Problem 1

Measurements on the Zaire River from Peters (1978) show dunes of 1.2–1.9m in amplitude and 95–400m in length. At a flow depth of 13.2 m, the velocity is 1.3 m/s and the river slope is 4.83 cm/km. The bed material is d_{50} =0.34mm and d_{90} = 0.54 mm, and the water temperature is 27°C. Determine the following: (a) compare the bedform type and geometry with all bedform predictors; (b) estimate f'/f''; and (c) plot the results on Figure 8.13.

A) Bedform Geometry, wavelength (Λ) and dune height (Δ)

$$\Lambda = 2\pi h = 2\pi(13.2) \rightarrow \Lambda = 82.94m$$

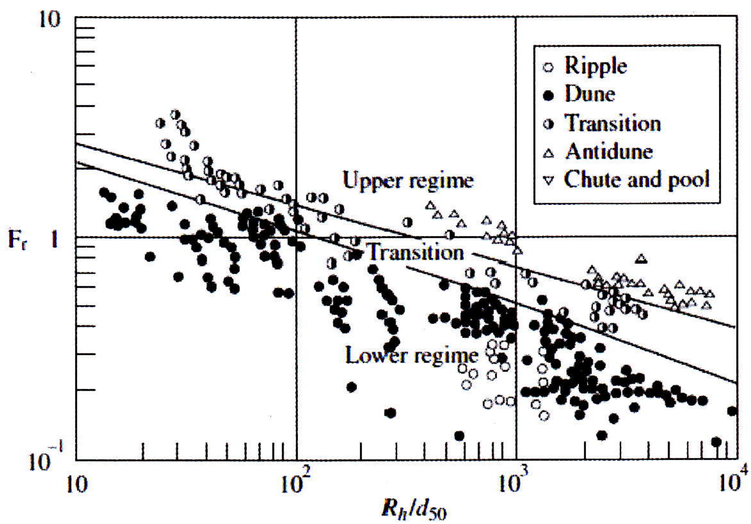
$$\Delta = 2.5h^{0.7}{d_s}^{0.3} = 2.5(13.2)^{0.7}(0.00034)^{0.3} \rightarrow \Delta = 1.39m$$

A) Comparison of bedform predictors

Athaullah 1968 needs the Froude Number and ratio of hydraulic radius over grain size:

$$Fr = \frac{v}{\sqrt{gh}} = \frac{1.3}{\sqrt{(9.81)(13.2)}} \rightarrow Fr = 0.114 \therefore Subcritical$$

$$\frac{R_h}{d_{50}} \cong \frac{h}{d_{50}} = \frac{13.2}{0.00034} = 38{,}823 \; or \; 3.8 \times 10^4$$



In this case, the predictors for bedforms are off the chart ($\frac{R_h}{d_{50}} > 10^4$). It is likely that when the chart is extrapolated that the bedforms would be dunes in the lower regime (since flow is subcritical), but it cannot be stated with confidence.

Liu 1957 requires comparison of the grain shear Reynolds number and the ratio of shear velocity to fall velocity. Shear velocity can be calculated using the bed shear where ($\rho = 997 kg/m^3, \gamma = 9.81(997) = 9780\ N/m^2$):

$$\tau_0 = \gamma R_b S_f \cong \gamma h S_0 = 9780(13.2)(0.0000483) \rightarrow \tau_0 = 6.24\ N/m^2$$

$$u_* = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{6.24}{997}} \rightarrow u_* = 0.079\ m/s$$

The fall velocity can be calculated as follows, assuming a C_D =1.5 and G = 2.65:

