CEE 716 HW 3

1. Apply the side wall correction to the clear flow dataset in problem 6.1.

Given: $u_* = 0.041 m/s$, $d_s = 0.105 mm$, $Q = 0.064 cms$, $h = 0.17 m$, $s_f = 0.002$, $W = 0.356 m$, C_{sed} negligible

$$R_h = \frac{Wh}{W+2h} = \frac{0.356m * 0.17m}{0.356m + 2 * 0.17m} = 0.0870m$$

$$V = \frac{Q}{Wh} = \frac{0.064cms}{0.356m * 0.17m} = 1.06\frac{m}{s}$$

$$R_e = \frac{4VR_h}{\nu_m} = \frac{4 * 1.06\frac{m}{s} * 0.087m}{10^{-6}\frac{m^2}{s}} = 368{,}000 \rightarrow fully\, turbulent$$

$$f = \frac{8u_*^2}{V^2} = \frac{8\left(0.041\frac{m}{s}\right)^2}{\left(1.06\frac{m}{s}\right)^2} = 0.0120$$

$$f_{wall} = 0.0026\left(log\left(\frac{R_e}{f}\right)\right)^2 - 0.0428\left(log\left(\frac{R_e}{f}\right)\right) + 0.1884 =$$

$$0.0026\left(log\left(\frac{368000}{0.0120}\right)\right)^2 - 0.0428\left(log\left(\frac{368000}{0.0120}\right)\right) + 0.1884 = 0.014$$

$$f_b = f + \frac{2h}{W}(f - f_{wall}) = 0.011$$

$$R_b = \frac{f_b}{f}R_h = 0.0661m$$

$$\tau_b = \gamma R_b s_f = 1.50\frac{N}{m^2}$$

$$u_{*b} = \sqrt{gR_b s_f} = 0.0016\ m/s$$

Problem 2

Compare the results from a) through j) for the plane bed and dune bedforms from the Low Flow Conveyance Channel (alongside the Rio Grande, NM). Final comparisons are presented at the end of the methodology for this problem in Table 4. Given:

$$Q = 625cfs, h = 5.6ft, R = 4.03\ ft, T_w = 50.1ft, S_f = 0.00038\ ft/ft,\ d_{50} = 0.15mm$$

Determine:

a) von Karman constant $\equiv \kappa$

b) Shear stress $\equiv \tau$

c) Mean flow velocity $\equiv$ v

d) Froude Number $\equiv Fr$

e) Darcy-Weisbach friction factor $\equiv f$

f) Manning's n $\equiv n$

g) Chézy coefficient $\equiv C$

h) Laminar sublayer thickness $\equiv \delta$

i) Friction slope $\equiv S_f$

j) Momentum Correction Factor $\equiv \beta_m$

Work for the mean flow velocity and momentum correction factor is included in Appendix A.

Tables 2 & 3: Plane and dune bed velocity profile data with concentrations

z [ft]	v [ft/s]	C [mg/L]	z [ft]	v [ft/s]	C [mg/L]
0.1	2.65		2.4	4.74	
0.17	3.16		2.6	4.83	223
0.27	3.44	1493	2.7	–	
0.3	–		2.8	4.97	
0.37	3.27	1,194	3	5.07	
0.47	3.62	975	3.2	4.78	
0.5	–	853	3.4	5.01	
0.6	3.51		3.6	5.07	
0.7	3.7	914	3.7	–	
0.8	3.9		3.8	5.03	
0.9	4.07		4	4.99	
1	3.91	776	4.2	5.06	
1.1	4.12		4.4	4.98	
1.2	4.22	648	4.5	–	
1.3	4.05		4.6	4.85	
1.4	4.31	463	4.8	4.78	
1.5	4.34		4.9	–	
1.6	4.45	459	5	4.56	
1.7	4.41		5.2	4.25	
1.8	4.58	283	5.4	4.2	
1.9	4.79		5.5	4.09	
2	4.73	271	5.5	4.41	
2.2	4.61	190			

