CE 716 Homework 2

1. Calculate the lift force in SI and English units on a 4 m-diameter semi-spherical tent under a 100 km/h wind. Compare with the lift force of a 4 m-long semi-cylindrical tent that has the same volume. (Hint: find the mass density of air and assume that there is no velocity at the base of the tent.)

For a semi-sphere:

$$F_L = \frac{27}{32}\rho v_\infty{}^2 \pi R^2$$

$$F_L = \frac{27}{32}\left(\frac{1.204bkg}{m^3}\right)\left(100\frac{km}{hr} * \frac{1000m}{1km} * \frac{1hr}{60^2 s}\right)^2 \pi * \left(\frac{4}{2}m\right)^2 = \boxed{\mathbf{9,850N}}$$

$$9850N * \left(0.225\frac{lb}{N}\right) = 2{,}240lb$$

For a cylinder, we can find the radius by using the volume of the half sphere

$$V = \frac{2}{3}\pi R^3 = \frac{2}{3}\pi\left(\frac{4}{2}m\right)^3 = 16.76\,m^3$$

$$16.76\,m^3 = \frac{1}{2}\pi R^2 L$$

$$16.76\,m^3 = \frac{1}{2}\pi R^2 4m$$

$$R = \sqrt{16.76\,m^3 * \frac{2}{\pi * 4m}} = 1.63m$$

Now we can plug into the lift equation E-4.2.2 from the book:

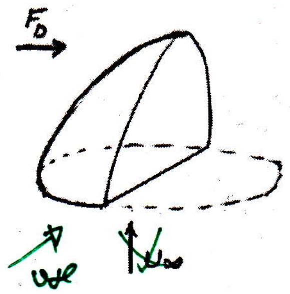
$$F_L = \frac{8}{3}\rho v_\infty{}^2 LR$$

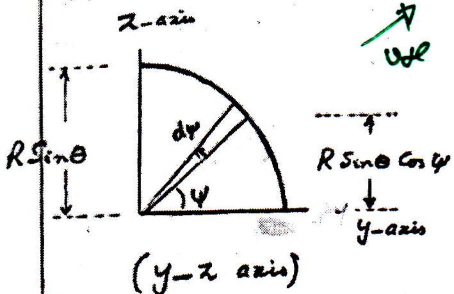
$$F_L = \frac{8}{3}\left(\frac{1.2kg}{m^3}\right)\left(100\frac{km}{hr} * \frac{1000m}{1km} * \frac{1hr}{60^2 s}\right)^2 4m * 1.63m = \boxed{\mathbf{16,200N}}$$

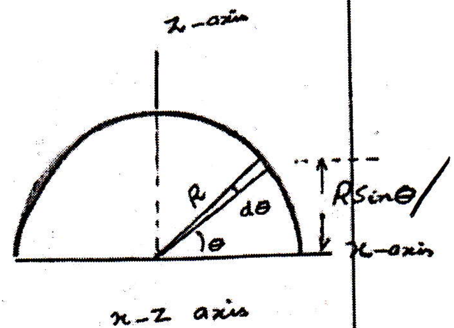
$$16{,}200N * \left(0.225\frac{lb}{N}\right) = \boxed{\mathbf{3,650lb}}$$

Problem # 2 | | | Page 04

Problem 4.4 p. 82 for quarter sphere. Integrate pressure and calculate the forces in all three directions.

