Assignment #5

May 3rd, 2023

Problem #1 Concentration Profiles (50 points) English and SI Units

Answer the questions from problem 10.2 on pg. 259 using the measurements of the Low Flow Conveyance Channel for the two profiles on pg. 138-139. Graphically determine the Ro and fall velocity ω, and use a spreadsheet to recalculate the mean flow velocity, momentum correction factor, unit discharge, unit sediment discharge q_s in lb./ft.s, and the flux-average concentration in mg/l. Compare the profiles and discuss the results.

Problem 10.2: Given the sediment concentration profile from Problem 6.1: (a) plot the concentration profile log *C* versus log *(h – z)/z*; (b) estimate the particle diameter from the Rouse number; and (c) determine the unit sediment discharge from the given data.

Cross Section:	**LF -11**		**Cross Section:**	**LF -11**	
Station:	**32ft)**		**Station:**	**47ft**	
Planform:	**Plane bed**		**Planform:**	**Dune**	
Date:	**Jun-99**		**Date:**	**May-01**	
Discharge:	**625**	**ft³/s**	**Discharge:**	**585**	**ft³/s**
Flow Area:	**202**	**ft²**	**Flow Area:**	**239**	**ft²**
Wetted Perimeter:	**52.69**	**ft**	**Wetted Perimeter:**	**53.66**	**ft**
Hydraulic Radius:	**3.95**	**ft**	**Hydraulic Radius:**	**4.04**	**ft**
Average Depth:	**5.04**	**ft**	**Average Depth:**	**7.34**	**ft**
Top Width:	**50.1**	**ft**	**Top Width:**	**50.2**	**ft**
Energy Slope:	**0.000382**		**Energy Slope:**	**0.000413**	
Froude Number:	**0.33**		**Froude Number:**	**0.28**	
Manning's:	**0.02**		**Manning's:**	**0.035**	
Total Depth:	**5.6**	**ft**	**Total Depth:**	**7.1**	**ft**
shear velocity	**0.22**	**ft/s**	**shear velocity**	**0.232**	**ft/s**

Concentration Profile

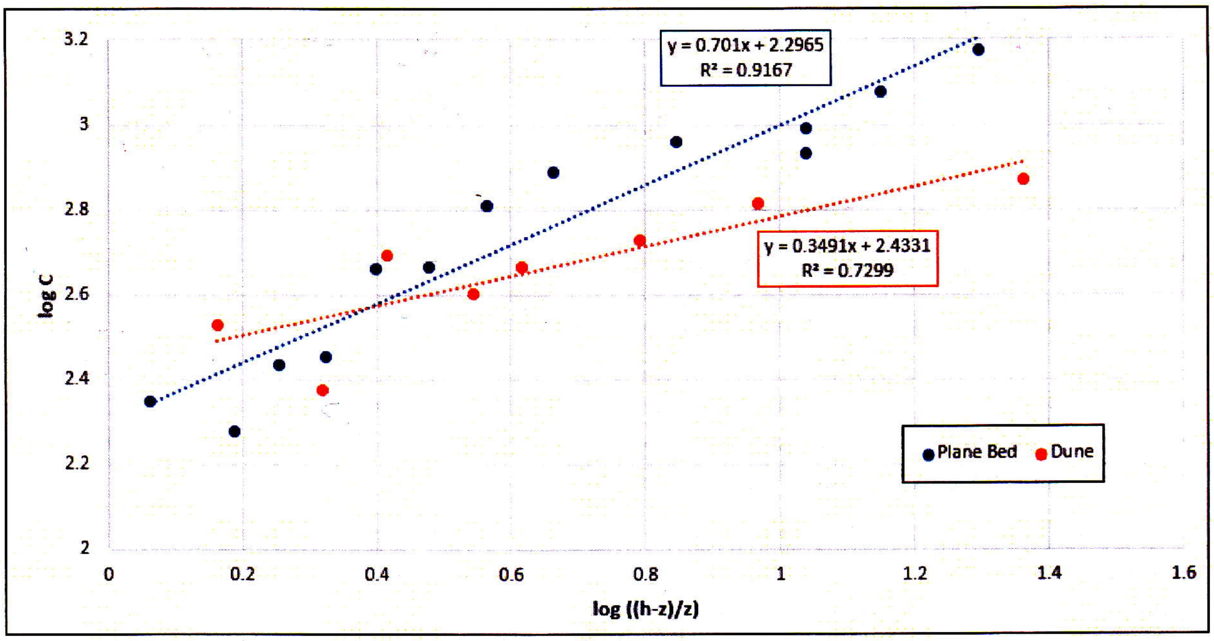


Figure 1. Concentration Profile

Very Nice.