z [ft]	v [ft/s]	C [mg/L]	z [ft]	v [ft/s]	C [mg/L]
0.0984	1.066		2.8864	3.198	337
0.1968	2.28944		3.1816	3.35544	
0.2952	2.3124	738	3.3784		
0.3936	2.39112		3.4768	3.34888	
0.492	2.44688		3.772	3.1324	
0.5904	2.47968		4.0672	3.5096	
0.6888	2.4764	654	4.2968		
0.7872	2.43704		4.3952	3.47024	
0.8856	2.66336		4.6904	3.54568	
0.984	2.58136	533	4.9856	3.53256	
1.1808	2.43376		5.2808	3.57848	
1.3776	2.7552	462	5.576	3.52272	
1.476	-		5.8712	3.47024	
1.5744	2.96512	401	6.1664	3.24392	
1.7712	2.83392		6.2976		
1.968	3.02416	491	6.4944	3.31608	
2.296	3.17832		6.7896	3.198	
2.3944	-	238	6.9864	3.37512	
2.5912	2.94872		7.0848		

Plane Bed Calculations

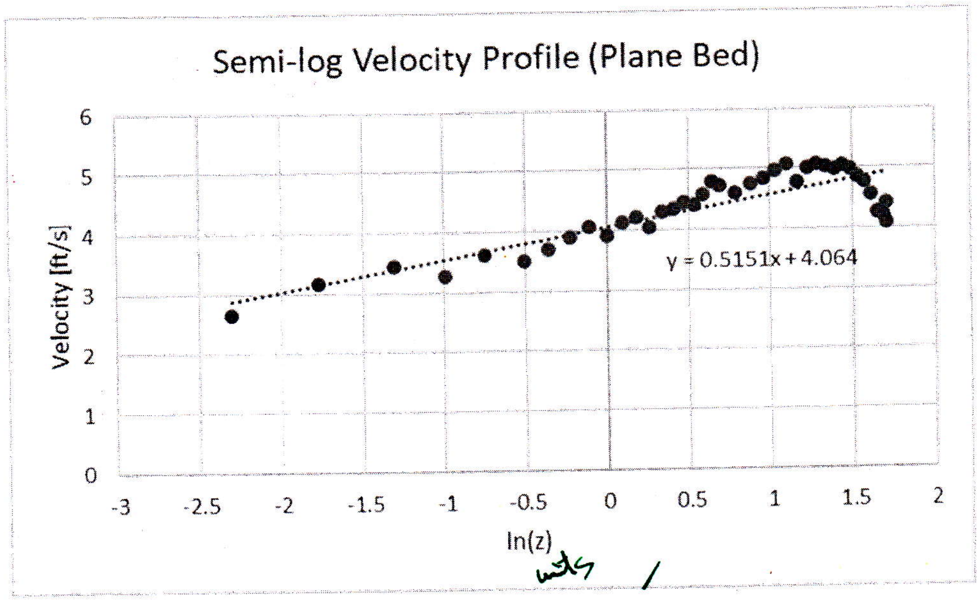

Figure 1: Plane Bed semi-logarithmic velocity profile

Extrapolating the linear regression equation from Figure 1 to the free surface at $h = 5.6ft$

$$v_x = 0.5151(ln(z)) + 4.064 = 0.5151(\ln(5.6)) + 4.064 \rightarrow \underline{\mathbf{v_x = 4.95\ ft/s}}$$

a) von Karman constant $\equiv \kappa$

$$u_* = \sqrt{gRS_f} = \sqrt{32.2(4.03)(0.00038)} \rightarrow u_* = 0.222\ ft/s$$

$$v_x = \frac{u_*}{\kappa} ln\left(\frac{z}{z_0}\right) = 0.5151(ln(z)) + 4.064 \therefore \frac{u_*}{\kappa} = 0.5151, \kappa = \frac{0.222}{0.5151} \rightarrow \underline{\mathbf{\kappa = 0.43}}$$

b) Shear stress $\equiv \tau$ (assume 70°F, $\rho = 1.935 \frac{slug}{ft^3}$)

$$\tau_0 = \rho u_*^2 = 1.935(0.222)^2 \rightarrow \underline{\mathbf{\tau_0 = 0.095psf}}$$