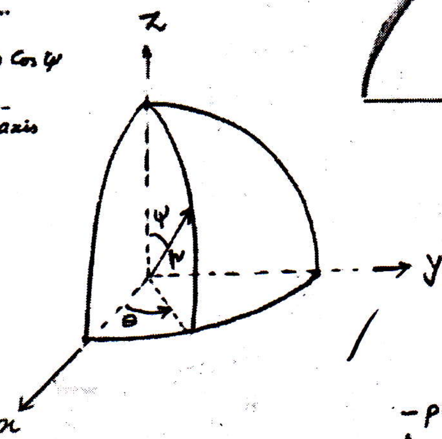


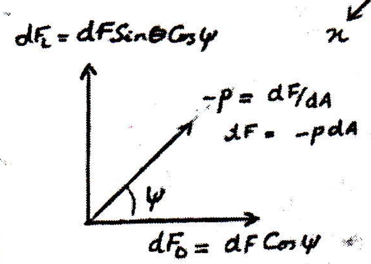


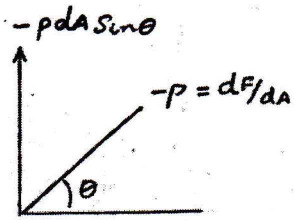


$0 \le \psi \le 90^\circ$

$0^\circ \le \theta \le 180^\circ$







Find :

$F_z = ?$

$F_y = ?$

$F_x = ?$

1) **Force in the z-direction F_z**

The pressure b/w z=0 and any point on the quarter sphere can be obtained by applying Bernoulli's equation

$$\frac{P}{\gamma} + \frac{V^2}{2g} + z = \frac{P_\infty}{\gamma} + \frac{V_\infty^2}{2g} + 0$$

$$P = \gamma\left(\frac{P_\infty}{\gamma} + \frac{U_\infty^2}{2g} - z - \frac{V^2}{2g}\right)$$

$$\boxed{P = P_\infty + \frac{\rho}{2}V_\infty^2 - z\gamma - \frac{\rho}{2}V^2} \quad \text{—— (A)}$$

This is is the general equation for the pressure on the curved surface

We have

$z = R\sin\theta\cos\psi$

$v = -\frac{3}{2}U_\infty \sin\theta$ Velocity at any point of sphere

$$P = P_\infty + \frac{\rho}{2}U_\infty^2 - \gamma R\sin\theta\cos\psi - \frac{\rho}{2}\left(\frac{-3}{2}U_\infty\sin\theta\right)^2$$

The force component on the curved surface in the z-direction can be estimated by integration as follows:

$$F_{z_s} = \int_F dF_z = \int_A -P\,dA\,\sin\theta\cos\psi\,dA$$

Where $dA = R^2\sin\theta\,d\psi\,d\theta$

and the integral limits are $\theta: 0$ to π, & $\psi: 0$ to $\pi/2$

$$F_{z_s} = -\int_0^\pi\int_0^{\pi/2}\left(P_\infty + \frac{\rho}{2}U_\infty^2 - \gamma R\sin\theta\cos\psi - \frac{\rho}{2}\frac{9}{4}U_\infty^2\sin^2\theta\right)\sin\theta\cos\psi R^2\sin\theta\,d\psi\,d\theta$$

$$F_{z_s} = -\int_0^\pi\int_0^{\pi/2}\left(P_\infty + \frac{\rho}{2}U_\infty^2 - \gamma R\sin\theta\cos\psi - \frac{9}{8}\rho U_\infty^2\sin^2\theta\right)\sin^2\theta.\cos\psi R^2\,d\psi\,d\theta$$

$$F_{z_s} = R^2\left(-P_\infty - \frac{\rho}{2}U_\infty^2\right)\int_0^\pi \sin^2\theta\left[\int_0^{\pi/2}\cos\psi\,d\psi\right]d\theta$$
$$+ \gamma R^3\int_0^\pi \sin^3\theta\left[\int_0^{\pi/2}\cos^2\psi\,d\psi\right]d\theta$$
$$+ \frac{9}{8}\rho U_\infty^2 R^2\int_0^\pi \sin^4\theta\left[\int_0^{\pi/2}\cos^2\psi\,d\psi\right]d\theta$$

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$$F_{z_s} = -\frac{1}{2} P_\infty \pi R^2 - \frac{1}{4}\rho U_\infty^2 \pi R^2 + \frac{1}{3}\gamma \pi R^3 + \frac{9}{8} * \frac{3}{8}\rho U_\infty^2 \pi R^2$$

$$F_{z_s} = -\frac{1}{2} P_\infty \pi R^2 - \frac{1}{4}\rho U_\infty^2 \pi R^2 + \frac{1}{3}\gamma \pi R^3 + \frac{27}{64}\rho U_\infty^2 \pi R^2 \quad \text{--- (i)}$$