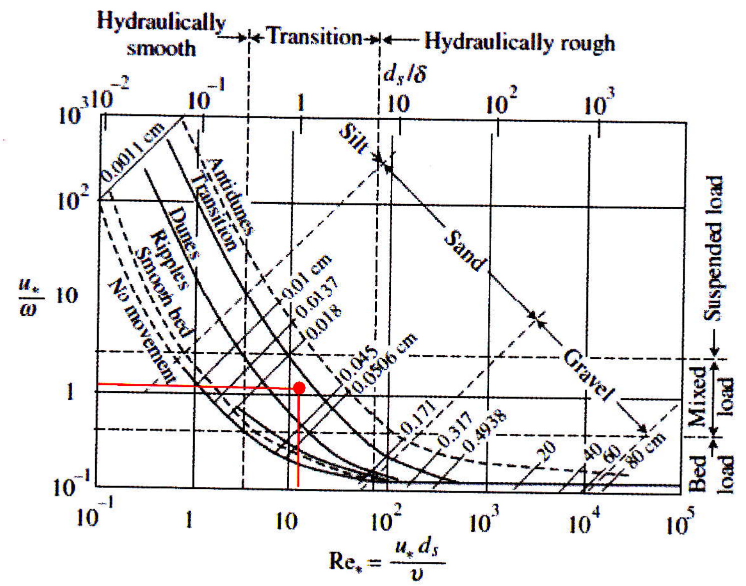
$$\omega = \left[\frac{4}{3}(G-1)\frac{g d_s}{C_D}\right]^{1/2} = \left[\frac{4}{3}(2.65-1)\frac{(9.81)0.00034}{1.5}\right]^{1/2} \rightarrow \omega = 0.07\ m/s$$

Assuming the kinematic viscosity at 27°C is approximately $1 \times 10^{-6} m^2/s$ (Table 2.3 in Julien, 2010). The grain shear Reynolds number can be calculated below:

$$Re_* = \frac{u_* d_s}{\upsilon} = \frac{0.079(0.00034)}{1 \times 10^{-6}} = 28.86$$

$\frac{u_*}{\omega}$ can be calculated below:

$$\frac{u_*}{\omega} = \frac{0.079}{0.07} = 1.128$$



The bedform predicted using Lui 1957 would predict dunes, matching the field observations.

Simons and Richardson 1963/1966 relates the grain size to the unit stream power, which is calculated below:

$$\Omega = \tau_0 v = 62.4(43.3)(0.0000483)(4.26) \rightarrow \Omega = 0.556 lb/ft-s$$

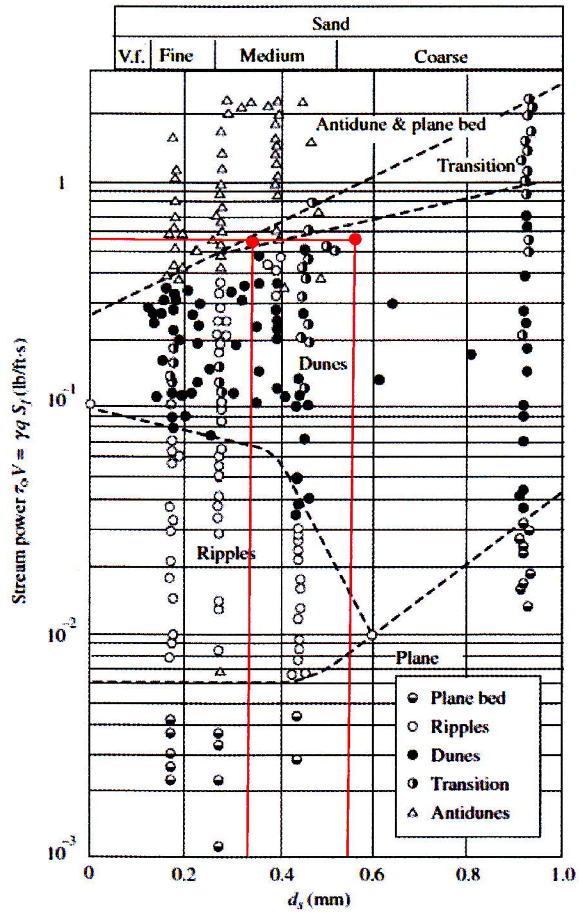


Figure 8.7. Bedform classification (after Simons and Richardson, 1963, 1966)