c) Mean flow velocity $\equiv \overline{v_x}$

$$\overline{v_x} = \frac{1}{h}\sum_{i=1}^{N} v_i \Delta z_i, \sum_{i=1}^{N} v_i \Delta z_i = 23.95\ (Table\ 2), \frac{1}{5.6}(23.95) \rightarrow$$

$$\underline{\mathbf{\overline{v_x} = 4.27\ ft/s}}$$

d) Froude Number $\equiv Fr$

$$A = \frac{Q}{\overline{v_x}} = \frac{625}{4.27} \rightarrow A = 146.37ft^2, D = \frac{A}{T_w} = \frac{146.37}{50.1} \rightarrow D = 2.92ft$$

$$Fr = \frac{\overline{v_x}}{\sqrt{gD}} = \frac{4.27}{\sqrt{32.2(2.92)}} \rightarrow \underline{\mathbf{Fr = 0.440\ \therefore\ Subcritcal}}$$

e) Darcy-Weisbach friction factor $\equiv f$

$$f = \frac{8u_*^2}{\overline{V_x}^2} = \frac{8(0.222^2)}{(4.27)^2} \rightarrow \underline{f = 0.022}$$

f) Manning's n $\equiv n$

$$\overline{V_x} = \frac{1.49}{n}R^{2/3}S_f^{1/2} \rightarrow 4.27 = \frac{1.49}{n}(4.03)^{2/3}(0.00038)^{1/2} \rightarrow \underline{n = 0.017}$$

g) Chézy coefficient $\equiv C$

$$C = \sqrt{\frac{8g}{f}} = \sqrt{\frac{8(32.2)}{0.026}} \rightarrow \underline{C = 122.29}$$

h) Laminar sublayer thickness $\equiv \delta$

The average concentration in the water column for the plane bed measurements is 672.46 mg/L, which can be converted to a concentration by volume as follows:

$$C_v = \frac{C_{mg/L}}{2{,}650{,}000}\ (Table\ 10.1\ in\ Julien, 2010) = \frac{672.46}{2{,}650{,}000} \rightarrow C_v = 2.54 \times 10^{-4}$$

Assuming 70°F where: $\rho = 1.935\frac{slug}{ft^3}$, $\mu = 2.05 \times 10^{-5}$, and $G = 2.56$, (Table 2.3 in Julien, 2010) the kinematic viscosity of the mixture for the plane bed system is calculated by:

$$\nu_m = \frac{\mu_m}{\rho_m} = \frac{\mu(1 + 2.5C_v)}{\rho(1 + (G-1)C_v)} = \frac{2.05 \times 10^{-5}\,(1 + 2.5(2.54 \times 10^{-4}))}{1.935(1 + (2.65 - 1)2.54 \times 10^{-4})} = 1.06 \times 10^{-5}\frac{ft^2}{s}$$

The laminar sublayer thickness can now be calculated by:

$$\delta = \frac{11.6\nu_m}{u_*} = \frac{11.6(1.06\times10^{-5})}{0.222} \rightarrow \delta = 0.00055ft\ or\ \underline{\delta = 0.152\ mm}$$

i) Friction slope $\equiv S_f$

$$\tau_0 = \gamma R S_f \rightarrow S_f = \frac{0.095}{62.4(4.03)} \rightarrow \underline{S_f = 0.00038\ ft/ft}$$

j) Momentum Correction Factor $\equiv \beta_m$

Show table value

$$\beta_m = \frac{1}{A\overline{v_x}^2}\int_A v_x^2 dA \cong \frac{1}{h\overline{v_x}^2}\sum_i v_{xi}^2\,\Delta z_i,\ \sum_i v_{xi}^2\,\Delta z_i = 109.4\ (Table\ 2)$$