Force on the base

$$F_{z_{base}} = P_b * \text{Area} \qquad \text{Area} = A_b = \frac{1}{2}\pi R^2$$

$$= \left(P_\infty + \frac{\rho U_\infty^2}{2} - \frac{\rho V_b^2}{2}\right) * \frac{\pi R^2}{2}$$

$$= \frac{1}{2} P_\infty \pi R^2 + \frac{1}{4}\rho U_\infty^2 \pi R^2 - \frac{1}{4}\rho V_b^2 \pi R^2$$

The net force in z-direction can be calculated by the addition of the component force on the base and the force on the curved surface

$$F_z = F_{z_s} + F_{z_{base}}$$

$$= \cancel{-\frac{1}{2} P_\infty \pi R^2} - \cancel{\frac{1}{4}\rho U_\infty^2 \pi R^2} + \frac{1}{3}\gamma \pi R^3 + \frac{27}{64}\rho U_\infty^2 \pi R^2 + \cancel{\frac{1}{2} P_\infty \pi R^2} + \cancel{\frac{1}{4}\rho U_\infty^2 \pi R^2} - \frac{1}{4}\rho V_b^2 \pi R^2$$

$$\boxed{F_z = \frac{1}{3}\gamma \pi R^3 + \frac{27}{64}\rho U_\infty^2 \pi R^2 - \frac{1}{4}\rho V_b^2 \pi R^2}$$

2) **Force in Y- direction**

The force on the curved surface in y-direction is computed by:

$$F_{y_s} = \int_F dF_y = \int_A - PdA \sin\theta \sin\psi \, dA$$

Where

$$dA = R^2 \sin\theta \, d\psi \, d\theta$$

and integral limits are $\theta : 0$ to π & $\psi : 0$ to $\pi/2$

$$F_{y_s} = -\int_0^{\pi}\int_0^{\pi/2} \left(P_\infty + \frac{\rho U_\infty^2}{2} - \gamma R \sin\theta \cos\psi - \frac{\rho}{2}\left(-\frac{3}{2} U_\infty \sin\theta\right)^2\right) * \sin\theta \sin\psi \, R^2 \sin\theta \, d\psi \, d\theta$$

$$F_{y_s} = R^2\left(-P_\infty - \rho\frac{U_\infty^2}{2}\right)\int_0^{\pi} \sin^2\theta \left[\int_0^{\pi/2} \sin\psi \, d\psi\right] d\theta + \gamma R^3 \int_0^{\pi} \sin^3\theta \left[\int_0^{\pi/2} \sin\psi \cos\psi \, d\psi\right] d\theta + \frac{9}{8}\rho U_\infty^2 R^2 \int_0^{\pi} \sin^4\theta \left[\int_0^{\pi/2} \sin\psi \, d\psi\right] d\theta$$

$$F_{y_s} = R^2\left(P_\infty - \rho\frac{U_\infty^2}{2}\right)\int_0^{\pi} \sin^2\theta [1] d\theta + \gamma R^3 \int_0^{\pi} \sin^3\theta \left[\frac{1}{2}\right] d\theta + \frac{9}{8}\rho U_\infty^2 R^2 \int_0^{\pi} \sin^4\theta [1] d\theta$$

$$F_{y_s} = R^2\left(P_\infty - \rho\frac{U_\infty^2}{2}\right)\left(\frac{\pi}{2}\right) + \gamma R^3\left(\frac{4}{3}\right)\left(\frac{1}{2}\right) + \frac{9}{8}\rho U_\infty^2 R^2 \left(\frac{3\pi}{8}\right)[1]$$

$$F_{y_s} = -\frac{1}{2}P_\infty \pi R^2 - \frac{1}{4}\rho U_\infty^2 \pi R^2 + \frac{2}{3}\gamma R^3 + \frac{9}{8} * \frac{3}{8}\rho U_\infty^2 \pi R^2$$

$$F_{y_s} = -\frac{1}{2}P_\infty \pi R^2 - \frac{1}{4}\rho U_\infty^2 \pi R^2 + \frac{2}{3}\gamma R^3 + \frac{27}{64}\rho U_\infty^2 \pi R^2$$

