Equation 3.13 was developed by combining Equations 3.5, 3.6, and 3.7, which are given below.

$$\hat{u} = \frac{1}{n} h^{2/3} S^{1/2}$$ Equation 3.5

$$\hat{u} = \sqrt{\frac{8g}{f}} h^{1/2} S^{1/2}$$ Equation 3.6

$$f = \frac{k_t}{Re}$$ Equation 3.7

To accomplish the task of deriving Equation 3.13 from Equations 3.5-3.7, two additional relationships were needed. The first is a resistance relationship $q = \alpha h^{\beta}$ for laminar overland flow, where $\beta = 3$ and the following equation describes α:

$$\alpha = \frac{8gS}{k_t v}$$ From Table 3.3

The second is a relationship between unit discharge and Reynolds number, where $Re = q/v$. The derivation process is described below:

Step 1) Set Equation 3.5 equal to Equation 3.6

$$\frac{1}{n} h^{2/3} \cancel{S^{1/2}} = \sqrt{\frac{8g}{f}} h^{1/2} \cancel{S^{1/2}}$$

Step 2) Replace f with Equation 3.7 and solve for n

$$\frac{1}{n} h^{2/3} = \sqrt{\frac{Re * 8g}{k_t}} h^{1/2} \quad \Rightarrow \quad n = \left(\frac{k_t}{Re * 8g}\right)^{1/2} h^{1/6}$$

Step 3) Solve for q such that $q = Re * v$

Step 4) Set the equation from Step 3) equal to the resistance relationship for laminar overland flow and solve for h

$$q = \underbrace{\frac{8gS}{k_t v}}_{\alpha} h^3 = Re * v \quad \Rightarrow \quad h = \left(\frac{k_t v^2 Re}{8gS}\right)^{1/3}$$

Step 5) Replace h from from Step 2) with the relationship for h from Step 4)

$$n = \left(\frac{Re * 8g}{k_t}\right)^{1/2} \left(\left(\frac{k_t v^2 Re}{8gS}\right)^{1/3}\right)^{1/6} = \left(\frac{Re * 8g}{k_t}\right)^{1/2} \left(\frac{k_t v^2 Re}{8gS}\right)^{1/18}$$

Straightforward.

Step 5) Simplify equation to derive Equation 3.13

$$n = \left[\left(\frac{k_t}{8g}\right)^{1/2} \left(\frac{k_t}{8g}\right)^{1/18}\right] \left[\frac{Re^{1/18}}{Re^{1/2}}\right] (v^2)^{1/18} \left(\frac{1}{S}\right)^{1/18}$$

fine.

$$\text{n} = \left(\frac{k_t}{8g}\right)^{5/9} \frac{v^{1/9}}{S^{1/18} Re^{4/9}}$$

Part b) Calculate Manning n (English units)

Step 1) Calculate unit discharge, q, using the relationship $q = i_e L$

$$q = i_e L = 1\frac{in}{hr} * 300\,ft * \frac{1ft}{12in} * \frac{1hr}{3600s} = 0.00694\,\frac{ft^2}{s}$$

Step 2) Calculate Re using $Re = q/v$. Viscosity, v, of 1 x 10^{-5} ft^2/s for clear water at 70°F was used

$$Re = q/v = \frac{\left(0.00694\,\frac{ft^2}{s}\right)}{1 * 10^{-5}\,\frac{ft^2}{s}} = 694.4$$

Step 3) Calculate Manning n using Equation 3.13, derived in Part a). Use the constant m = 1.49 to convert the equation from SI units to English units

$$\text{n} = \left(\frac{k_t}{8g}\right)^{5/9} \frac{1.49 * v^{1/9}}{S^{1/18} Re^{4/9}} = \left(\frac{50{,}000}{8(32.2)}\right)^{5/9} * \frac{1.49 * (1 * 10^{-5})^{\frac{1}{9}}}{(0.05)^{\frac{1}{18}}(694.4)^{\frac{4}{9}}} = \mathbf{0.54}$$

PROBLEM #2)

The first task for Problem #2 was to select a gage station with at least 20 years of discharge and sediment data. After evaluating USGS gages within Colorado, 5 were located that had a sufficient period of record for both discharge and sediment data, and most of these were located along Fountain Creek and Monument Creek in and around Colorado Springs.

The USGS gage at station 07103970 along Monument Creek above Woodmen Road at Colorado Springs, Colorado, was selected for this evaluation. This gage has nearly 25 years of discharge and sediment collection record and has operated since October of 1996. Discharge data was collected year-round, while sediment data was collected during the summer months between April 1 and September 30. Figure 1 gives discharge (cms) measurements during the period of record, and Figure 2 gives a semi-log plot of both discharge (cms) and sediment discharge (metric T/d).

Part a) Superposed hydrographs

Hydrographs for each year were superposed by day of the year to compare the timing and relative magnitude of the flows, shown by Figure 3. The three largest flow events to happen during the 25-year period of record occurred in the month of May. The largest magnitude storm to happen occurred in 1999 and had a peak daily flow rate of 57 cms. The second and third largest peak flow rates both occurred in 2015 and had peak daily flow rates of 46 cms and 33 cms, respectively.

Nice

The corresponding peak sediment discharges, also superposed by day of the year for the period of record, are shown in Figure 4. Generally, the largest peak sediment discharges correlate in timing and magnitude to the largest peak flow discharges. The largest daily peak sediment discharge, corresponding to the 1999 flood, was measured to be 44,544 T/d.