Rouse Number:

The Rouse number can be determined by the slope of the linear trendline for each set of data. Therefore:

- **Plane Bed: Rouse number = 0.701**
- **Dune: Rouse number = 0.3491**

1/

Settling Velocity, ω:

$$\frac{u_*}{\omega} = \frac{2.5}{Ro} \quad \text{From pg. 231.}$$

$$\omega = \frac{Ro * u_*}{2.5}$$

Plane Bed:

$$\omega = \frac{(0.701) * (0.22\frac{ft}{s})}{2.5} = \mathbf{0.062\frac{ft}{s} = 0.019\frac{m}{s}}$$

Dune:

$$\omega = \frac{(0.3491) * (0.232\frac{ft}{s})}{2.5} = \mathbf{0.032\frac{ft}{s} = 0.010\frac{m}{s}}$$

Particle Diameter:

$$\omega = \frac{8v_m}{d_s}((1 + 0.0139d_*^3)^{0.5} - 1) \qquad \text{(Eqn. 5.23d)}$$

$$d_* = d_s\left[\frac{(G-1)g}{v_m^2}\right]^{\frac{1}{3}} \qquad \text{(Eqn. 5.23e)}$$

$$\omega = \frac{8v_m}{d_s}(\left(1 + 0.0139\left(d_s\left[\frac{(G-1)g}{v_m^2}\right]^{\frac{1}{3}}\right)^3\right)^{0.5} - 1)$$

Plane Bed:

$$\boldsymbol{d_s = 5.47 * 10^{-4} ft = 0.167\ mm}$$

Dune:

$$\boldsymbol{d_s = 3.77 * 10^{-4} ft = 0.115\ mm}$$

Mean Flow Velocity:

Plane Bed:

$$V = \frac{1}{h}\sum_{i=1}^{N} v_i \Delta z_i = \frac{24.6982}{5.6} = \boldsymbol{4.41\frac{ft}{s} = 1.34\frac{m}{s}}$$

Dune:

$$V = \frac{1}{h}\sum_{i=1}^{N} v_i \Delta z_i = \frac{21.844926}{7.1} = \boldsymbol{3.08\frac{ft}{s} = 0.938\frac{m}{s}}$$

Momentum Correction Factor:

Plane Bed:

Table.

$$\beta_m = \frac{1}{hV_x^2}\sum_i v_{xi}^2 dh_i = \frac{1}{(5.04ft)\left(4.41\frac{ft}{s}\right)^2}(112.64412) = \mathbf{1.15}$$

Dune:

$$\beta_m = \frac{1}{hV_x^2}\sum_i v_{xi}^2 dh_i = \frac{1}{(7.34ft)\left(3.08\frac{ft}{s}\right)^2}(69.65937) = \mathbf{1.00}$$

Unit Discharge:

Plane Bed:

$$q = Vh = \left(4.41\ \frac{ft}{s}\right)(5.6\ ft) = \mathbf{24.7}\frac{\boldsymbol{ft^2}}{\boldsymbol{s}} = \mathbf{7.53}\frac{\boldsymbol{m^2}}{\boldsymbol{s}}$$

Dune:

$$q = Vh = \left(3.08\ \frac{ft}{s}\right)(7.1\ ft) = \mathbf{21.8}\frac{\boldsymbol{ft^2}}{\boldsymbol{s}} = \mathbf{6.66}\frac{\boldsymbol{m^2}}{\boldsymbol{s}}$$

Unit Sediment Discharge, q_s:

$$q_s = \sum_{i=1}^{N} C_i v_i \Delta z_i$$

Where C_i = concentration in mg/l, v_i = flow velocity in ft/s, and Δz_i = change in depth in ft.