Force on face of quarter sphere

$$F_{y_f} = \int dF_y = PA_y \qquad (y = \text{Center of pressure})$$

$$F_{y_f} = \left(P_\infty + \frac{\rho}{2}\frac{U_\infty^2}{2} - \gamma z - \rho\frac{V_b^2}{2}\right)\left(\frac{\pi R^2}{2}\right)$$

$$F_{y_f} = P_\infty \frac{\pi R^2}{2} + \frac{\rho U_\infty^2}{2}\left(\frac{\pi R^2}{2}\right) - \gamma\left(\frac{4R}{3\pi}\right)\left(\frac{\pi R^2}{2}\right) - \frac{\rho V_b^2}{2}\left(\frac{\pi R^2}{2}\right)$$

$$F_{y_f} = P_\infty \frac{\pi R^2}{2} + \frac{\rho U_\infty^2}{4}\pi R^2 - \frac{2\pi R^3}{3} - \frac{\rho}{4} V_b^2 \pi R^2$$

The net force in y-direction can be calculated by summing the components forces on curved surface and forces on face of quarter circle

$$F_y = F_{y_s} + F_{y_f}$$

$$= \cancel{-\frac{1}{2} P_\infty \pi R^2} - \cancel{\frac{1}{4}\rho U_\infty^2 \pi R^2} + \cancel{\frac{2}{3}\gamma R_3} + \frac{27}{64}\rho U_\infty^2 \pi R^2$$

$$+ \cancel{\frac{P_\infty \pi R^2}{2}} + \cancel{\frac{\rho U_\infty^2 \pi R^2}{4}} - \cancel{\frac{2\pi R^3}{3}} - \frac{\rho V_b^2 \pi R^2}{4}$$

$$F_y = \frac{27}{64}\rho U_\infty^2 \pi R^2 - \rho \frac{V_b^2 \pi R^2}{4}$$

OK. Velocity outside is not ~~base~~

Further, if $V_b \approx 0$ (assumed)

$$\boxed{F_y = \frac{27}{64}\rho U_\infty^2 \pi R^2}$$

DK

(3) **Force in x-direction**

Force component on the curved surface in x-direction can be estimated as:

$$F_{x_s} = \int_F dF_x = \int_A -P\,dA\cos\theta\,dA$$

Where

$$dA = R^2 \sin\theta\, d\psi\, d\theta$$

and integral limits are $\theta : 0$ to π, $\psi : 0$ to $\pi/2$

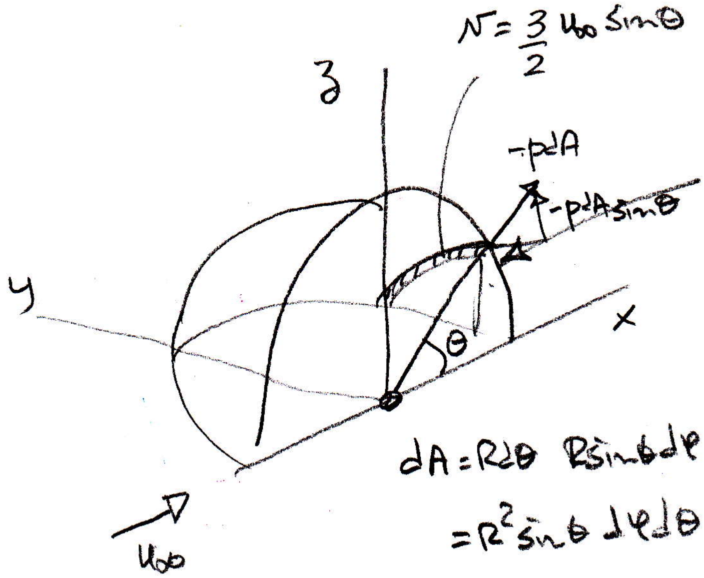
$$F_{x_s} = -\int_0^{\pi}\int_0^{\pi/2}\left(P_\infty + \frac{\rho}{2}U_\infty^2 - \gamma R\sin\theta\cos\psi - \frac{\rho}{2}\left(-\frac{3}{2}U_\infty\sin\theta\right)^2\right) \times \cos\theta\, R^2\sin\theta\, d\psi\, d\theta$$