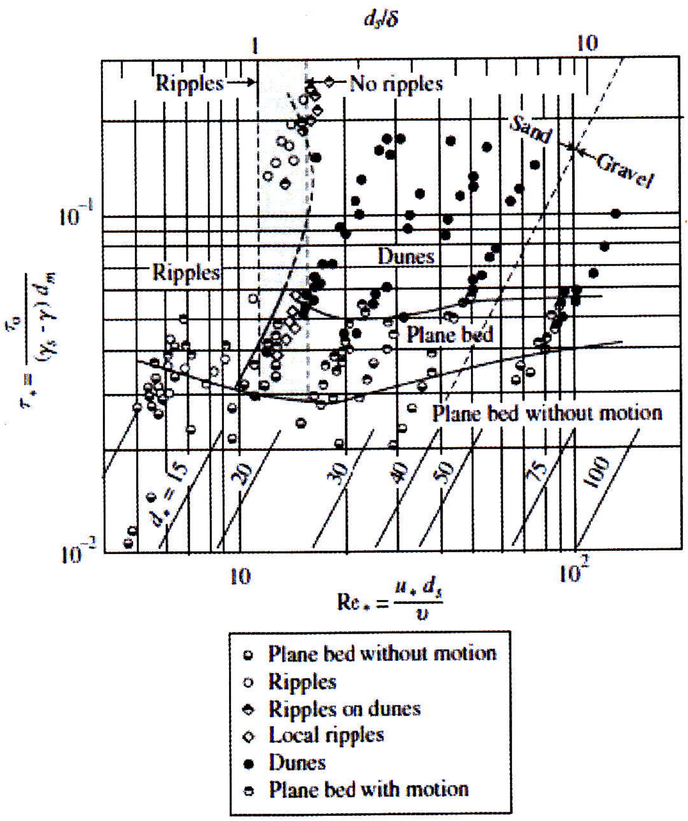
Plotting on Simons and Richardson 1963 for either d_{50} or d_{90} predicts dunes, matching observations.

Chabert and Chauvin 1963 compares the dimensionless shear to the grain shear Reynolds number, where dimensionless shear can be calculated:

$$\tau_* = \frac{\tau_0}{(\gamma_s - \gamma)d_m} = \frac{6.24}{(2.65(9780) - 9780)(0.00034)} = 1.13$$

Assuming the kinematic viscosity at 27°C is approximately $1 \times 10^{-6} m^2/s$ (Table 2.3 in Julien, 2010). The grain shear Reynolds number can be calculated below:

$$Re_* = \frac{u_* d_s}{v} = \frac{0.079(0.00034)}{1 \times 10^{-6}} = 28.86$$



Chabert and Chauvin 1963 yields values of the provided chart. However, it is likely that the predicted bedform would be in the dune zone, matching observations.

Van Rijn 1984 compares the transport stage parameter to the dimensionless particle diameter. The dimensionless particle diameter can be calculated as:

$$d_* = d_{50}\left[\frac{(G-1)g}{{\upsilon_m}^2}\right]^{1/3} = 0.00034\left[\frac{(2.65-1)9.81}{(1\times10^{-6})^2}\right]^{1/3} \rightarrow d_* = 8.6$$

The transport stage parameter requires the critical shear and shear associated with form drag. The dimensionless critical shear can be obtained from the following approximation, and where ϕ is 30° from Table 7.1 in Julien, 2010.

$$\tau_{*c} \approx 0.3e^{-d_*/3} + 0.06tan\phi\left(1-e^{-d_*/20}\right) = 0.3e^{-8.6/3} + 0.06tan(30)\left(1-e^{-8.6/20}\right)$$

$$\rightarrow \tau_{*c} = 0.0292$$

The dimensionless shear associated with form drag can be calculated below:

$$\tau_*' \approx 0.04\left(\frac{d_{50}}{h}\right)^{1/3}\left(\frac{v^2}{(G-1)gd_{50}}\right) = 0.04\left(\frac{0.00034}{13.2}\right)^{1/3}\left(\frac{1.3^2}{(1.65)(9.81)0.00034}\right)$$

$$\rightarrow \tau_*' = 0.363$$

The transport stage parameter can be calculated as follows:

$$T = \frac{\tau_*' - \tau_{*c}}{\tau_{*c}} = \frac{0.363 - 0.0292}{0.0292} \rightarrow T = 11.44$$