$$\beta_m = \frac{1}{5.6(4.27)^2}109.4 \rightarrow \underline{\beta_m = 1.071}$$

Dune Bed Calculations

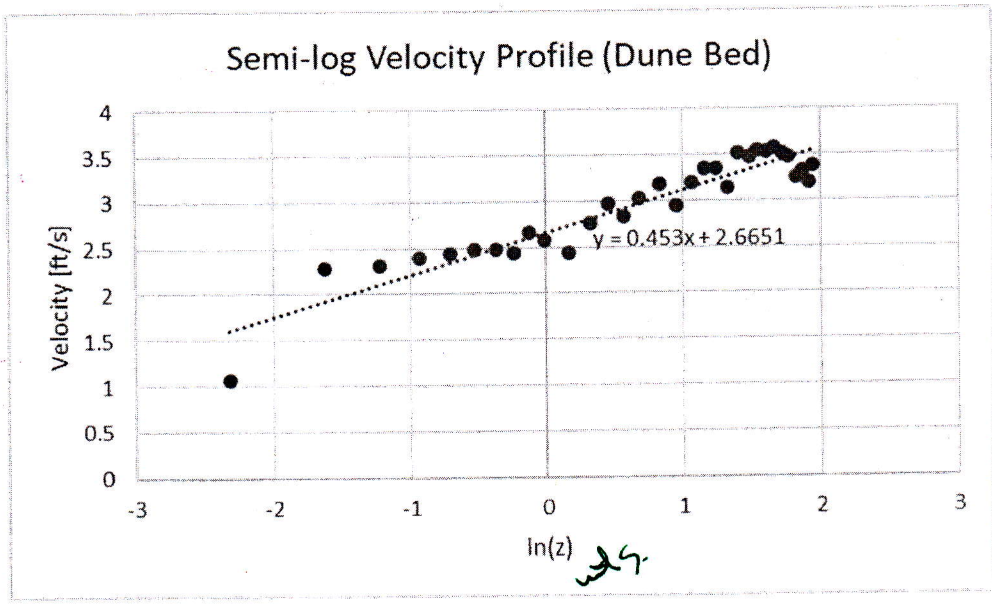

Figure 2: Dune Bed semi-logarithmic velocity profile

Extrapolating the linear regression equation from Figure 2 to the free surface at $h = 7.08ft$

$$v_x = 0.453(ln(z)) + 2.6651 = 0.453(\ln(7.08)) + 2.6651 \rightarrow \underline{\mathbf{v_x = 3.55\ ft/s}}$$

a) von Karman constant $\equiv \kappa$

Using a ratio, we can approximate the hydraulic radius of the dune morphology system:

$$\frac{h_{plane}}{h_{dune}} \propto \frac{R_{plane}}{R_{dune}} \rightarrow R_{dune} = 4.03\left(\frac{7.08}{5.6}\right) \rightarrow R_{dune} = 5.09ft$$

$$u_* = \sqrt{gRS_f} = \sqrt{32.2(5.09)(0.00038)} \rightarrow u_* = 0.250\ ft/s$$

$$v_x = \frac{u_*}{\kappa} ln\left(\frac{z}{z_0}\right) = 0.1381(ln(z)) + 0.9766 \therefore \frac{u_*}{\kappa} = 0.1381, \kappa = \frac{0.250}{0.453} \rightarrow \underline{\mathbf{\kappa = 0.55}}$$

b) Shear stress $\equiv \tau$ (assume 70°F, $\rho = 1.935 \frac{slug}{ft^3}$)

$$\tau_0 = \rho u_*^2 = 1.935(0.250)^2 \rightarrow \underline{\mathbf{\tau_0 = 0.121psf}}$$

c) Mean flow velocity $\equiv \overline{v_x}$

$$\overline{v_x} = \frac{1}{h}\sum_{i=1}^{N} v_i \Delta z_i, \sum_{i=1}^{N} v_i \Delta z_i = 19.52\ (Table\ 2), \frac{1}{7.08}(21.72) \rightarrow$$

$$\underline{\mathbf{\overline{v_x} = 3.07\ ft/s}}$$

d) Froude Number ≡ Fr

$$\frac{h_{plane}}{h_{dune}} \propto \frac{T_{w,plane}}{T_{w,dune}} \rightarrow T_{w,dune} = 50.1\left(\frac{7.08}{5.6}\right) \rightarrow T_{w,dune} = 63.3ft$$

$$A = \frac{Q}{\overline{V_x}} = \frac{625}{3.07} \rightarrow A = 205.58ft^2, D = \frac{A}{T_w} = \frac{205.58}{63.3} \rightarrow D = 3.22\ ft$$