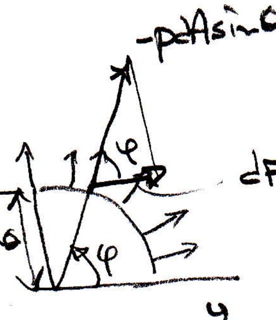
$$F_{x_s} = R^2\left(-P_\infty - \frac{\rho}{2}U_\infty^2\right)\int_0^{\pi}\cos\theta\sin\theta\left[\int_0^{\pi/2}d\psi\right]d\theta + \gamma R^3\int_0^{\pi}\sin^2\theta\cos\theta\left[\int_0^{\pi/2}\cos\psi\, d\psi\right]d\theta + \frac{9}{8}\rho U_\infty^2 R^2\int_0^{\pi}\sin^3\theta\cos\theta\left[\int_0^{\pi/2}\cos^2\psi\, d\psi\right]d\theta$$

$$F_{x_s} = R^2\left(-P_\infty - \frac{\rho}{2}U_\infty^2\right)[0] + \gamma R^3[0] + \frac{9}{8}\rho U_\infty^2 R^2[0]$$

$$\boxed{F_{x_s} = 0}$$

Due to symmetrical geometry, the forces in x-direction will be zero.





$dF_y = -p\,dA \sin\theta \cos\varphi$

$p = p_\infty + \rho \frac{u_\infty^2}{2} - \rho \frac{v^2}{2} - \gamma R \sin\theta \sin\varphi$

$p = p_\infty + \rho\frac{u_\infty^2}{2} - \rho \frac{9}{4} u_\infty^2 \sin^2\theta$

$dA = R\,d\theta\; R\sin\theta\, d\varphi$

$= R^2 \sin\theta\, d\varphi\, d\theta$

$p_\infty + \rho \frac{u_\infty^2}{2}$

<u>curved surface.</u>

$dF_y = -p\,dA \sin\theta\cos\varphi.$

$$F_y = -\int_{\theta=0}^{\pi}\int_{\varphi=0}^{\pi/2}\Big(\underset{①}{p_\infty} + \underset{②}{\rho\frac{u_\infty^2}{2}} - \underset{③}{\frac{9}{4}\rho u_\infty^2 \sin^2\theta} - \underset{④}{\gamma R\sin\theta\sin\varphi}\Big) R^2 \sin\theta\, d\varphi\, d\theta\, \sin\theta\cos\varphi$$

$$① = -p_\infty R^2 \int_0^{\pi} \sin^2\theta \Big[\underbrace{\int_0^{\pi/2} \cos\varphi\, d\varphi}_{1}\Big] d\theta = -p_\infty \frac{\pi R^2}{2}$$

$$② \quad -\rho \frac{u_\infty^2}{2} R^2 \int_0^{\pi} \underbrace{\sin^2\theta}_{\pi/2} \Big[\underbrace{\int_0^{\pi/2} \cos\varphi\, d\varphi}_{1}\Big] d\theta = -\rho\frac{u_\infty^2}{2}\frac{\pi R^2}{2}$$

$\frac{\pi}{2}$

$$③ \quad +\frac{9}{4}\rho u_\infty^2 R^2 \int_0^{\pi} \underbrace{\sin^4\theta}_{\frac{3\pi}{16}} \Big[\underbrace{\int_0^{\pi/2} \cos\varphi\, d\varphi}_{1}\Big] d\theta = \frac{27}{64}\rho u_\infty^2 \pi R^2$$