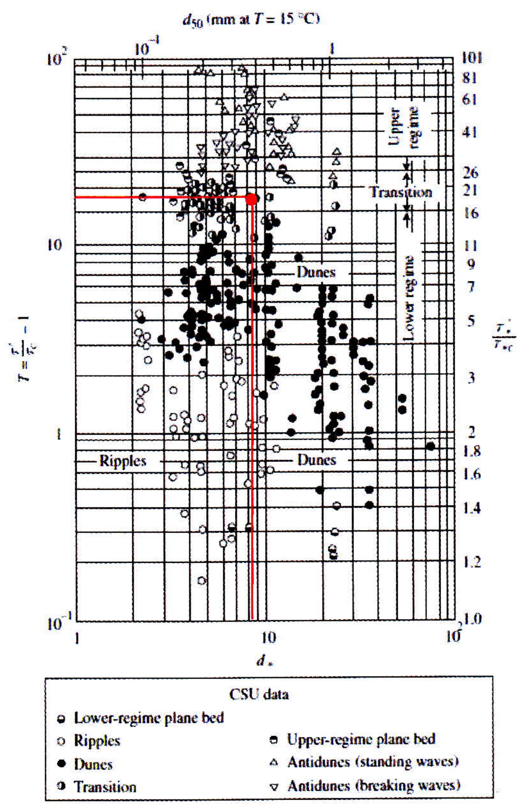


Figure 8.9. Bedform classification (after van Rijn, 1984b)

Van Rijn predicts dunes at the transition, which is relatively consistent with observations.

B) The Darcy-Weisbach friction factor can be calculated using the following where S_f is approximated as S_0:

$$f = \frac{8ghS_0}{v^2} = \frac{8(9.81)13.2(0.0000483)}{1.3^2} \rightarrow f = 0.0296$$

The Darcy-Weisbach friction factor due to form drag can be approximated using:

$$f' \approx 0.32\left(\frac{d_{50}}{h}\right)^{1/3} = 0.32\left(\frac{0.00034}{13.2}\right)^{1/3} = 0.0095$$

By subtraction, we can find the Darcy-Weisbach friction factor f'':

$$f'' = f - f' = 0.0296 - 0.0095 \rightarrow f'' = 0.0201, \therefore \frac{f'}{f''} = \frac{0.0095}{0.0201} \rightarrow \frac{f'}{f''} = 0.469$$

C) Using the predicted dune height, we can find:

$$\frac{\Delta}{h}\left(\frac{h}{d_{50}}\right)^{0.3} = \frac{1.39}{13.2}\left(\frac{13.2}{0.00034}\right)^{0.3} = 2.507$$

