shown in figure 4. It is clear that ST_L increases rapidly in the buffer layer in the near-wall region with a maximum value just greater than 18 at a distance of $y^+ \approx 8$. In essence, ST_L serves as the anisotropic correction scale in the near-wall region to the original velocity scale based on k that is used in the k–ϵ model. We also note that further away from the wall (in the far outer region, $y^+ \sim 1000$), the agreement between the exact curve and $(1/ST_L)^2$ shown in figure 3 diverges. This is clearly expected as the mean shear rapidly drops to zero beyond the log-law region. The effects of this singularity in the context of numerical modelling may be handled by adding a very small (positive) perturbation to ST_L.

2.3. *Relevant length and time scales*

Here we extend our discussion to the relevant length scale and time scale that are inherent in the eddy viscosity formulation that was presented in the previous section. From (2.5) in § 2.2, it is clear that a number of different length, time and velocity scales can be combined to obtain a dimensionally consistent eddy viscosity. However, the critical issue in the context of near-wall modelling is that the classical scales (i.e. $L = k^{3/2}/\epsilon$, $T_L = k/\epsilon$ and $U = k^{1/2}$) that two-equation models are based on do not seem to capture the near-wall behaviour of the eddy viscosity in a wall-bounded shear flow. Using the appropriate velocity scale obtained in (2.12), the corresponding length

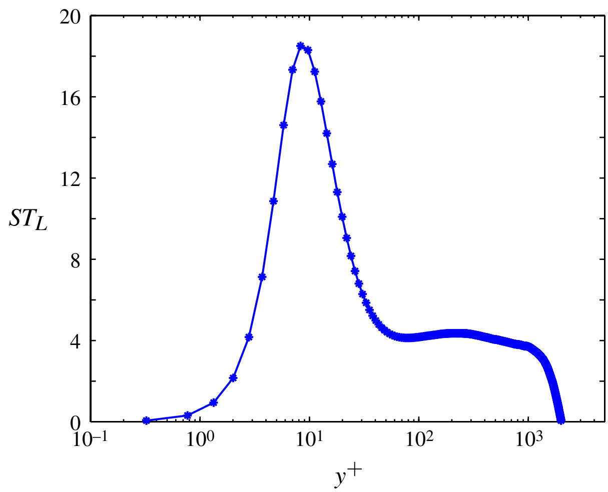

FIGURE 4. (Colour online) Profile of ratio of turbulence decay to mean shear time scales ST_L in a turbulent channel flow at $Re_\tau = 2003$, computed from the DNS data of Hoyas & Jiménez (2006).

scale l^* can be back calculated from (2.4) as follows:

$$l^* = \frac{\nu_t}{u^*} = \frac{\epsilon/S^2}{(k/ST_L)^{1/2}} = \left(\frac{\epsilon}{S^3}\right)^{1/2}. \tag{2.13}$$

This is indeed the shear length scale L_S, sometimes referred to as the Corrsin scale as he was the first to allude to this scale in his discussion on local isotropy in turbulent shear flows (Corrsin 1958). It is considered as the relevant scale that marks the start of the inertial subrange in turbulent shear flows (e.g. see Pope 2000). Conceptually, it can be thought of as the smallest scale at which eddies are strongly deformed by mean shear. The corresponding time scale is given by

$$T^* = \frac{\nu_t}{u^{*2}} = \frac{1}{S}. \tag{2.14}$$

We shall denote this time scale as T_S. This might perhaps appear a surprising result since it implies that the relevant time scale is governed by the mean shear rate S and not T_L as used in the formulations for the models given in (1.2) and (1.4). However, it is not at all surprising once we recognize that if $k^{1/2}$ is the wrong velocity scale (as suggested by Durbin 1991), then it must also mean that T_L will be the wrong time scale. Furthermore, this is obvious once we recall that S is implicit in the eddy viscosity hypothesis. In other words, T_L needs a similar anisotropic correction to that for the velocity scale, i.e. in this case by a factor of $(ST_L)^{-1}$.