$$Fr = \frac{\overline{v_x}}{\sqrt{gD}} = \frac{3.07}{\sqrt{32.2(3.22)}} \rightarrow \underline{\boldsymbol{Fr = 0.301\ \therefore\ Subcritcal}}$$

e) Darcy-Weisbach friction factor ≡ f

$$f = \frac{8u_*^2}{\overline{V_x}^2} = \frac{8(0.250^2)}{(2.76)^2} \rightarrow \underline{\boldsymbol{f = 0.053}}$$

f) Manning's n ≡ n

$$\overline{V_x} = \frac{1.49}{n}R^{2/3}S_f^{1/2} \rightarrow 3.07 = \frac{1.49}{n}(5.09)^{2/3}(0.00038)^{1/2} \rightarrow \underline{\boldsymbol{n = 0.027}}$$

g) Chézy coefficient ≡ C

$$C = \sqrt{\frac{8g}{f}} = \sqrt{\frac{8(32.2)}{0.066}} \rightarrow \underline{\boldsymbol{C = 69.72}}$$

h) Laminar sublayer thickness ≡ δ

The average concentration in the water column for the plane bed measurements is 481.75 mg/L, which can be converted to a concentration by volume as follows:

$$C_v = \frac{C_{mg/L}}{2{,}650{,}000} (Table\ 10.1\ in\ Julien, 2010) = \frac{481.75}{2{,}650{,}000} \rightarrow C_v = 1.82 \times 10^{-4}$$

Assuming 70°F where: $\rho = 1.935\frac{slug}{ft^3}$, $\mu = 2.05 \times 10^{-5}$, and $G = 2.56$, (Table 2.3 in Julien, 2010) the kinematic viscosity of the mixture for the plane bed system is calculated by:

$$\nu_m = \frac{\mu_m}{\rho_m} = \frac{\mu(1 + 2.5C_v)}{\rho(1 + (G - 1)C_v)} = \frac{2.05 \times 10^{-5}\ (1 + 2.5(1.82 \times 10^{-4}))}{1.935(1 + (2.65 - 1)1.82 \times 10^{-4})} = 1.06 \times 10^{-5}\frac{ft^2}{s}$$

The laminar sublayer thickness can now be calculated by:

$$\delta = \frac{11.6\nu_m}{u_*} = \frac{11.6(1.06 \times 10^{-5})}{0.250} \rightarrow \delta = 0.00049ft\ or\ \underline{\boldsymbol{\delta = 0.127\ mm}}$$

i) Friction slope ≡ S_f

$$\tau_0 = \gamma RS_f \rightarrow S_f = \frac{0.121}{62.4(5.09)} \rightarrow \underline{\boldsymbol{S_f = 0.00038\ ft/ft}}$$

j) Momentum Correction Factor $\equiv \beta_m$

$$\beta_m = \frac{1}{A\overline{v_x}^2}\int_A v_x^2 dA \cong \frac{1}{h\overline{v_x}^2}\sum_i v_{xi}^2\,\Delta z_i, \sum_i v_{xi}^2\,\Delta z_i = 68.90\ (Table\ 2)$$

$$\beta_m = \frac{1}{7.08(3.07)^2}68.9 \rightarrow \underline{\underline{\boldsymbol{\beta_m = 1.045}}}$$

The following table summarizes the above results for comparison between the systems. For the dune bed system, it is observed that roughness increases due to the presence of the dunes, and for the same flow, velocity decreases and roughness increases, as seen in the total depth of the profile.

Plot 2 profiles side by side on linear scale

Parameter	Plane Bed	Dune Bed
Free Surface Velocity [ft/s]	4.95	3.55
von Karman	0.43	0.55
Shear Stress [psf]	0.095	0.121
Mean Velocity [ft/s]	4.27	3.07
Froude Number	0.44	0.301
Darcy-Weisbach *f*	0.022	0.053
Manning's *n*	0.017	0.027
Chézy *C*	122.29	69.72
Laminar Sublayer [mm]	0.152	0.127
Friction Slope [ft/ft]	0.00038	0.00038
Momentum Correction	1.071	1.045

higher

Nice.