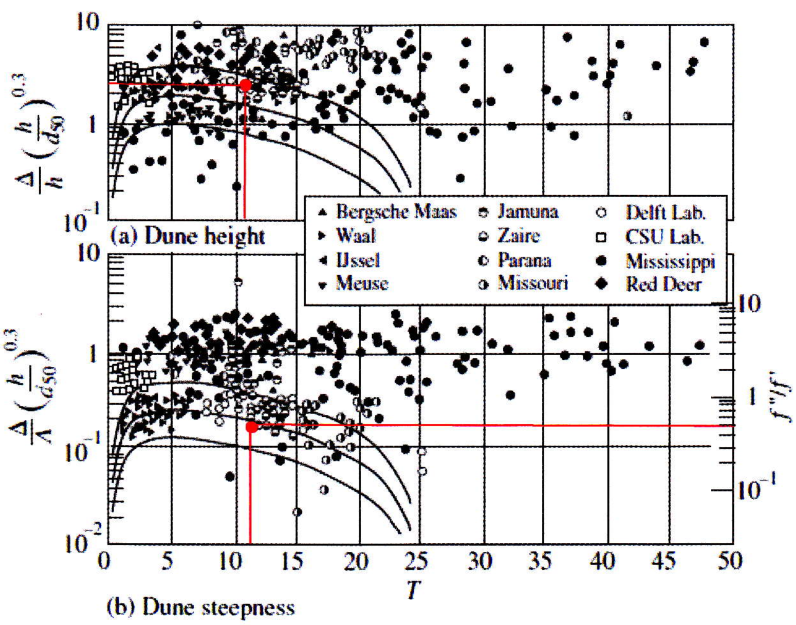


Figure 8.13. Dune height and steepness in rivers (after Julien and Klaassen, 1995)

Most of the bedform predictors matched the observations, with a high degree of variablility between predictors.

Problem 2

Solve problems 9.1, 9.2, and 9.4 in SI units and compile the results. The channel has a slope $S_0 = 0.01$, flow depth h = 20 cm, and a grain size d_{50} = 15mm.

Problem 9.1 – DuBoys Equation (Find q_{bw} and q_{bv})

The general form of the DuBoys Equation can be written as follows:

$$q_{bv} = \frac{0.173}{{d_s}^{3/4}}\tau_0(\tau_0 - 0.0125 - 0.019d_s)$$

The shear stress exerted on the bed can be calculated as, assuming a temperature of 25°C ($\rho = 997kg/m^3, \gamma = 9.81(997) = 9780\ N/m^2$):

$$\tau_0 = \gamma R_b S_f \cong \gamma h S_0 = 9780(0.2)(0.01) = 19.56\ Pa = 0.409\ psf$$

Plugging into the DuBoys equation:

$$q_{bv} = \frac{0.173}{(15)^{3/4}}(0.409)((0.409) - 0.0125 - 0.019(15)) \rightarrow q_{bv} = 0.001 ft^2/s$$

$$\rightarrow \underline{\mathbf{q_{bv} = 9.66 \times 10^{-5} m^2/s}}$$

Converting q_{bv} to q_{bw} multiplies by $G\rho g$, assuming G = 2.65:

$$q_{bw} = \rho g q_{bv} = 2.65(997)(9.81)(9.66 \times 10^{-5}) \rightarrow \underline{\mathbf{q_{bw} = 2.504\ kg/s^3}}$$

Problem 9.1 – Meyer-Peter Müller Equation (Find q_{bm} and q_{bv})

The general form of the MPM Equation can be written as follows:

$$q_{bv} = 8(\tau_* - \tau_{*c})^{3/2}\sqrt{(G-1)g{d_s}^3}$$

The dimensionless shear can be calculated below using the above for τ_0 where d_s = 0.049ft:

$$\tau_* = \frac{\tau_0}{(\gamma_s - \gamma_m)d_s} = \frac{0.409}{(2.65(62.4) - 62.4)0.049} = 0.0808\ psf$$

Plugging into the MPM Equation, assuming $\tau_{*c} = 0.047$:

$$q_{bv} = 8(\tau_* - \tau_{*c})^{3/2}\sqrt{(G-1)g{d_s}^3} = 8(0.0808 - 0.047)^{3/2}\sqrt{(2.65-1)(32.2)(0.049)^3}$$

$$\rightarrow q_{bv} = 0.00396\ ft^2/s \rightarrow \underline{\mathbf{q_{bv} = 3.68 \times 10^{-4} m^2/s}}$$

Converting q_{bv} to q_{bm} multiplies by $G\rho$:

$$q_{bm} = \rho q_{bv} = 2.65(997)(3.68 \times 10^{-4}) \rightarrow \underline{\mathbf{q_{bm} = 0.972\ kg/(ms)}}$$

Problem 9.4 – Einstein-Brown Equation (Find q_{bm} and q_{bw})

The general Einstein-Brown equation is given by when d_s > 1mm:

$$q_{bv*} = \frac{q_{bv}}{\sqrt{(G-1)g{d_s}^3}} = 2.15e^{\frac{-0.391}{\tau_*}}$$