Unit Conversion:

$$\frac{1mg}{L} * \frac{2.2046 * 10^{-6}\ lb}{1mg} * \frac{1L}{0.035315 ft^3}$$

Plane Bed:

Plane Bed					
z (ft)	Δz (ft)	v (ft/s)	C (mg/l)	qsi (mg*ft^2/s*L)	qsi (lb/ft-s)
0.27	0.27	3.44	1493	1386.6984	0.086568569
0.37	0.1	3.27	1194	390.438	0.024374196
0.47	0.1	3.62	975	352.95	0.022033902
0.7	0.23	3.7	914	777.814	0.048557238
1	0.3	3.91	776	910.248	0.056824805
1.2	0.2	4.22	648	546.912	0.034142528
1.4	0.2	4.31	463	399.106	0.024915321
1.6	0.2	4.45	459	408.51	0.025502392
1.8	0.2	4.58	283	259.228	0.016183041
2	0.2	4.73	271	256.366	0.016004372
2.2	0.2	4.61	190	175.18	0.010936107
2.6	0.4	4.83	223	430.836	0.026896155
			qs	0.392938626	lb/ft-s

The unit sediment discharge for the plane bed is: $q_s = \mathbf{0.393}\frac{lb}{ft*s}$

Dune:

			Dunes		
z (m)	Δz (ft)	v (ft/s)	C (mg/l)	qsi (mg*ft^2/s*L)	qsi (lb/ft-s)
0.09	0.09	2.3129921	738	153.628937	0.0095907
0.21	0.12	2.4770341	654	194.3976378	0.0121358
0.3	0.09	2.582021	533	123.8595472	0.0077323
0.42	0.12	2.7559055	462	152.7874016	0.0095382
0.48	0.06	2.9658793	401	71.35905512	0.0044548
0.6	0.12	3.0249344	491	178.2291339	0.0111265
0.7	0.1	3.1791339	238	75.66338583	0.0047235
0.88	0.18	3.1988189	337	194.0403543	0.0121135
				qs	0.0714153 lb/ft-s

The unit sediment discharge for the dunes is: $\boldsymbol{q_s = 0.071\frac{lb}{ft*s}}$

low.

Flux-averaged Concentration, C_f:

$$C_f = \frac{q_s}{q} \quad \text{(Eqn. 11.30b)}$$

Plane Bed:

$$C_f = \frac{q_s}{q} = \frac{0.071\frac{lb}{ft*s}}{21.8\frac{ft^2}{s}} = 0.0159\frac{lb}{ft^3} * \frac{1mg}{2.2046*10^{-6}\,lb} * \frac{0.035315\,ft^3}{1L}$$

$$\boldsymbol{C_f = 254.8\frac{mg}{L}}$$

Dune:

$$C_f = \frac{q_s}{q} = \frac{0.393\frac{lb}{ft*s}}{24.7\frac{ft^2}{s}} = 0.0033\frac{lb}{ft^3} * \frac{1mg}{2.2046*10^{-6}\,lb} * \frac{0.035315\,ft^3}{1L}$$

$$\boldsymbol{C_f = 52.37\frac{mg}{L}}$$

Discussion:

The plane bed bedform has a much steeper concentration profile, resulting in a higher Rousse number when compared to the dune bedform. The plane bed bedform has a greater sediment yield than the dune bedform as well as a higher settling velocity, unit discharge, momentum correction factor, mean flow velocity, and particle diameter compared to the dune bedform.

plot profiles | short.

Velocity profiles:

The high unit discharge at each velocity profile tells me that they were taken near the thalweg of the channel and is not representative of the average conditions along the cross section. Our estimate of the suspended sediment discharge likely overpredicts the true value because we only have data from one profile which is in the thalweg where there is high sediment discharge and no data from the lower velocity, lower transport regions of the flow.