Part b) Sediment Rating Curve

A sediment rating curve was developed by relating the sediment discharge with the flow discharge for the period of record using a log plot, shown by Figure 5. A trendline was extracted from the data. Using this trendline, sediment discharge, Q_s, was related to discharge, Q, by the following power equation.

$$Q_S = 13.159Q^{2.225}$$

Part c) Flow Duration Curve (like Figure 4.6)

A flood frequency analysis was performed for the gage data using the maximum daily flow rate for each year between 1997 and 2021, shown in Figure 6. The following method was used to calculate the probability of exceedance:

Step 1) Determine the maximum daily flow rate for each complete year during the period of record

Step 2) Rank flow rates in order of largest magnitude to smallest magnitude

Step 3) Assign a Rank [m] according to magnitude, from 1 to number of values [n]

Step 4) Calculate the exceedance probability using the equation $E(Q) = \frac{m}{n+1} * 100\%$

A log plot of flow duration curve, modeled after Figure 4.6 from the River Mechanics manual, is given in Figure 7 below. Calculations for exceedance probability is provided in **Appendix A.**

Show graph here

Part d) Flow Duration Curve (like Figure 4.7)

A flow duration curve was then plotted after Figure 4.7 of the River Mechanics manual, which relates ln(Q) to Π(Q), which defined by the following equation:

$$\Pi(Q) = \ln[-\ln E(Q)]$$

This flow duration curve is shown in Figure 8. Extracting a trendline gives the following linear equation:

$$\Pi(Q) = 1.2512 \ln(Q) - 2.8285$$

Where $a = e^{-2.8285} = 0.059$ and $b = 1.2512$

The exponent $\hat{b}$ measures the non-linearity between rainfall intensity and parameter x, which in this case is discharge. The exponents $\hat{a}$ and $\hat{b}$ can be calculated using the following equations:

$$\hat{a} = \left(\frac{1}{a}\right)^{1/b} = 9.6$$

$$\hat{b} = \frac{1}{b} = 0.799$$

Part e) Flood Frequency Curve with Gumbel and Log-Pearson III distributions

To develop a flood frequency curve from the gage data, return period, T, was determined by taking the inverse of the exceedance probability for the daily discharge for each year from Part c):

$$T = \frac{1}{E(Q)}$$

The Gumbel distribution requires the mean $\bar{Q}$ and standard deviation, σ, to calculate discharge. For the maximum daily peak flow for each year, the mean $\bar{Q} = 10.1\ cfs$ and standard deviation, $\sigma = 13.08$. The frequency factor, K_G, for a return period T is calculated from the following equation:

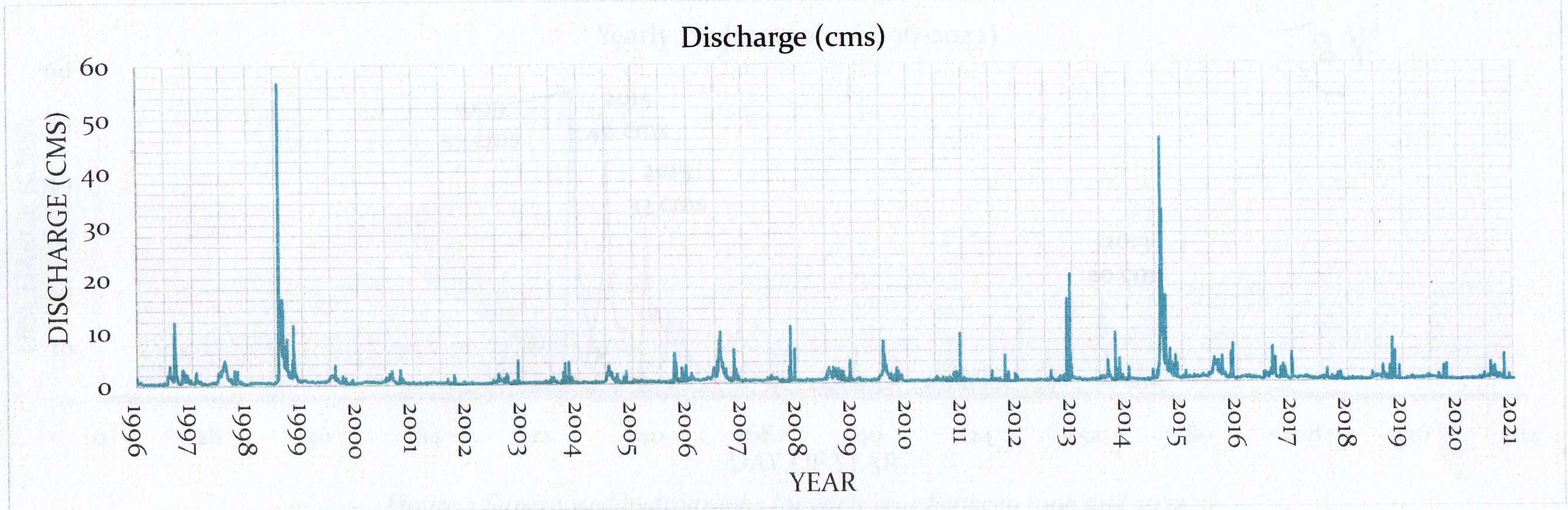


Figure 1. Flow rate at USGS gage 07103970 Monument Cr Abv Woodmen Rd at Colorado Springs, CO