Using the dimensionless shear under the MPM method, we can solve for the Einstein-Brown:

$$q_{bv} = 2.15e^{\frac{-0.391}{\tau_*}}\sqrt{(G-1)g{d_s}^3} = 2.15e^{\frac{-0.391}{0.0808}}\sqrt{(2.65-1)(32.2)(0.049)^3}$$

$$\rightarrow q_{bv} = 0.00134 ft^2/s \rightarrow \underline{\boldsymbol{q_{bv} = 1.26 \times 10^{-4} m^2/s}}$$

Converting q_{bv} to q_{bw} multiplies by $G\rho g$:

$$q_{bw} = \rho q_{bv} = 2.65(997)9.81(1.26 \times 10^{-4}) \rightarrow \underline{\boldsymbol{q_{bw} = 3.27\ kg/s^3}}$$

Now, repeating the process for the case where the channel has a slope S_0 = 0.01, flow depth h = **50** cm, and a grain size d_{50} = 15mm.

Problem 9.1 – DuBoys Equation (Find q_{bw} and q_{bv})

The general form of the DuBoys Equation can be written as follows:

$$q_{bv} = \frac{0.173}{{d_s}^{3/4}}\tau_0(\tau_0 - 0.0125 - 0.019d_s)$$

The shear stress exerted on the bed can be calculated as, assuming a temperature of 25°C ($\rho = 997 kg/m^3, \gamma = 9.81(997) = 9780\ N/m^2$):

$$\tau_0 = \gamma R_b S_f \cong \gamma h S_0 = 9780(0.5)(0.01) = 48.90\ Pa = 1.023\ psf$$

Plugging into the DuBoys equation:

$$q_{bv} = \frac{0.173}{(15)^{3/4}}(1.023)((1.023) - 0.0125 - 0.019(15)) \rightarrow q_{bv} = 0.017 ft^2/s$$

$$\rightarrow \underline{\boldsymbol{q_{bv} = 1.57 \times 10^{-3} m^2/s}}$$

Converting q_{bv} to q_{bw} multiplies by $G\rho g$:

$$q_{bw} = \rho g q_{bv} = 2.65(997)(9.81)(1.57 \times 10^{-3}) \rightarrow \underline{\boldsymbol{q_{bw} = 40.62\ kg/s^3}}$$

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Problem 9.1 – Meyer-Peter Müller Equation (Find q_{bm} and q_{bv})

The general form of the MPM Equation can be written as follows:

$$q_{bv} = 8(\tau_* - \tau_{*c})^{3/2}\sqrt{(G-1)g{d_s}^3}$$

The dimensionless shear can be calculated below using the above for τ_0 where d_s = 0.049ft:

$$\tau_* = \frac{\tau_0}{(\gamma_s - \gamma_m)d_s} = \frac{1.023}{(2.65(62.4) - 62.4)0.049} = 0.202\ psf$$

Plugging into the MPM Equation, assuming $\tau_{*c} = 0.047$:

$$q_{bv} = 8(\tau_* - \tau_{*c})^{3/2}\sqrt{(G-1)gd_s{}^3} = 8(0.202 - 0.047)^{3/2}\sqrt{(2.65-1)(32.2)(0.049)^3}$$

$$\rightarrow q_{bv} = 0.0388\ ft^2/s \rightarrow \underline{q_{bv} = 3.61 \times 10^{-3} m^2/s}$$

Converting q_{bv} to q_{bm} multiplies by $G\rho$:

$$q_{bm} = \rho q_{bv} = 2.65(997)(3.61 \times 10^{-3}) \rightarrow \underline{q_{bw} = 9.54\ kg/(ms)}$$

Problem 9.4 – Einstein-Brown Equation (Find q_{bm} and q_{bw})

The general Einstein-Brown equation is given by when $d_s > 1$mm:

$$q_{bv*} = \frac{q_{bv}}{\sqrt{(G-1)gd_s{}^3}} = 2.15e^{\frac{-0.391}{\tau_*}}$$