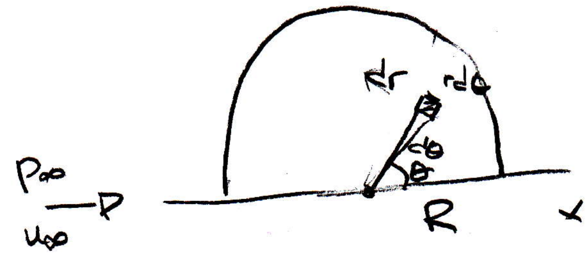
$$④ \quad +\gamma R^3 \int_0^{\pi} \underbrace{\sin^3\theta}_{\frac{2}{3}} \Big[\underbrace{\int_0^{\pi/2} \sin\varphi\cos\varphi\, d\varphi}_{\frac{\sin^2\varphi}{2}\Big|_0^{\pi/2} = \frac{1}{2}}\Big] d\theta = \frac{2}{3}\gamma R^3$$

$dA = r\,dr\,d\theta$

F_y. On side.

$v = u_\infty$

$p = p_\infty - \gamma r \sin\theta$



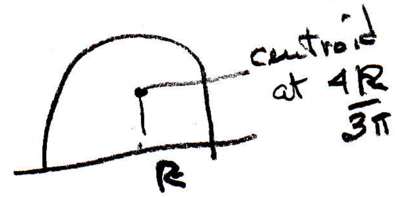
$$F_{y\,side} = \int p\,dA$$

$$= \int_0^R \int_0^\pi \left(\underset{①}{p_\infty} - \underset{②}{\gamma r \sin\theta} \right) r\,d\theta\,dr$$

Bernoulli

$$p_\infty + \frac{\rho u_\infty^2}{2} = p + \frac{\rho u_\infty^2}{2} - \gamma R \sin\theta$$

$$① = p_\infty \int_0^R r \left[\underbrace{\int_0^\pi d\theta}_{\pi} \right] dr = p_\infty \frac{\pi R^2}{2}$$



$$② = -\gamma \int_0^R r^2 \left[\underbrace{\int_0^\pi \sin\theta\,d\theta}_{2} \right] dr = -\gamma\, 2 \frac{R^3}{3} = -\frac{2}{3}\gamma R^3$$

$F = A \times P_{centroid}$

$$\frac{\pi R^2}{2} \times \left(p_\infty + \gamma \frac{4R}{3\pi} \right)$$

$$p_\infty \frac{\pi R^2}{2} - \gamma \frac{2R^3}{3}$$

Net Force in y

$$\cancel{-p_\infty \frac{\pi R^2}{2}} - \frac{\rho u_\infty^2 \pi R^2}{4} + \frac{27}{64} \rho u_\infty^2 \pi R^2 \cancel{+\frac{2}{3}\gamma R^3} \cancel{+ p_\infty \frac{\pi R^2}{2}} \cancel{- \frac{2}{3}\gamma R^3}$$

$$F_y = \frac{27}{64} \rho u_\infty^2 \pi R^2 - \frac{\rho u_\infty^2}{4} \pi R^2$$

roblem 03

Derive Rubey's fall velocity equation in equations (5.23 a and b) from combining equation (5.22 b) and $C_D = 2 + 24/Re_p$

$$\omega_0 = \left[\frac{4}{3}(G-1)g\frac{d_s}{C_D}\right]^{\frac{1}{2}} \quad — \quad 5.22b$$

$$C_D = 2 + \frac{24}{Re_p} \qquad Re = \frac{\omega_0 d_s}{\nu}$$

$$C_D = 2 + \frac{24\nu}{\omega_0 d_s}$$

$$\omega_0^2 = \frac{\frac{4}{3}(G-1)g d_s}{C_D}$$

$$\omega_0^2 C_D = \frac{4}{3}(G-1)g d_s$$

$$\omega_0^2\left(2 + \frac{24\nu}{\omega_0 d_s}\right) - \frac{4}{3}(G-1)g d_s = 0$$

$$2\omega_0^2 + \frac{24\nu}{d_s}\omega_0 - \frac{4}{3}(G-1)g d_s = 0$$

$$ax^2 + bx + c = 0$$

$$a = 2 \qquad b = \frac{24\nu}{d_s} \qquad c = -\frac{4}{3}(G-1)g d_s$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\omega_0 = \frac{-\frac{24\nu}{d_s} \pm \sqrt{\left(\frac{24\nu}{d_s}\right)^2 - 4\times 2\left(-\frac{4}{3}(G-1)g d_s\right)}}{2\times 2}$$