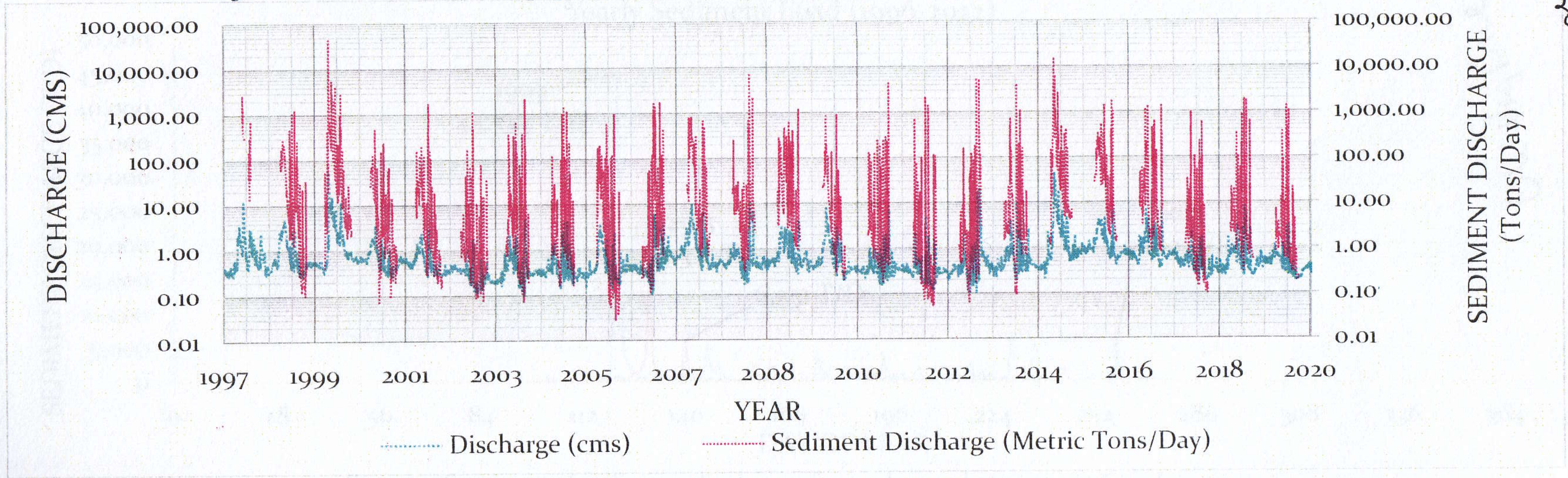


Figure 2 Flow rate and sediment data at USGS gage 07103970

Superpose each year

-3.

Cool!