Using the dimensionless shear under the MPM method, we can solve for the Einstein-Brown:

$$q_{bv} = 2.15e^{\frac{-0.391}{\tau_*}}\sqrt{(G-1)gd_s{}^3} = 2.15e^{\frac{-0.391}{0.202}}\sqrt{(2.65-1)(32.2)(0.049)^3}$$

$$\rightarrow q_{bv} = 0.0247 ft^2/s \rightarrow \underline{q_{bv} = 2.30 \times 10^{-3} m^2/s}$$

Converting q_{bv} to q_{bw} multiplies by $G\rho g$:

$$q_{bw} = \rho q_{bv} = 2.65(997)9.81(2.30 \times 10^{-3}) \rightarrow \underline{q_{bw} = 59.47\ kg/s^3}$$ ✓

DuBoys qbv		DuBoys qbw	
0.001039	ft^2/s	0.171834	slug/s^3
9.66E-05	m^2/s	2.503463	kg/s^3
MPM qbv		MPM qbm	
0.003956	ft^2/s	0.020285	slug/ft*s
0.000368	m^2/s	0.971475	kg/m*s
E-B qbv		E-B qbw	
0.001354	ft^2/s	0.223923	slug/s^3
0.000126	m^2/s	3.262348	kg/s^3

DuBoys qbv		DuBoys qbw	
0.01686	ft^2/s	2.787979	slug/s^3
0.001567	m^2/s	40.61826	kg/s^3
MPM qbv		MPM qbm	
0.038841	ft^2/s	0.199167	slug/ft*s
0.00361	m^2/s	9.538572	kg/m*s
E-B qbv		E-B qbw	
0.024689	ft^2/s	4.082547	slug/s^3
0.002295	m^2/s	59.47893	kg/s^3

Table 1: Comparison of bedload calculation methods at varying flow depths (flow depth of 20cm at left, 50cm at right, bedload increases by an order of magnitude)

15/15

Problem 3

Visit the Engineering Research Center and stop by the Hydraulics Laboratory. Study the posters from Dr. Kristin Bunte (ERC 2nd floor hallway of the West wing). Select two posters and write one page (per poster) describing what you learned about the experimental methods to measure sediment transport in gravel-bed streams.

Critical Shields values in coarse-bedded steep streams. ← authors.

Critical Shields values (τ_c) are back calculated using a flow competence/critical flow technique and realistic bed load measurements from mountain streams. For steep and coarse-grained channels, numerous Shields-type parameters with varying computational details and numerical values have been proposed. However, there is little guidance regarding which values should be chosen, and large errors may result when critical particle size or critical flow is computed using an inappropriate Shields value. These are the unexpected results that bring into the question the physical basis for shield values in steep coarse -bedded mountain streams,

1. Shield values τ^*_{c50} for a given stream gradient are smaller in shallower are rougher channels.
2. τ^*_{c50} is unaffected by the stability and mobility of the bed.
3. Shield values for the bed D_{16} size are larger than for the D_{50} size even though D_{16} particles are entrained at lower flow.

Predicting flows that entrain gravel and cobble particle in steep, coarse -bedded mountain streams were investigated because typical Shield's curve ends at $Re_p = 500$ and Particle Reynolds numbers Re_p reach 5 000–50 000 for the mountain study streams. Field data used for the computations of Shields values comprised bed load samples collected with bed load traps in 11 Rocky Mountain streams using bedload traps and net frame sampler during snowmelt high flow season and 22 data sets from North America and Swiss Alps also considered to study wide range of channel condition and stream types.