Problem # 3 (20 points)

In English Units, solve Problem 7.4: An angular 10 mm sediment particle is submerged on an embankment inclined at $\Theta_1 = 20°$ and $\Theta_0 = 0°$. Calculate the critical shear stress from the moment stability method when the streamlines near the particle are: a) $\lambda = 15°$ (deflected downward); b) $\lambda = 0°$ (horizontal flow); and c) $\lambda = -15°$ (deflected upward).

Table 1. Conversion Table of Given Values

	Given	
Particle Size	10 mm	0.032808399 ft
Θ_1	20 degrees	0.34906585 radians
Θ_0	0 degrees	0 radians
Downstream Channel Slope	0 degrees	0 radians
λ_a	15 degrees	0.261799388 radians
λ_b	0 degrees	0 radians
λ_c	-15 degrees	-0.261799388 radians
SF	1	
ϕ	37 degrees	0.645771823 radians
*From Figure 7.4		
v (viscosity)	1.06E-05 ft^2/s	
*Table 2.3-70F		

Table 2. Calculated Constant Values

Θ	0 degrees	0 radians
a_Θ	0.93969	
d_*	255.6 *pg160	
M/N	5.63524 *eq 7.14b	
M+N	6.63524	

*The page number refers to the page in textbook that value was calculated from.

Next, the following equations were entered in Excel and assigned a random τ_0 value:

$$\eta_0 = \frac{21\tau_0}{(\gamma_s - \gamma_m)d_s} \quad \text{(Eqn. 7.11a)}$$

$$\beta = \tan^{-1}\left(\frac{\cos(\lambda+\Theta)}{\frac{(M+N)\sqrt{1-a_\Theta^2}}{N\eta_0\tan(\phi)}+\sin(\lambda+\Theta)}\right) \quad \text{(Eqn. 7.13)}$$

$$\eta_1 = \eta_0\left(\frac{\left(\frac{M}{N}\right)+\sin(\lambda+\beta+\Theta)}{1+\left(\frac{M}{N}\right)}\right) \quad \text{(Eqn. 7.10)}$$

$$SF_0 = \frac{a_\Theta\tan(\phi)}{\eta_1\tan(\phi)+\sqrt{1-a_\Theta^2}\cos(\beta)} \quad \text{(Eqn. 7.8a)}$$

Finally, I used Solver to calculate the τ_c by setting the safety factor to 1. I set the safety factor to 1 because we are looking for the critical shear stress that the particle will begin to move. The values for the critical shear stress for each λ are below.

Table 3. Critical Shear Stress Solutions

Solver	
λ	**τ_c (lb/ft^2)**
15	0.086804
0	0.090525
-15	0.094765

Given				
Particle Size	10	mm	0.032808399	ft
Θ_1	20	degrees	0.34906585	radians
Θ_0	0	degrees	0	radians
Downstream Channel Slope	0	degrees	0	radians
λ_a	15	degrees	0.261799388	radians
λ_b	0	degrees	0	radians
λ_c	-15	degrees	-0.261799388	radians
SF	1			
ϕ	37	degrees	0.645771823	radians
*From Figure 7.4				
v (viscosity)	1.06E-05	ft^2/s		
*Table 2.3-70F				

Calculated Parameters

λ_a				
Θ	0 degrees			0 radians
a_0	0.93969			
d.	255.6 *pg160			
M/N	5.63524 *eq 7.14b			
M+N	6.63524			
β	0.16393			
η_0	0.53964			
η_1	0.4919			
SF	1 Should be 1			
	β calcs	η_0	η_1	SF
numerator	0.96593	1.82289	6.04823	0.70811
left denominator	5.58068	-	-	0.37067
right denominator	0.25882	-	-	0.33743
denominator	5.8395	3.37795	6.63524	0.70811

λ_b				
Θ	0 degrees			0 radians
a_0	0.93969			
d.	255.6 *pg160			
M/N	5.63524 *eq 7.14b			
M+N	6.63524			
β	0.18474			
η_0	0.56278			
η_1	0.49354			
SF	1 Should be 1			
	β calcs	η_0	η_1	SF
numerator	1	1.90103	5.81893	0.70811
left denominator	5.35129	-	-	0.37191
right denominator	0	-	-	0.3362
denominator	5.35129	3.37795	6.63524	0.70811