$$= \frac{-6\nu}{d_s} \pm \frac{1}{4}\sqrt{\frac{16\times 36\nu^2}{d_s^2} + 16\times\frac{2}{3}(G-1)g\frac{d_s}{d_s^2}\times d_s^2}$$

$$= \frac{-6\nu}{d_s} \pm \frac{4}{4d_s}\sqrt{36\nu^2 + \frac{2g}{3}(G-1)d_s^3}$$

$$\omega_o = \frac{-6\nu}{d_s} \pm \frac{1}{d_s}\sqrt{\frac{2g}{3}(G-1)d_s^3 + 36\nu^2}$$

$$\omega_o = \frac{1}{d_s}\left[\sqrt{\frac{2g}{3}(G-1)d_s^3 + 36\nu^2} - 6\nu\right] \quad \text{— 5.23 a}$$

or

$$\omega_o = \frac{-6\nu}{d_s} + \frac{1}{d_s}\sqrt{\frac{2g}{3}(G-1)d_s^3 + 36\nu^2}$$

$$= -\sqrt{\frac{36\nu^2}{d_s^2}} \times \sqrt{\frac{(G-1)gd_s}{(G-1)gd_s}} + \sqrt{\frac{2g(G-1)d_s^3}{3d_s^2} + \frac{36\nu^2}{d_s^2}\,\frac{(G-1)gd_s}{(G-1)gd_s}}$$

$$= \sqrt{(G-1)gd_s}\left[\sqrt{\frac{2}{3} + \frac{36\nu^2}{(G-1)gd_s^3}} - \sqrt{\frac{36\nu^2}{(G-1)gd_s^3}}\right] \quad \text{— 5.23b}$$

$$\omega_o = \frac{-6\upsilon}{d_s} \pm \frac{1}{d_s}\sqrt{\frac{2g}{3}(G-1)d_s^3 + 36\upsilon^2}$$

$$\omega_o = \frac{1}{d_s}\left[\sqrt{\frac{2g}{3}(G-1)d_s^3 + 36\upsilon^2} - 6\upsilon\right] \quad \text{— 5.23 a}$$

or

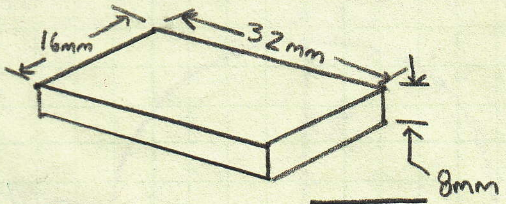
$$\omega_o = \frac{-6\upsilon}{d_s} + \frac{1}{d_s}\sqrt{\frac{2g}{3}(G-1)d_s^3 + 36\upsilon^2}$$

$$= -\sqrt{\frac{36\upsilon^2}{d_s^2}} \times \sqrt{\frac{(G-1)gd_s}{(G-1)gd_s}} + \sqrt{\frac{2g(G-1)d_s^3}{3d_s^2} + \frac{36\upsilon^2}{d_s^2}\,\frac{(G-1)gd_s}{(G-1)gd_s}}$$

$$= \sqrt{(G-1)gd_s}\left[\sqrt{\frac{2}{3} + \frac{36\upsilon^2}{(G-1)gd_s^3}} - \sqrt{\frac{36\upsilon^2}{(G-1)gd_s^3}}\right] \quad \text{— 5.23b}$$

PROBLEM 4

ESTIMATE THE VOLUME, SPHERICITY, & COREY SHAPE FACTOR OF THE PARTICLE. ESTIMATE THE SETTLING VELOCITY OF THE PARTICLE FROM FIGURE 5.2A & COMPARE WITH THE SETTLING VELOCITY OF AN EQUIVALENT VOLUME SPHERE



$$\forall_s = 16(32)8 = 4096\,mm^3 = 4.10\,cm^3 = \forall_s$$

$$S_p = \left(\frac{l_c(l_b)}{l_a^2}\right)^{1/3} = \left(\frac{8(16)}{32^2}\right)^{1/3} \Rightarrow \underline{S_p = 0.5}$$