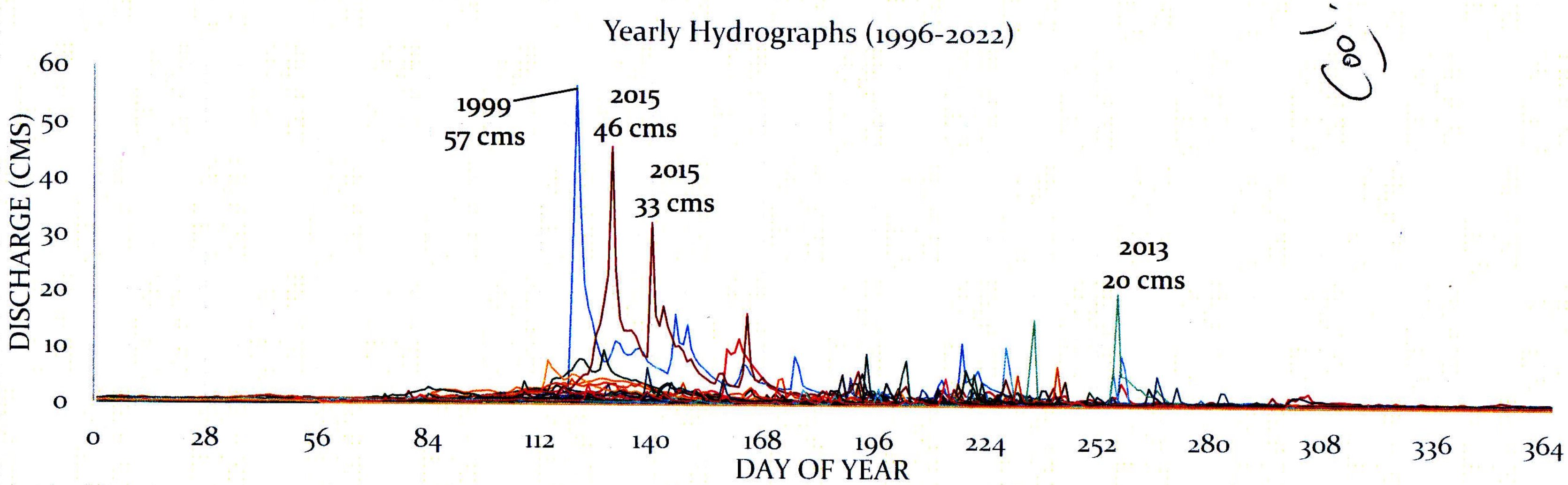


Figure 3 Superposed hydrographs for each year between 1996 and 2022

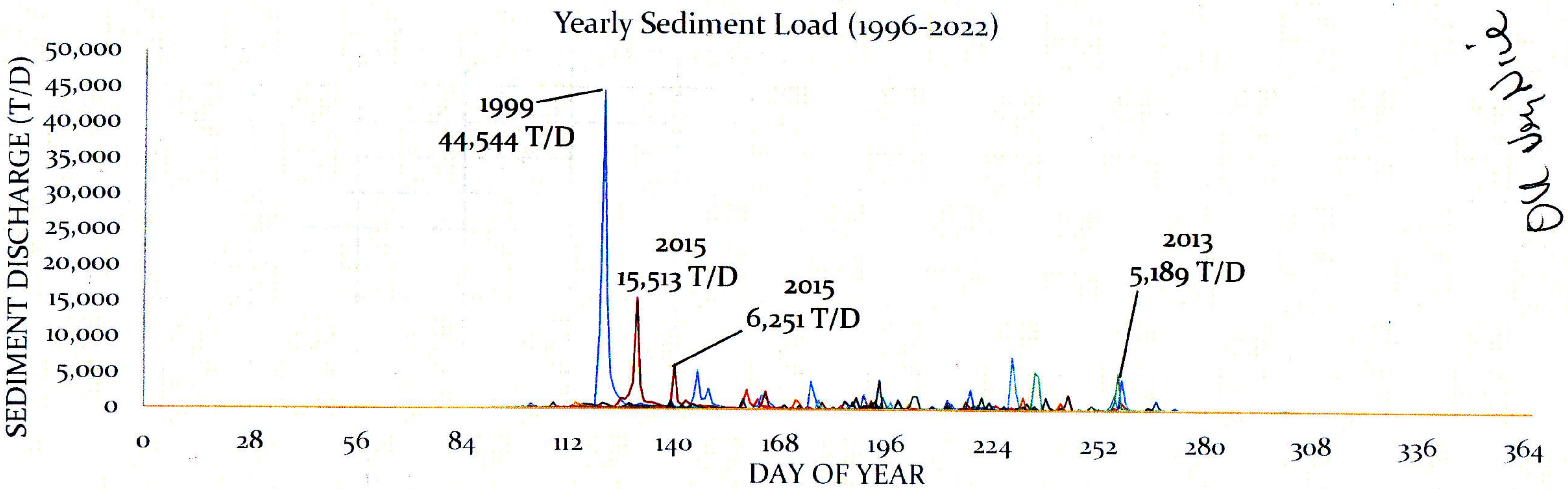


Figure 4 Superposed sediment discharge for each year between 1996 and 2022

Oh very nice

+3

So Nice

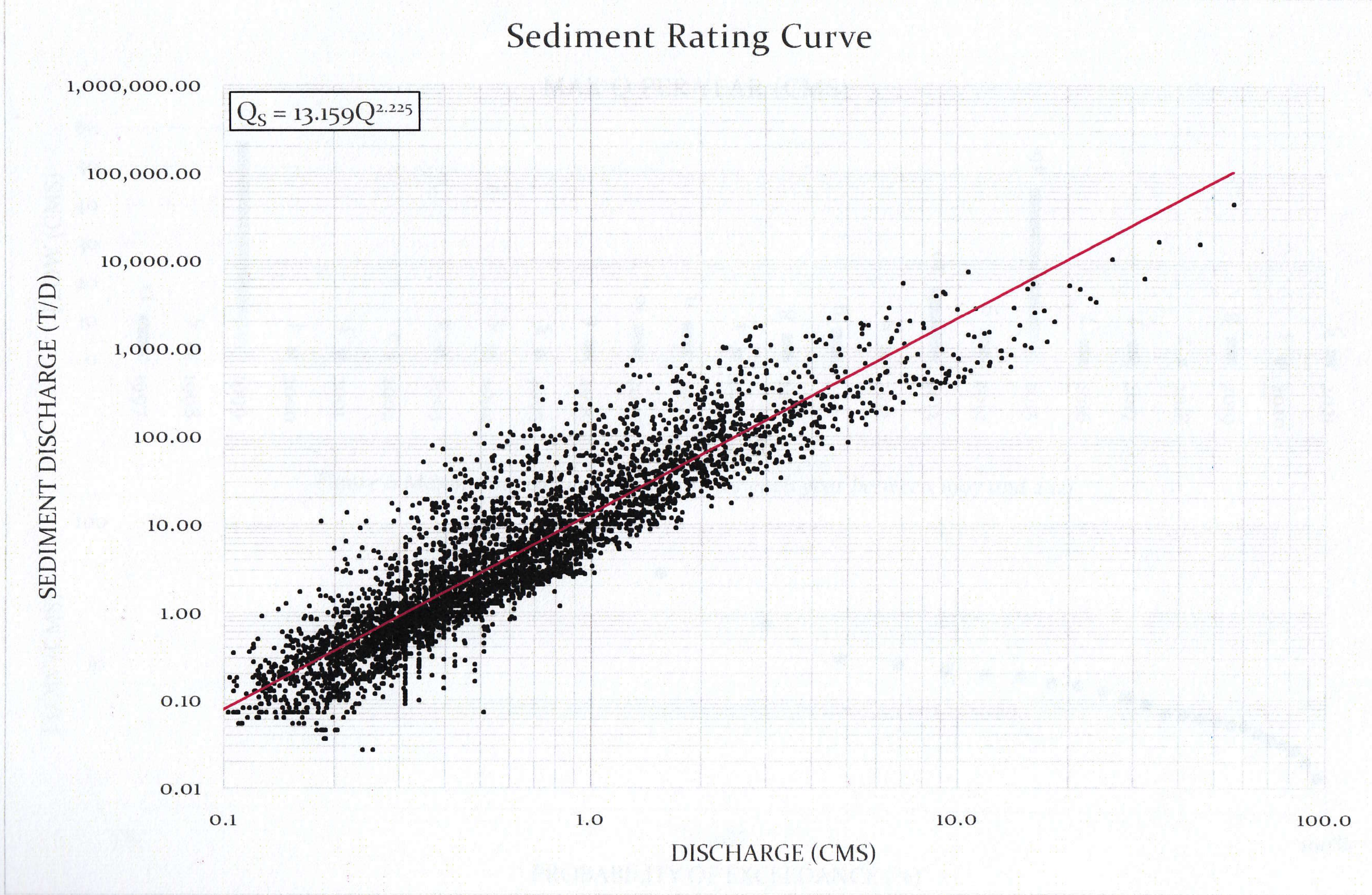


Figure 5 Sediment Rating Curve

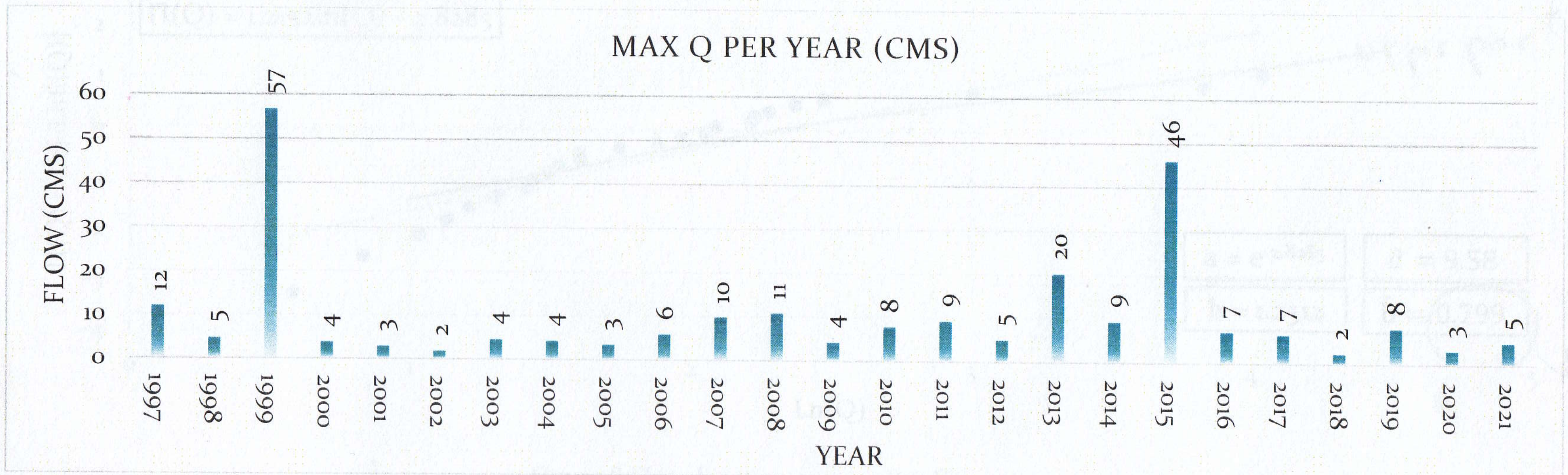


Figure 6 Maximum peak daily flow rate for each year between 1997 and 2021

So nice

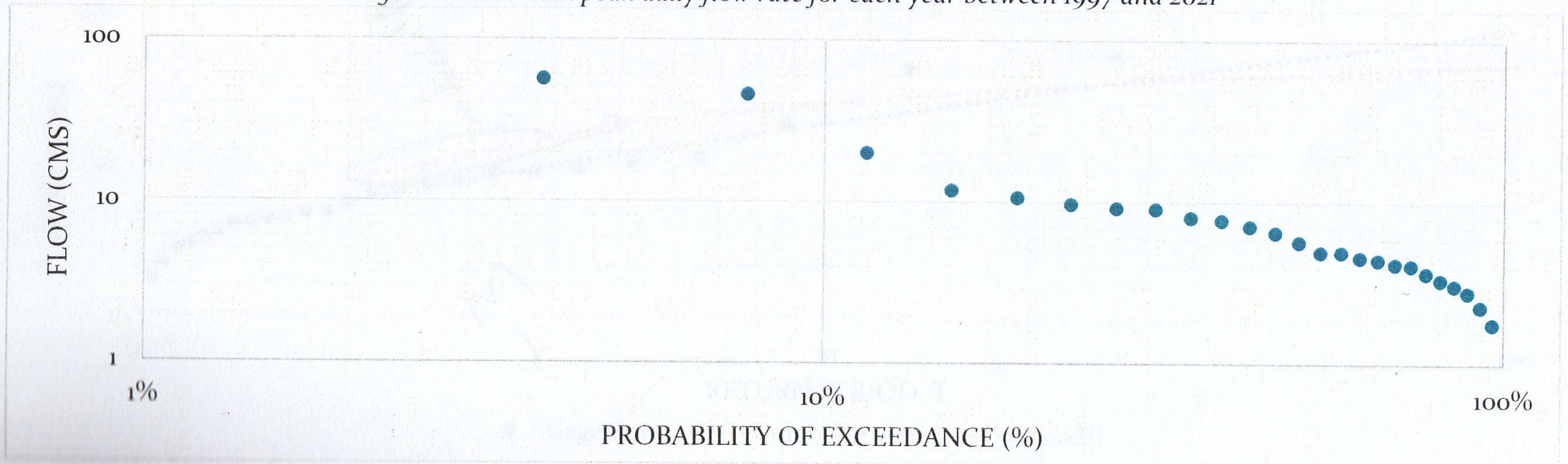


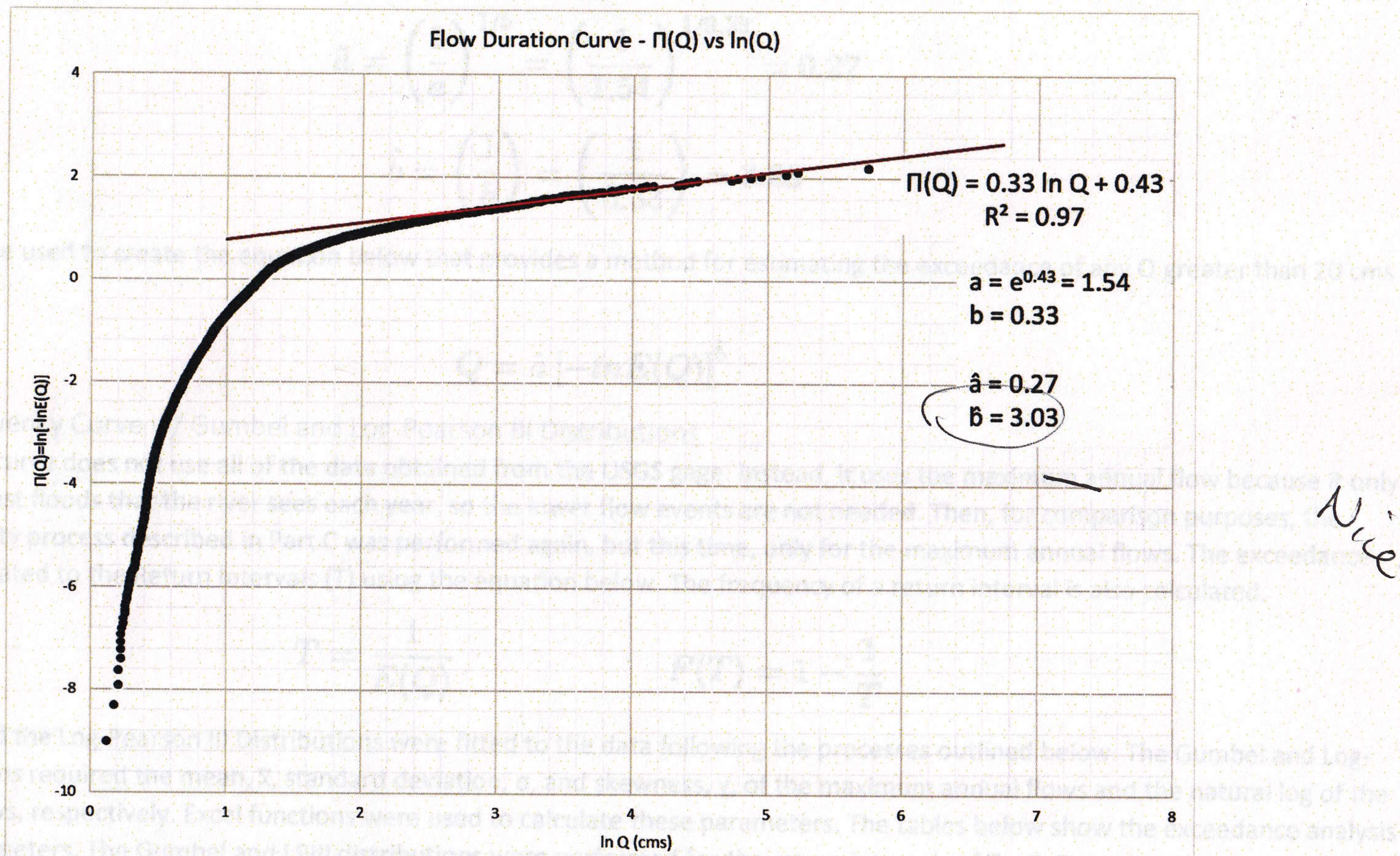
Figure 7 Flow Duration Curve, after Figure 4.6

Part D – Flow Duration Curve in Terms of ln Q and Π(Q)

Another way to represent a Flow Duration Curve is through the comparison of the ln Q and the Π(Q). The Π(Q) is calculated using the exceedance probability calculated in Part C above and the equation below.

$$\prod(Q) = ln\left[-lnE(Q)\right]$$

Once graphed, with the ln Q on the x-axis and the Π(Q) on the y-axis, a linear trendline can be fit for the higher flows, which is the zone of interest for this analysis because