It is constructive here to revisit the eddy viscosity formulation proposed by Durbin (1991) as shown in (1.4). We will assume that (2.4) is valid for now and test this validity later (in § 3) using DNS data. Using (2.4) and assuming (for the purpose of this exercise) that T_L is the appropriate time scale (noting that this is the time scale assumed in the k–ϵ model), the corresponding velocity scale can be calculated as follows:

$$\nu_t = \frac{\epsilon}{S^2} = U_M^2 T_L = U_M^2 \frac{k}{\epsilon}, \tag{2.15}$$

which can be rearranged to give the expression for velocity as

$$U_M = \frac{1}{ST_L} k^{1/2}. \tag{2.16}$$

The corresponding length scale can also be obtained in a similar manner as

$$L_M = \frac{k^{1/2}}{S} = \frac{1}{ST_L}\frac{k^{3/2}}{\epsilon}. \tag{2.17}$$

Venayagamoorthy & Stretch (2010) indicate that L_M can be considered as a rough measure of the active turbulent fluctuations in momentum and can be interpreted as the approximate measure of the average eddy size. Note that, using L_M and T_L, (2.4) can also be expressed as $\nu_t = L_M^2/T_L$. Now if (2.4) is indeed a good approximation to the actual eddy viscosity, then it should also be approximately equal to the eddy viscosity obtained from (1.4). Equating these two equations reveals the following relationships:

$$U_M \approx \left(c'_\mu \overline{v'^2}\right)^{1/2}, \tag{2.18}$$

$$L_M \approx \left(c'_\mu \overline{v'^2} T_L^2\right)^{1/2}. \tag{2.19}$$

Essentially, the terms on the right-hand side of (2.18) and (2.19) can be considered as the pertinent (effective) velocity and length scales in the k–ϵ–$\overline{v^2}$ model of Durbin (1991). We test the validity of these relationships in the next section.

3. *A priori* tests using DNS data

In this section we test the validity of the proposed model for the eddy viscosity given by (2.4). We then compare the velocity scale U_S and length scale L_S with the exact velocity scale $U_{mix} = u^*$, with u^* given previously in (2.6), and the exact mixing length scale $L_{mix} = |\overline{u'v'}|^{1/2}/S$ using DNS data of turbulent channel flow. We also compare the relationships proposed in (2.18) and (2.19). Comparisons with DNS data of turbulent boundary layer flows are also presented to highlight the applicability of the proposed model to other canonical wall-bounded turbulent flows.

3.1. *Eddy viscosity comparisons in turbulent channel flow*

Figure 5 shows the '*a priori*' comparison between the exact eddy viscosity obtained from (1.1) and the proposed approximation given in (2.4). The excellent agreement in the near-wall region is remarkable, especially given the fact that all the transport terms were neglected in arriving at (2.4). This implies that the transport terms are not as important as the production and dissipation terms in the near-wall region, at least as far as modelling the mixing in the near-wall region is concerned. We note that the eddy viscosity given by (2.4) can be expressed in non-dimensional form as a shear Reynolds number $Re_S = \epsilon/(\nu S^2)$. Therefore, Re_S provides a very good measure of the intensity of turbulent mixing in unstratified shear flows.

3.2. *Comparisons of velocity and length scales in turbulent channel flow*

Figure 6 shows the comparison of velocity scales and length scales discussed in § 2. First, the comparison between the exact velocity scale U_{mix} and the proposed velocity U_S given by (2.12) is very good in the near-wall region (see figure 6*a*). The corresponding comparisons between L_{mix} and L_S given by (2.13) as shown in figure 6(*b*) is almost perfect in the near-wall region. In essence, these results clearly

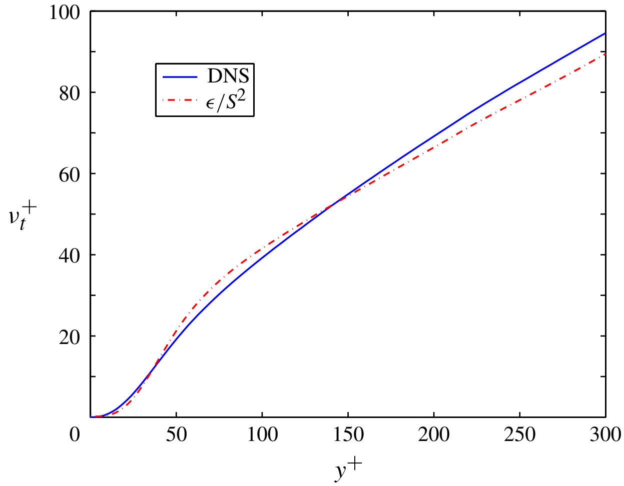

FIGURE 5. (Colour online) Comparison of the exact eddy viscosity and the prediction given by (2.4) in a turbulent channel flow at $Re_\tau = 2003$, computed from the DNS data of Hoyas & Jiménez (2006).

show that L_S, T_S and U_S are the appropriate turbulent scales that capture the behaviour of the near-wall turbulence.