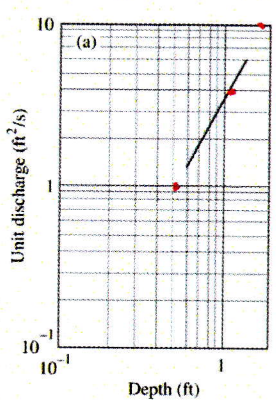
Sediment load:

The relatively low calculated rouse number of these two profiles indicates that this system is transporting the majority of its sediment in suspension, especially for profile 2 where dunes are observed, and concentrations are higher. Due to this low Rouse Number the modified Einstein approach is recommended for total sediment discharge calculation. This approach is a top-down approach which represents systems with high suspended transport best. The back calculated diameter of the particles in suspension is close to or more than the d50 of the bed. This indicates that the system is capacity limited, which is helpful for prediction of future sediment loading. Using the two selected methods of estimating the Einstein integral I found very similar results. The small differences could be attributed to the relatively large discretization in method 1 or the biases in the fit to the velocity profile in method 2. The larger of the two estimates was reported in the results section to provide the most conservative estimate for sedimentation. In most engineering applications it would be better to air on the side of higher sediment transport for our designs as it makes for more conservative designs. With that said, the differences between the methods were minimal with only a ~15% difference for profile 1 and ~3% in profile 2. This is a drop in the ocean of error which surrounds these estimates of sediment transport.

Problem 2:

Consider the data from the Niobrara River from Computer Problem 11.1 p. 317-18. For these conditions, calculate the sediment transport in lb/ft.s for three values of unit discharge (q = 1, 3, and 10 ft2 /s) using the methods of Brownlie, Yang, Simons-Li-Fullerton, and Engelund-Hansen based on d50 only (no size fractions). Plot the results on the sediment-rating curve p. 318, and compare with the field measurements. Discuss the results of your analysis.

First, we need to extrapolate the depth off of the stage unit discharge relationship and back calculate velocity. This step is likely to introduce some error.



q (ft²/s)	h (ft)	v (ft/s)
10	1.8	5.555556
3	0.9	2.727273
1	0.5	2

The first step for Brownlie's method is to find the critical velocity as follows:

$$\frac{V_c}{\sqrt{(G-1)gd_s}} = 4.596\tau_{*c}^{0.529} S_f^{-0.1405} \sigma_g^{-0.1606}$$

For this calculation I assumed s_f= channel slope and $\tau_{*c} = 0.047$

$$V_c = 0.47\frac{ft}{s}$$

This Vc gives us all we need to use their equation for average sediment concentration. Here I assumed R_h= h, and c_B=1.268.

$$C_{ppm} = 7,115c_B\left(\frac{V-V_c}{\sqrt{(G-1)gd_s}}\right)^{1.978} S_f^{0.6601}\left(\frac{R_h}{d_s}\right)^{-0.3301}$$

q (ft²/s)	C (ppm)
10	4550
3	1834
1	644

With the average concentration for each flow condition we can find q_s easily.

$$q_s\left(\frac{lb}{ft\,s}\right) = q\left(\frac{ft^2}{s}\right) * 62.4\frac{lb}{ft^3} * \frac{C(ppm)}{10^6}$$

Yang:

Yang uses a similar method to Brownlie in that the concentration by weight is calculated empirically using a velocity.

Using the sand equation for concentration we first need to find the settling velocity. For this I interpolated table 5.4 in the text book.

$$\omega = 32\frac{mm}{s} = 0.10\frac{ft}{s}$$

Now we need to calculate the incipient motion parameter $\frac{V_c}{\omega}$

$$\frac{V_c}{\omega} = \frac{2.5}{\left[\log\left(\frac{u_* d_s}{\nu}\right) - 0.06\right]} + 0.66;$$

q (ft²/s)	u*ds/ν	Vc/ω

	10	4.853498	4.65326
	3	2.966027	6.725384
	1	1.348194	36.50108

With this parameter calculated, we can calculate the concentration by weight:

$$\log C_{ppm} = 5.435 - 0.286 \log \frac{\omega d_s}{\nu} - 0.457 \log \frac{u_*}{\omega} + \left(1.799 - 0.409 \log \frac{\omega d_s}{\nu} - 0.314 \log \frac{u_*}{\omega}\right) \log \left(\frac{VS}{\omega} - \frac{V_c S}{\omega}\right)$$

q (ft²/s)	C (ppm)
10	2383.979
3	400.2463
1	0

The sediment load is calculated from the concentration by weight with the same method as the Bronlie method.