most analyses want to know exceedance probabilities of higher flows or floods. For this exercise, the flows greater than 20 cms (1.8% Exceedance) were used to develop the trendline. The graph is shown below.



The equation from the trendline is shown below.

$$\prod(Q) = 0.33ln(Q) + 0.43$$

The a and b values are taken from the trendline equation. The values of $\hat{a}$ and $\hat{b}$ are calculated from the a and b values using the equations below.

$$a = e^{0.43} = 1.54$$

$$b = 0.33$$

$$\hat{a} = \left(\frac{1}{a}\right)^{1/b} = \left(\frac{1}{1.54}\right)^{1/0.33} = 0.27$$

$$\hat{b} = \left(\frac{1}{b}\right) = \left(\frac{1}{0.33}\right) = 3.03$$

The $\hat{a}$ and $\hat{b}$ values are used to create the equation below that provides a method for estimating the exceedance of any Q greater than 20 cms (1.8% Exceedance).

$$Q = \hat{a}\left[-lnE(Q)\right]^{\hat{b}}$$

Part E – Flood Frequency Curve w/ Gumbel and Log-Pearson III Distributions

The flood frequency curve does not use all of the data obtained from the USGS gage. Instead, it uses the maximum annual flow because it only cares about the highest floods that the river sees each year, so the lower flow events are not needed. Then, for comparison purposes, the exceedance probability process described in Part C was performed again, but this time, only for the maximum annual flows. The exceedance probabilities were related to the Return Intervals (T) using the equation below. The frequency of a return interval is also calculated.

$$T = \frac{1}{E(Q)} \qquad F(T) = 1 - \frac{1}{T}$$

Then, the Gumbel and the Log-Pearson III Distributions were fitted to the data following the processes outlined below. The Gumbel and Log-Pearson III distributions required the mean, $\bar{x}$, standard deviation, σ, and skewness, γ, of the maximum annual flows and the natural log of the maximum annual flows, respectively. Excel functions were used to calculate these parameters. The tables below show the exceedance analysis and distribution parameters. The Gumbel and LPIII distributions were performed for the return intervals of T = 2, 5, 10, 20, 25, 50, 100, 200.