Figure 6(*a,b*) also shows comparisons between U_M given by (2.16) with the right-hand-side term in (2.18) and between L_M given by (2.17) with the right-hand-side term in (2.19), respectively. Note that we have used $c'_\mu = 0.18$, since this value gave the best agreement with the DNS results shown in figure 2. The close agreement between these scales indicates that L_M and U_M may be considered to be the pertinent length and velocity scales embedded in the k–ϵ–$\overline{v^2}$ model. However, if a comparison is done between $(\overline{v'^2})^{1/2}$ and U_{mix} (not shown here), it becomes evident that $(\overline{v'^2})^{1/2}$ is a good choice for the velocity scale in the near-wall region but it deviates faster from U_{mix} than U_S does. This means that the constant c'_μ in the k–ϵ–$\overline{v^2}$ model is equivalent to a time scale correction factor such that $c'_\mu T_L$ is by construction designed to mimic the behaviour of the appropriate time scale $T_S = 1/S$, in order to predict the correct eddy viscosity in the near-wall region.

3.3. *Comparisons in turbulent boundary layer flow*

Figure 7(*a*) shows the '*a priori*' comparison between the exact eddy viscosity obtained from (1.1) and the proposed approximation given in (2.4) using DNS data of turbulent boundary layer flow (Jiménez *et al.* 2010) at a Reynolds number based on the momentum thickness of $Re_\theta \approx 2000$. Similar to the channel flow comparison shown in figure 5, there is excellent agreement in the near-wall region. Furthermore, the agreement between the exact velocity scale U_{mix} and the proposed velocity scale U_S is very good as shown in figure 7(*b*). The corresponding comparisons between the length scales L_{mix} and L_S shown in figure 7(*c*) are in excellent agreement in the near-wall region similar to the channel flow comparisons shown in figure 6(*b*). We note that comparisons (not shown) with DNS data of pipe flow at the relatively low Reynolds number of $Re_\tau = 190$ based on the pipe radius show good agreement (Loulou *et al.* 1997). The highest-Reynolds-number DNS of pipe flows to date were

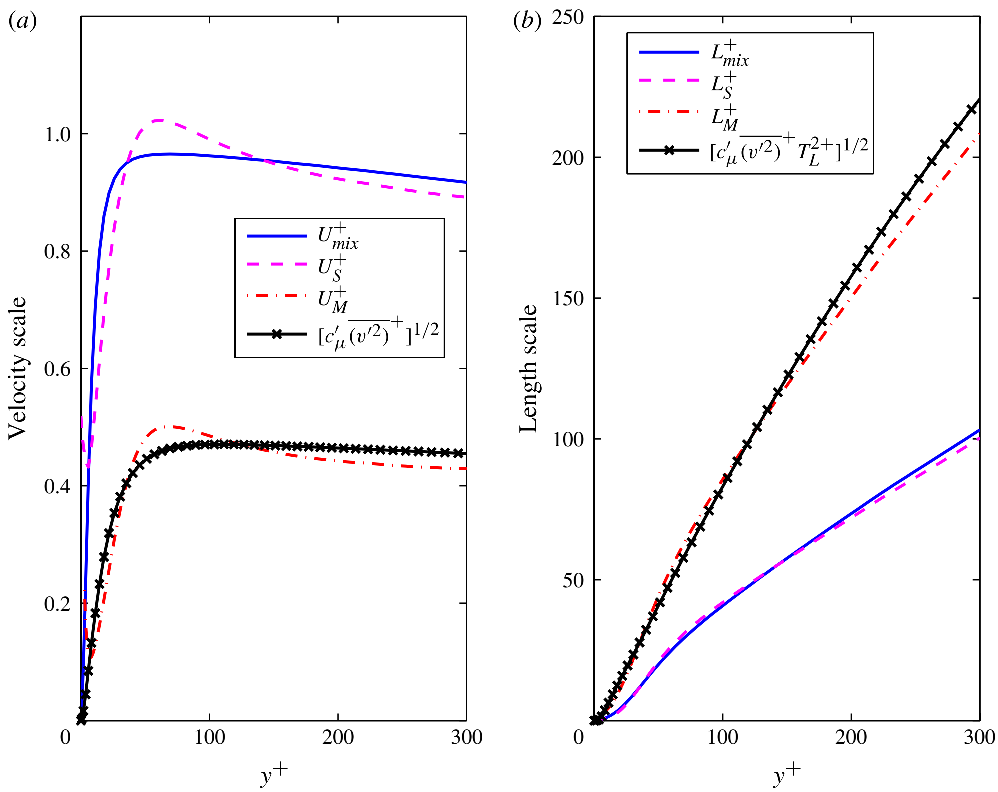

FIGURE 6. (Colour online) Comparison of (*a*) velocity scales and (*b*) length scales in a turbulent channel flow at $Re_\tau = 2003$, computed from the DNS data of Hoyas & Jiménez (2006). Note that $c'_\mu = 0.18$ was used in both (2.18) and (2.19).