$d_s^3 = abc \Rightarrow d_s = \sqrt[3]{32(16)(8)}$

$$C_o = \frac{l_c}{\sqrt{l_a l_b}} = \frac{8}{\sqrt{32(16)}} \Rightarrow \underline{C_o = 0.354}$$

FIGURE 4: Approximate Particle Diagram

Assuming high Re_p gives:

$$C_D = \cancel{\frac{24}{Re_p}}^{\to 0} + \frac{0.5}{C_o^2}, \rightarrow C_D = \frac{0.5}{0.354^2} \Rightarrow C_D = 4$$

Nice.

First iteration assuming high Rep gives a fall velocity:

$$\omega = \left(\frac{4}{3}(G-1)\,g\frac{d_s}{c_D}\right)^{1/2} = \left(\frac{4}{3}(2.65-1)\frac{9.81(0.016)}{4}\right)^{1/2} \Rightarrow \omega = 0.293\,m/s$$

To verify the assumption of high Rep, we can use the following:

$$Re_p = \frac{\omega d_s}{\nu} = \frac{0.293(0.016)}{1\times10^{-6}} \Rightarrow Re_p = 4701.06 > 10^3 \therefore Re_p \text{ Fully Turbulent}$$

Correct

Since our assumption of turbulent Rep, the fall velocity is:

$$\underline{\omega_o = 0.294\,m/s}$$

For high Rep, we can see that $C_D = \frac{0.5}{C_o^2}$, which plotted below on Figure 5:

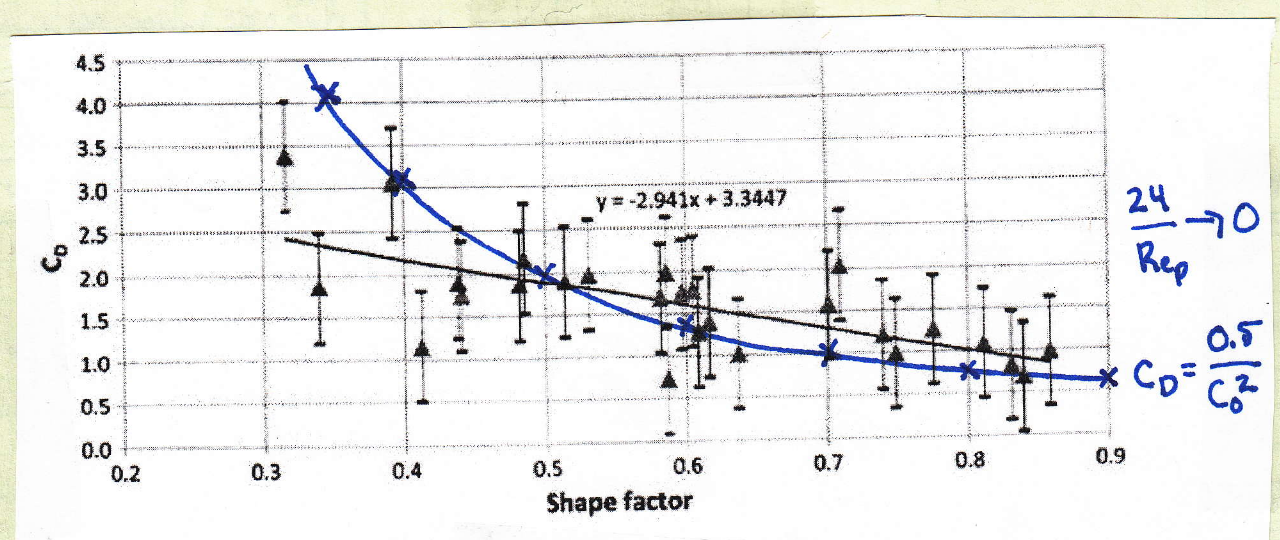


Fig. 5. Shape factor versus drag coefficient