Simons Li Fullerton:

Simons li and Fullerton created an empirical relationship for total load based on river depth, velocity and sediment characteristics. To use their relationship, we need to find d84, d16 and d50 to calculate the gradation coefficient (Gr). I interpolated these from the particle size distribution provided in the problem statement.

$$d_{16} = 0.166mm, \qquad d_{84} = 0.47mm$$

$$Gr = \frac{1}{2}\left(\frac{d_{84}}{d_{50}} + \frac{d_{50}}{d_{16}}\right) = 1.7 \approx 2 \; rounded$$

From here we can select the empirical coefficents from table 11.1:

cs_1	1.59×10^{-5}
cs_2	0.51
cs_3	3.55

With all these defined it is a quick calculation to find q_s (ft²/s) at each flow condition using their equation and then use a conversion factor to calculate q_s (lb/ft*s):

$$q_s = cs_1\, h^{cs_2}\, V^{cs_3} \qquad q_s\left(\frac{lb}{ft\,s}\right) = q_s\left(\frac{ft^2}{s}\right) * G * 62.4\frac{lb}{ft^3}$$

Engelund Hans~~o~~en:

This method is the most straightforward. Calculate concentration by weight with this simple equation:

$$C_w = 0.05\left(\frac{G}{G-1}\right)\frac{V\,S_f}{[(G-1)g d_s]^{1/2}}\frac{R_h\,S_f}{(G-1)d_s}$$

Then calculate q_s the same way as the Yang and Brownline.

Problem 2 Results:

q (ft²/s)	Brownlie	Yang	Simons Li Fullerton	Engelund Hans~~o~~en
10	2.84	1.49	1.56	2.53
3	0.34	0.09	0.18	0.23
1	0.04	0.00	0.02	0.03

Table 2: Unit sediment discharge in lb/(ft*s) as predicted by each method.