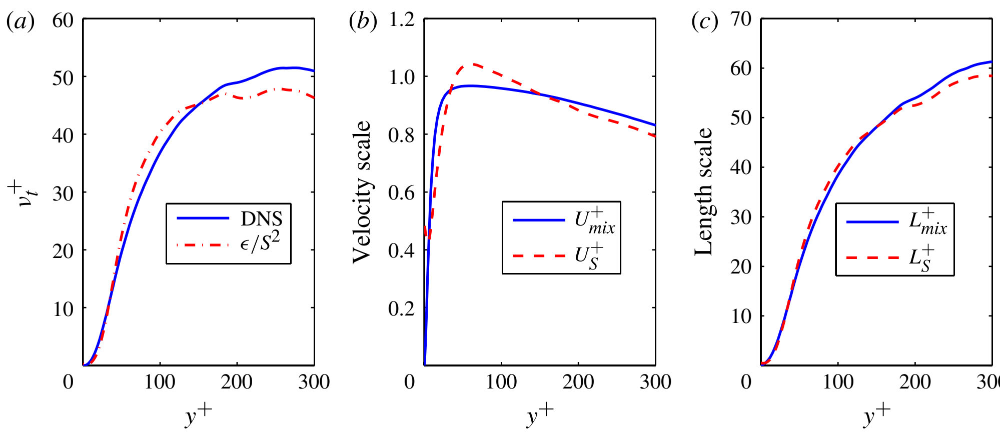

FIGURE 7. (Colour online) Comparison of (*a*) the exact eddy viscosity and the prediction given by (2.4), (*b*) velocity scales and (*c*) length scales in a turbulent boundary layer flow at $Re_\theta \approx 2000$, computed from the DNS data of Jiménez *et al.* (2010).

done by Wu & Moin (2008) but, unfortunately, the dissipation rate was not sampled and hence we were unable to verify the proposed scaling at higher Reynolds numbers that are comparable to the channel and boundary layer flows discussed in this paper. Nevertheless, these results indicate that the proposed scaling is widely applicable to turbulent wall-bounded flows.

3.4. *Implications at higher Reynolds numbers*

The turbulent channel and boundary layer flow DNS data that have been used to test the proposed scaling have (to our knowledge) the highest Reynolds numbers investigated to date. However, they are still well below the Reynolds numbers of most practical flows. There are higher-Reynolds-number pipe flow experiments (see e.g. the Princeton Superpipe experiments by Zagarola & Smits 1998 and McKeon *et al.* 2004) that have a significant logarithmic region but due to constraints of measurements very close to the wall did not include turbulence fluctuation statistics near the wall. However, we performed comparisons (not shown here in order to avoid repetition) of the proposed scaling with DNS of channel flow data at lower Reynolds numbers (Kim *et al.* 1987; Moser, Kim & Mansour 1999; del Álamo *et al.* 2004). The agreement becomes consistently better with increasing Re_τ, which is a promising trend, suggesting that the proposed prediction should hold true at even the higher Reynolds numbers that typify relevant practical flows.

4. Concluding remarks

In this study, we have made the equilibrium assumption (i.e. $P \approx \epsilon$) to propose that the eddy viscosity $\nu_t \approx \epsilon/S^2$. We have then argued by revisiting the eddy viscosity formulation that the appropriate velocity scale is $U_S = (ST_L)^{-1/2} k^{1/2} = (\epsilon/S)^{1/2}$ as opposed to the classical scale of $k^{1/2}$. We then extended our analysis to show that the corresponding appropriate length and time scales are $L_S = (\epsilon/S^3)^{1/2}$ and $T_S = 1/S$, respectively. The comparisons between the proposed scales and the exact scales computed from the most highly resolved turbulent channel flow DNS dataset to date show remarkable agreement. The agreement with DNS data of turbulent boundary layer flow is also very good. To our knowledge, this appears to be the first time such results have been reported for describing the behaviour of near-wall turbulence. We have also provided some insights into the pertinent velocity, length and time scales that are inherent in the k–ϵ–$\overline{v^2}$ model proposed by Durbin (1991).

In essence, these results highlight how well the equilibrium assumption holds in the near-wall region. We believe these findings may be useful in developing near-wall turbulence models. However, we recognize that this requires work well beyond the scope of this present study. Another obvious extension to this work is to study the effects of density stratification in wall-bounded shear flows. This adds another level of complexity through the coupling between the equations for the turbulent kinetic energy and density fluctuations via the buoyancy flux term.

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