

Computer Problem 3: Field Trip

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Problem Statement:

Marcos Palu (pers. Comm.) reported that after the collapse of Fundao Dam in Brazil, the sediment concentration in the Doce River reached a value of 580g/l for about 6 hours. Consider the following river characteristics: river width 130m, flow depth 3.5 m, flow velocity 1.1 m/s, bed slope 0.0005, and shear velocity 0.13 m/s. Use Eq. 13.8 with the following dispersion coefficient $K_d = 150 \text{ m}^2/\text{s}$ and the sediment settling rate $k = 0.0000036 \text{ s}^{-1}$ to estimate the sediment concentration as a function of time at Oculos station located 94 km downstream. Compare the results with $k = 0$.

Analysis:

Sediment Concentration was calculated using Equation 13.8, seen below:

$$C(x, t) = \frac{C_0}{2} \left\{ e^{\frac{Ux(1-\Gamma)}{2K}} \left[\operatorname{erfc} \left(\frac{x - U t \Gamma}{2\sqrt{Kt}} \right) - \operatorname{erfc} \left(\frac{x - U(t-T)\Gamma}{2\sqrt{K(t-T)}} \right) \right] + e^{\frac{Ux(1+\Gamma)}{2K}} \left[\operatorname{erfc} \left(\frac{x + U t \Gamma}{2\sqrt{Kt}} \right) - \operatorname{erfc} \left(\frac{x + U(t-T)\Gamma}{2\sqrt{K(t-T)}} \right) \right] \right\} \quad (13.8)$$

Where parameters are as follows:

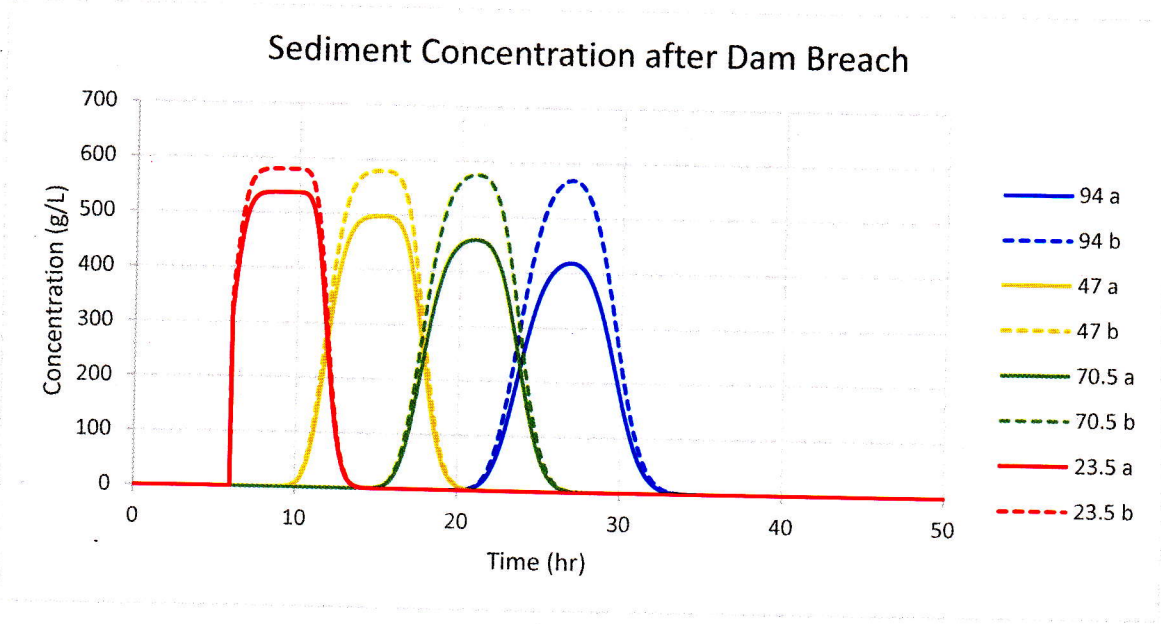
Parameter	Variable	Value
C_0	Initial Concentration	580 g/L
U	Flow Velocity	1.1 m/s
x	Station Downstream	Varied
t	Time	Varied
K	Dispersion Coefficient	$150 \text{ m}^2/\text{s}$
T	Duration of Spill	21,600 seconds

Gamma was defined using the following equation:

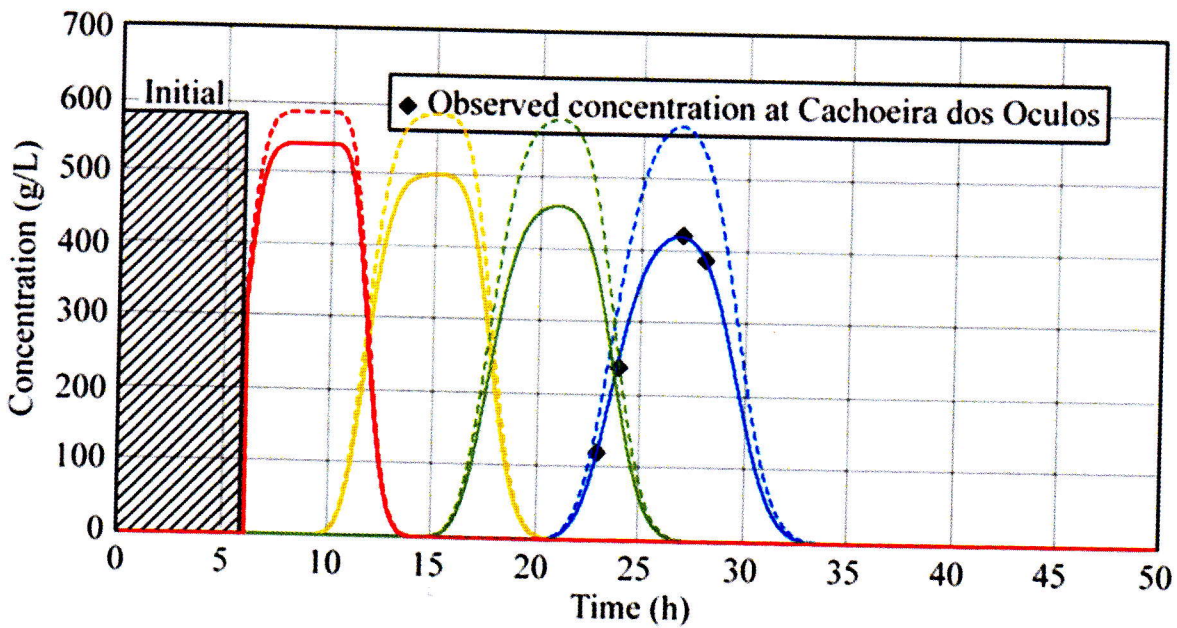
$$\Gamma = \sqrt{1 + 4kK / U^2}$$

Where k was either 0.0000036 per second's or 0 , for cases a and b respectively. Gamma for case a was 1.000089 and gamma for case b was 1 .

Concentration was calculated at stations 23.5, 47, 70.5 and 94 at each minute. The concentrations were then plotted against time in the figure below:



When overlaid against Figure P.13.10, the following figure was acquired:



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Case a, represented by the solid lines, fits the observed data well at station 94. Case b, where no sediment settles out in transport, dramatically overestimates the concentration, with the peak hardly decreasing at all from the initial value. A table of Sediment Concentration and time for station 94 can be found attached.

Station	Time (hr)	C a g/L	C b g/L
94000	0		
94000	1		
94000	2		
94000	3		
94000	4		
94000	5		
94000	6		
94000	7	6E-126	6E-126
94000	8	2.4E-97	2.7E-97
94000	9	9E-76	1E-75
94000	10	3.9E-59	4.4E-59
94000	11	4.3E-46	4.9E-46
94000	12	9.7E-36	1.1E-35
94000	13	1.9E-27	2.2E-27
94000	14	8.5E-21	1E-20
94000	15	2E-15	2.4E-15
94000	16	4E-11	4.9E-11
94000	17	1.1E-07	1.4E-07
94000	18	6.7E-05	8.5E-05
94000	19	0.00876	0.01118
94000	20	0.35595	0.45955
94000	21	5.42629	7.08817
94000	22	36.4552	48.1507
94000	23	125.213	167.052
94000	24	254.983	343.038
94000	25	359.464	486.489
94000	26	408.492	554.68
94000	27	417.889	568.415
94000	28	389.618	531.283
94000	29	301.217	412.897
94000	30	171.479	236.958
94000	31	66.9918	93.4995
94000	32	17.6161	24.8606
94000	33	3.14906	4.49656
94000	34	0.39147	0.56579
94000	35	0.03477	0.05087
94000	36	0.00227	0.00336
94000	37	0.00011	0.00017
94000	38	5E-06	7.6E-06