Ranked

Year	Peak Daily Flow (cms)	Ln Q	Rank	E(Q)	T
1999	311.30	5.74	1	0.04	24.00
2013	184.52	5.22	2	0.08	12.00
2015	140.37	4.94	3	0.13	8.00
2008	83.77	4.43	4	0.17	6.00
2011	78.11	4.36	5	0.21	4.80
2016	54.34	4.00	6	0.25	4.00
2004	51.79	3.95	7	0.29	3.43
2014	46.70	3.84	8	0.33	3.00
2007	42.45	3.75	9	0.38	2.67
2005	40.75	3.71	10	0.42	2.40
2001	39.05	3.66	11	0.46	2.18
2006	34.53	3.54	12	0.50	2.00
2012	32.55	3.48	13	0.54	1.85
2017	31.70	3.46	14	0.58	1.71
2010	30.00	3.40	15	0.63	1.60
1998	28.30	3.34	16	0.67	1.50
2003	25.84	3.25	17	0.71	1.41
2002	22.41	3.11	18	0.75	1.33
2009	20.09	3.00	19	0.79	1.26
2019	19.39	2.96	20	0.83	1.20
2018	19.02	2.95	21	0.88	1.14
2000	17.38	2.86	22	0.92	1.09
2020	14.46	2.67	23	0.96	1.04

	Gumbel	Log Pearson III
Mean, $\bar{x}$ =	59.51	3.72
Standard Deviation, σ =	68.39	0.78
Skew, γ =	2.78	1.10

Gumbel Distribution

The Gumbel Distribution method consists of calculating a frequency factor for each return interval using the equation below.

$$K_G(T) = -\frac{\sqrt{6}}{\pi}\left\{0.5772 + ln\left[ln\left(\frac{T}{T-1}\right)\right]\right\}$$

Then, the following equation is used to calculate a flow rate, Q, for each of the frequency factors using the equation below.

$$Q_G(T) = \bar{x} + \sigma K_G(T)$$

Log Pearson II Distribution

The normalized variable for each of the return intervals is calculated using the equation below.

$$z_p = \frac{F(T)^{0.135} - (1 - F(T))^{0.135}}{0.1975}$$

Then, similar to the Gumbel Distribution, a frequency factor for each return interval is calculated using the skewness and normalized variable.

$$K_p(\gamma) = \frac{2}{\gamma}\left(1 + \frac{\gamma z_p}{6} - \frac{\gamma^2}{36}\right)^3 - \frac{2}{\gamma}$$

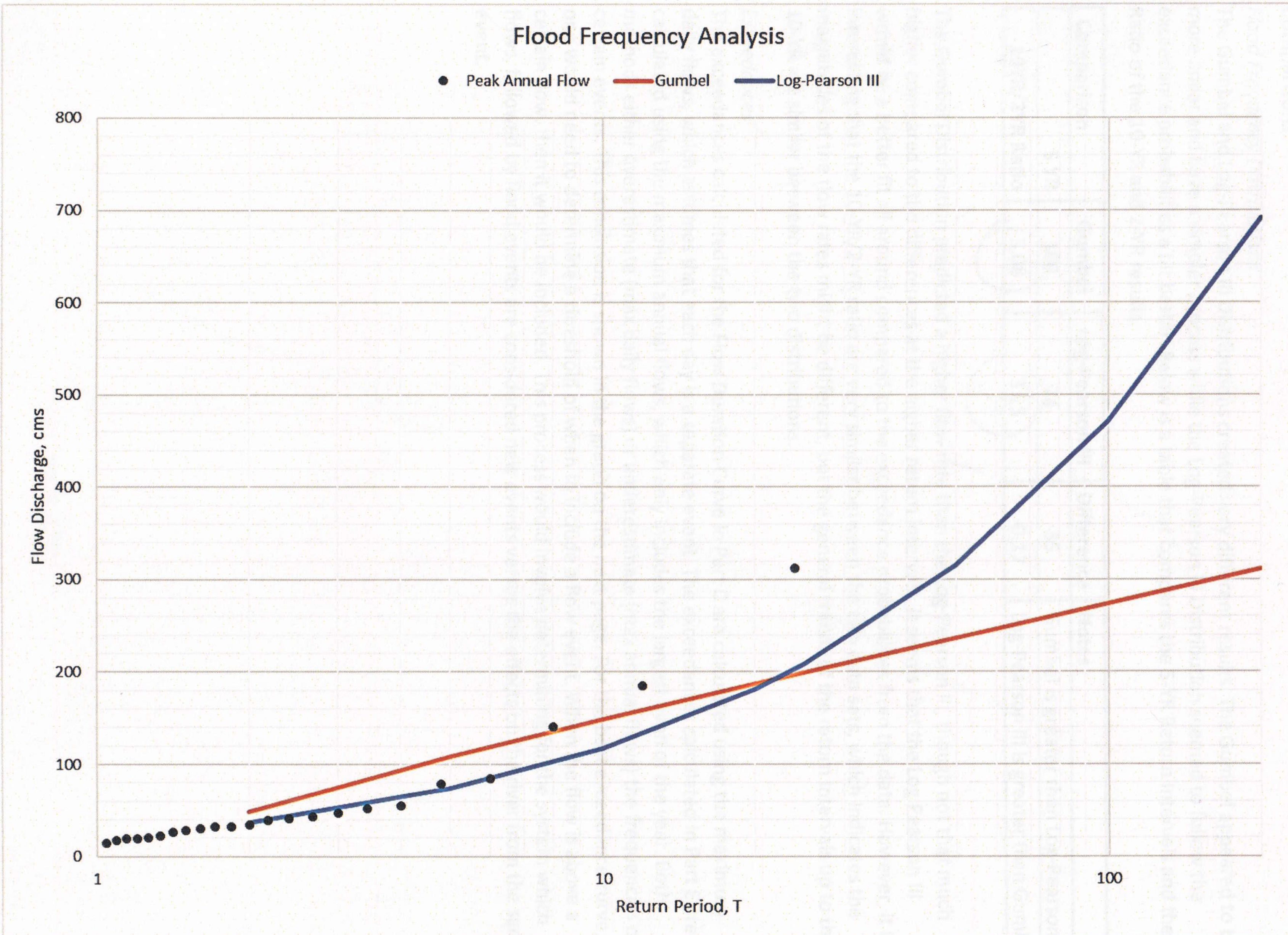
To obtain the flow rate at a particular return interval, the equation below is used.