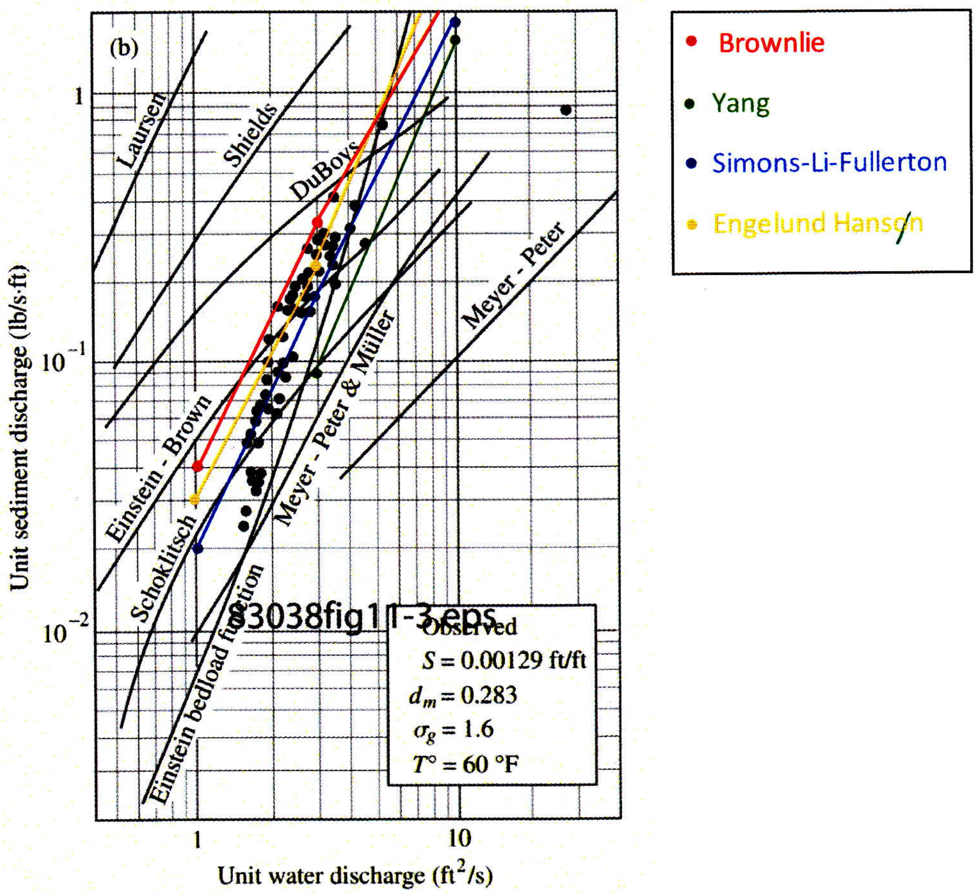


Figure 3: Results of the total sediment discharge analysis with recorded data and all 4 above-described methods.

Great.

Problem 2 Discussion:

All the methods provided estimates within the same order of magnitude. The largest difference observed was between the Yang and Brownlie methods for estimation at 3 ft^2/s with Brownlie predicting 3.7 times more sediment transport than Yang. This may be because the Yang method is nearing its critical threshold where it predicts less transport than the other methods. The Yang method is the only method used which predicted no sediment transport during this analysis. The Yang method consistently predicted lower than the observed sediment transport. This underprediction may limit its use as a conservative estimate of sediment load for engineering design.

The solution which fits the data best visually is the Simons-Li-Fullerton method. This method is a best fit to the solutions of the combined suspended and bedload transport of the Einstein integral and the Meyer-Peter-Muller bedload equation. This was also one of the easiest to model and will be useful in the future as it requires minimal input data to create an estimate which is relatively close at least for this application. Looking at these methods in only one application is sure to skew our perspective on their accuracy as these methods all rely on the assumption of similitude to their underlying data which will be appropriate depending on the application. Through reading about the underlying data for each equation or plotting these functions against data from more river systems at varying flows we could better determine the conditions in which each method works best.

N.G.

Appendix:

Calculations for profile 2

$$V = \frac{u_*}{k}\ln(z) + c = 0.453 * \ln(z) + 2.67\ (from\ figure\ 1b)$$

$$\frac{u_*}{k} = 0.453$$

$$k = \frac{0.260}{0.453} = a_1)\ 0.574$$

Mean flow velocity and the momentum correction factor are calculated with the same sums described for profile 1:

$$\bar{V} = \frac{\sum_{i=0}^{n} V_i * \Delta z_i}{h} = 2.97\frac{ft}{s}$$

$$\beta_m = \frac{1}{hv^2}\sum_{i=1}^{n} v_i{}^2 \Delta z = 1.07$$

Unit discharge:

$$q_{avg} = \frac{Q}{w} = \frac{585\frac{ft^3}{s}}{50.2ft} = 11.7\frac{ft^2}{s}$$

$$q_{prof} = h * \bar{V} = 7.34ft * 2.97\frac{ft}{s} = 21.8\frac{ft^2}{s}$$

From figure 2

$$R_o = 0.35$$

$$\omega = R_o \beta_s k u_*$$

$$k = \frac{0.260}{0.453} = 0.574$$

$$assume\ \beta_s \approx 1$$

$$\omega = 0.35 * 0.43 * \frac{0.26ft}{s} = 0.042\ ft/s$$

$$\omega = 0.042\ \frac{ft}{s} * \frac{304.8\ mm}{ft} = 12.8\frac{mm}{s}$$

$$Interpolate\ Fine \rightarrow medium\ d_s = 0.14mm = 0.00046ft$$

$$q_s = \sum_{i=1}^{n} V_i\left(\frac{ft}{s}\right) * C_{fit}\left(\frac{lb}{ft^3}\right)\Delta z\ (ft) = 0.4\frac{lb}{ft * s}$$

$$\frac{\left(\int_{0.000921}^{6.986} 10^{0.3528 \log_{10}((6.986-z)/z)+2.4249}\ (0.453 \log(z) + 2.6651)\, dz\right) 6.24}{10^5} = 0.392624$$