Uncertainty of c values as well as dispersion in relationships of τ^*_c with S_x and other variables are caused by random errors in the input parameters to the Shields equation within and between trials. Re_p is poor predictor for τ^*_{c50}. The average τ^*_{c50}. values for streams with Sx = 0.01 were close to 0.05 and rose to 0.20 for Sx = 0.1. Individual τ^*_{c50}. values span a greater than 6-fold range, indicating that a single value cannot accurately characterize c50 in steep streams. The concept of hiding and exposure as the primary mechanism to explain particle entertainment in coarse-bedded streams appears oversimplified. An abundance of active gravel bars with particle sizes finer than the thalweg bed material, obvious near-stream sediment sources, a high percentage of surface and subsurface sand and pea gravel, and relatively many large particles that lie fully exposed on top of the bed are all signs of low bank full bed stability in steep coarse bedded streams. Critical shield levels are determined by simple arithmetic relations within $\tau^*_c = f(d.SID$ rather than physical processes. The critical shield values change on a weekly basis and are connected to bed movement and stability. Without significant field research, hiding function exponents are difficult to estimate since they are negatively correlated with the steepness of the flow competency curve and positively correlated with the critical flow. The fact that smaller particles have higher τ_c^* values can also be explained by a numerical artifact. Compared to the average largest mobile particle size, shield values derived from the absolute largest mobile particle size are systematically lower. To establish prediction relationships of critical Shields values with stream parameters, more reliable field data sets of flow competence and other stream characteristics from varied stream conditions are desired. From what we've seen in the field, hiding and exposing is not the main process in coarse-bedded mountain streams, where the stability of the bed slows the movement of large rocks until very high flow.

Gravel transport rates in Rocky Mountain streams for normal annual highflow event

Numerous stream studies, ranging from channel restoration to watershed management, necessitate knowledge of the sediment transport rate during the average annual high flow event. In mountain streams, bedload transport equations fail at this task because they are not designed for coarse and uneven beds or variable sediment supply. It takes a lot of time and can be dangerous to measure gravel transport accurately. Using equations to figure out how fast gravel moves is very hard, uncertain, and not very reliable. This research takes a unique approach by conducting a comprehensive comparison study of gravel transport rates as assessed in worldwide steep mountain streams (S>0.007 m/m). Those data were collected using bedload traps, vortex, baskets, and pit-type samplers. Power functions $Q_B = a\,Q^{\,b}$ were fitted to the sampled transport rates Q_B (g/s) and the discharge Q (m^3/s) at the time of sampling; two functions were fitted for curved trends. At each site, we calculated the percent of subsurface fines 8 mm and the estimated bankfull flow Q. The bankfull gravel transport rate was calculated by extrapolating the bankfull width. Modified stream power expression (ω') was developed. $\omega' = \rho \cdot q_{bf} \cdot S^{0.5} \cdot \%Dsub{<}8$. as predictor of $q_{B,bf}$ and includes the percentage of subsurface sediment < 8 mm ($\%Dsub{<}8$).

When measured values of $q_{B,bf}$ were plotted vs. ω' in log-log space, and the data fell inside an envelope two orders of magnitude wide, a positive, straight trend developed for the streams in the Rocky Mountains. This pattern indicated that the stream flow was getting higher. Central rock mountain streams yield a fairly well-defined trend with variation of 2 orders. The few outliers showed that the transport rates were temporarily high (when the log jam broke up) and temporarily low (when gravel got stuck upstream). Most of the data from around the world that was added to the plot of $q_{B,bf}$ vs. w' fell into the extrapolated envelope of Rocky Mountain streams, such as Alpine step-pool and plane-bed streams near tree line. An r^2 of 0.75 was found when regression was used to fit the data inside the envelope.

Applicability of the output is highly important. Using specific sites, refinement of $q_{B.bf}$ predicted on channel's likely sediment supply that based on the water sediment production, hillslope - channel connectivity, sediment contribution from bank erosion, overbank sediment storage and downstream conveyance blockages is assessed from Google Earth to categorize a stream to extremely high, moderate high, moderate, moderate low and extremely low. Streams draining basins with a rich gravel supply have been shown to have bankfull transport rates that are significantly higher than those found in Rocky Mountain streams. For Rocky Mountain streams, this technique can estimate q to within +- and order of magnitude. Estimation of $q_{B,bf} = f(\omega')$ extends to many worldwide streams for which sediment supply is not extremely low or high. The estimation is refined, or a stream is placed inside or outside the central envelope in extreme circumstances where aerial photography is used to visually examine the watershed sediment supply (e.g., hillslope-channel link), active bank erosion, and downstream gravel conveyance capability.