Nice

$$Q_p(T) = e^{\bar{x} + \sigma K_p(\gamma)}$$

The table below and the graph on the next page show the results of the distribution calculations.

		Gumbel		Log-Pearson III		
T	F(T)	KG	QG	zp	Kp	Qp
2	0.5	-0.16	48	0.00	-0.18	36
5	0.8	0.72	109	0.84	0.74	74
10	0.9	1.30	149	1.28	1.33	117
20	0.95	1.87	187	1.65	1.89	182
25	0.96	2.04	199	1.76	2.07	208
50	0.98	2.59	237	2.06	2.60	315
100	0.99	3.14	274	2.34	3.11	470
200	0.995	3.68	311	2.58	3.61	692


Flood Frequency Analysis
Peak Annual Flow
Gumbel
Log-Pearson III
Flow Discharge, cms
800
700
600
500
400
300
200
100
0
1
10
100
Return Period, T

Comparisons

Flood Frequency Distributions

The Gumbel and Log-Pearson III Distributions created very different results. The Gumbel appeared to be more linear and have a simpler process while the Log-Pearson III Distribution seemed to follow the exceedance probabilities a bit better. Below is a table that compares the 5-YR Return Intervals and the Ratio of the 10-YR and 2-YR results.

Comparison	Gumbel	Log-Pearson III	Difference	Notes
5 YR	109	74	35	Gumbel is greater than Log-Pearson III
10YR/2YR Ratio	3.08	3.25	-0.17	Log-Pearson III is greater than Gumbel

The Gumbel Distribution predicted a higher flow rate than the Log-Pearson III, though not that much higher compared to the differences at the higher return intervals. It seems that the Log Pearson III would be a better fit all around compared to the exceedance probabilities from the data. However, it is interesting that the 10-YR/2-YR ratio are very similar between the two data sets, which indicates the magnitudes of the flow rates might be different, but the general trends of the return intervals up to the 10-YR are similar between the two distributions.

Exceedances

Nice

The exceedances calculated for the Flow Duration Curve in Part C are calculated using the maximum daily flows, which assumes that each day is a separate event. The exceedances calculated in Part E are calculated using the maximum annual flows, which only includes the largest even of the year. Both methods either overestimate (max daily flows) or underestimate (max annual flows) the frequencies of certain events. This conclusion is shown in the graph on the next page. For the best exceedance curve, one would need to determine a threshold of when to include a flow event. When the flow is above a certain flow, then it would be included. This process would involve determining for the system which flows, followed by flood events are considered new events versus the effects on the river from the same event.

Introduction:

Problem #1 covers the concept of resistance to flow and Problem #2 uses gage data of daily flow and sediment loading to plot a sediment rating curve, perform a flood frequency analysis, and compare the Gumbel and Log-Pearson distributions to peak flow data.

Given:

The tasks for Problem #1 and Problem #2 of Homework Assignment #1 are described below.

PROBLEM #1 – RESISTANCE TO FLOW

a) Combine equations 3.5-3.7 for overland flow to develop the relationship between n and k in Eq. 3.13.
b) What would be the n value for a grassed surface with k = 50,000 on a 5% hill slope 300 ft long under a 1 inch per hour rainfall intensity.

Nice!

PROBLEM #2 – FLOW-DURATION/SEDIMENT-RATING CURVES (SI UNITS)

Access the USGS web site from your home state and select a station with at least 20 years of discharge and sediment data. Determine the following:

a) Superpose the hydrographs for each year for your period of record
b) Plot the sediment rating curve in metric tons/day vs Q cms and define $Q_S = AQ^B$
c) Plot the flow duration curve in terms of log Q – log exceedance probability (like Fig. 4.6)
d) Plot the same flow duration curve (like Fig. 4.7a) and evaluate $\hat{a}$ and $\hat{b}$
e) Plot a flood frequency curve based on the same data and fit the Gumbel and LPIII distributions
f) Compare the 5 year discharge from both methods
g) What is the ratio of the 10 year flood to the 2 year flood from both methods?

Discuss the results!

Solution:

PROBLEM #1)

Part a) Derive Equation 3.13

Equation 3.13 gives the following relationship between Manning n and the resistance factor k_t:

$$n = \left(\frac{k_t}{8g}\right)^{5/9} \frac{\upsilon^{1/9}}{S^{1/18} Re^{4/9}} \qquad \text{Equation 3.5}$$