λ_c				
Θ	0 degrees			0 radians
a_0	0.93969			
d.	255.6 *pg160			
M/N	5.63524 *eq 7.14b			
M+N	6.63524			
β	0.19647			
η_0	0.58913			
η_1	0.49455			
SF	1 Should be 1			
	β calcs	η_0	η_1	SF
numerator	0.96593	1.99007	5.56996	0.70811
left denominator	5.11187	-	-	0.37267
right denominator	-0.25882	-	-	0.33544
denominator	4.85306	3.37795	6.63524	0.70811

Solver	
λ	τ_0 (lb/ft^2)
15	0.086804
0	0.090525
-15	0.094765

Problem # 4 (20 points)

Solve Problem 7.10: Based on Equation (7.2c), consider that $\tau_{*c} = 0.03$ when $\frac{F_L}{F_D} = 0$, then combine $\frac{F_L}{F_D}$ from figure 7.15 with $\frac{l_4}{l_3} \approx 2.6$. Compare the values of τ_{*c} that are obtained with the Shields diagram value in Figure 7.8.

$$\tau_{*C} \sim \frac{F_D}{F_s} = \frac{l_2}{l_3} \frac{1}{1+\left(\frac{F_L l_4}{F_D l_3}\right)} \qquad \text{(Eqn. 7.2c)}$$

If $\tau_{*c} = 0.03$ when $\frac{F_L}{F_D} = 0$, then $\frac{l_2}{l_3} = 0.03$.

$$\Pi_{ld} = \frac{F_L}{F_D} \approx -0.38 + \left[2.6e^{-\frac{d_*}{3}} + 0.5\tan(\theta)\left(1 - e^{-\frac{d_*}{20}}\right)\right]^{-1} \qquad \text{(Eqn. 7.14a)}$$

Where $\tan(\theta) = 0.8$.

Next, combine equations 7.2c and 7.14a and insert stated values for $\frac{l_2}{l_3}$, $\frac{l_4}{l_3}$, and $\tan(\theta)$.

$$\tau_{*C} = (0.03)\frac{1}{1 + \left((2.6)\left(-0.38 + \left[2.6e^{-\frac{d_*}{3}} + 0.5(0.8)\left(1 - e^{-\frac{d_*}{20}}\right)\right]^{-1}\right)\right)}$$

Finally, plot τ_{*C} as a function of d*:

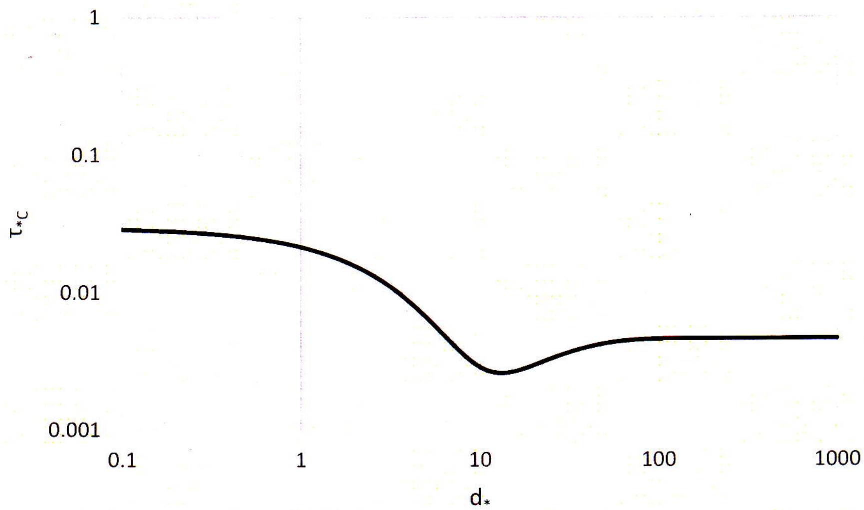

*Figure 1. τ_{*C} as a function of d**

The values calculated in this problem are approximately one degree of magnitude different than the modified shields diagram (Figure 7.8). This is because the modified shields diagram uses a ratio of forces to determine the curve while this problem uses stability factors.