1) Solve Problem 2.3 at a concentration $C_v = 0.2$. The volumetric sediment concentration of a sample is $C_v = 0.2$. Determine the corresponding: a) porosity p_o; b) void ratio e; c) specific weight γ_m; d) specific mass ρ_m; e) dry specific weight γ_{md}; f) dry specific mass ρ_{md}.

a. $\boldsymbol{p_o} = 1 - C_v = 1 - (0.2) = \mathbf{0.8}$

b. $\boldsymbol{e} = \frac{p_o}{(1-p_o)} = \frac{(0.8)}{1-(0.8)} = \mathbf{4}$

c. $\boldsymbol{\gamma_m} = \gamma(1 + (G-1)C_v) = \left(9810\frac{N}{m^3}\right)(1 + (2.65-1)(0.2)) = \mathbf{13.05\frac{kN}{m^3}}$

d. $\boldsymbol{\rho_m} = \frac{\gamma_m}{g} = \frac{13.05\frac{kN}{m^3}}{\frac{9.81m}{s^2}} = \mathbf{1330\frac{kg}{m^3}}$

e. $\boldsymbol{\gamma_{md}} = \gamma_s C_v = \left(25996.5\frac{N}{m^3}\right)(0.2) = \mathbf{5.2\frac{kN}{m^3}}$

i. $\gamma_s = G\gamma = (2.65)\left(9810\frac{N}{m^3}\right) = 25996.5\frac{N}{m^3}$

f. $\boldsymbol{\rho_{md}} = \frac{\gamma_{md}}{g} = \frac{5199.3\frac{N}{m^3}}{9.81\frac{m}{s^2}} = \mathbf{530\frac{kg}{m^3}}$

3.14 c

$$a_z = \frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z} = g_z + \frac{1}{\rho_m}\left(\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y}\right)$$

Work:

$$a_z = \frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}$$

add & subtract $v_x\frac{\partial v_x}{\partial z}$ and $v_y\frac{\partial v_y}{\partial z}$ to get

$$a_z = \frac{\partial v_z}{\partial t} + \underbrace{v_x\frac{\partial v_x}{\partial z} + v_y\frac{\partial v_y}{\partial z} + v_z\frac{\partial v_z}{\partial z}}_{\frac{\partial}{\partial z}\left(\frac{v_x^2}{2} + \frac{v_y^2}{2} + \frac{v_z^2}{2}\right)} + v_x\underbrace{\left(\frac{\partial v_z}{\partial x} - \frac{\partial v_x}{\partial z}\right)}_{-\otimes y} + v_y\underbrace{\left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)}_{\otimes x}$$

$$a_z = \frac{\partial v_z}{\partial t} + \frac{\partial}{\partial z}\left(\frac{v_x^2}{2} + \frac{v_y^2}{2} + \frac{v_z^2}{2}\right) - v_x \otimes y + v_y \otimes x$$

$$v_x^2 + v_y^2 + v_z^2 = v^2$$

$$\boxed{a_z = \frac{\partial v_z}{\partial t} + \frac{\partial v^2}{2\partial z} + v_y \otimes - v_x \otimes y}$$

$\Sigma F = ma$

3) Solve Problem 3.9.

Consider a quarter-sphere of radius R under hydrostatic pressure. Determine the pressure distribution around the surface if the pressure on the flat base is p_o. Integrate the pressure distribution to determine the buoyancy force components in the x and z directions.

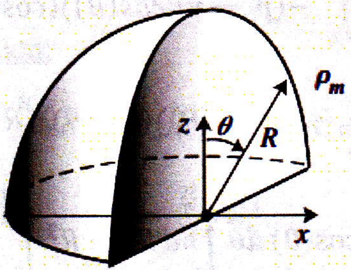
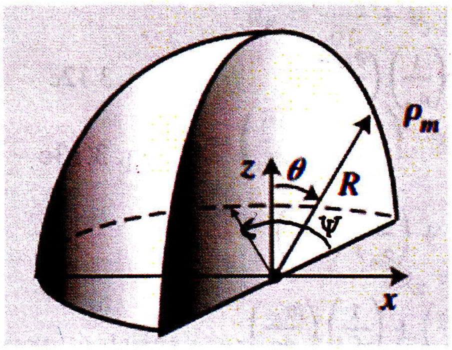
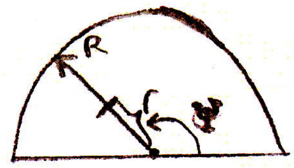


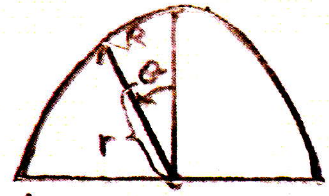
Figure P-3.9 Quarter-sphere

Defining Variables:





Base of Quarter Sphere



Side of Quarter Sphere

Where R is the radius, r is some distance between the base of R and R. θ and Ψ are defined above.

Z Direction:

Surface of the quarter-sphere:

$$\frac{\partial z}{\partial n} = \cos(\theta)$$

$$p = p_o - \rho_m g R cos(\theta) = p_o - \gamma R cos(\theta)$$

$$dA = (R d\theta)(R sin(\theta) d\Psi) = R^2 \sin(\theta)\, d\Psi d\theta$$

$$F_{B_z} = \int_A -p\left(\frac{\partial z}{\partial n}\right) dA$$

$$F_{B_z} = \int_A -(p_o - \gamma R cos(\theta))(\cos(\theta)) dA$$

$$F_{B_z} = \iint -(p_o - \gamma R cos(\theta))(\cos(\theta))(R^2 \sin(\theta)\, d\Psi d\theta)$$

$$F_{B_z} = -p_o R^2 \int_0^{\frac{\pi}{2}} \sin(\theta)\cos(\theta)\, d\theta \int_0^{\pi} d\Psi + \gamma R^3 \int_0^{\frac{\pi}{2}} \sin(\theta)\cos^2(\theta)\, d\theta \int_0^{\pi} d\Psi$$

$$F_{B_z} = \frac{\pi}{3}\gamma R^3 - p_o R^2 \frac{\pi}{2}$$

Base of the quarter-sphere:

$\frac{\partial z}{\partial n} = -1$, because the normal force is perpendicular to the surface.

$p = p_o$

$dA = (rd\Psi)(dr)$

$$F_{B_z} = \int_A -p\left(\frac{\partial z}{\partial n}\right) dA$$

$$F_{B_z} = \int_A -(p_o)(-1)dA$$

$$F_{B_z} = \iint -(p_o)(-1)\,(rd\Psi)(dr)$$

$$F_{B_z} = p_o \int_0^R \mathrm{r\,dr} \int_0^\pi d\Psi$$

$$F_{B_z} = p_o R^2 \frac{\pi}{2}$$

/

Flat side of the quarter-sphere:

$\frac{\partial z}{\partial n} = 0$, because the normal force parallel to the side.

Since the normal force is zero:

$$F_{B_z} = 0$$

Summation of forces in the z-direction:

$$F_{B_z} = \frac{\pi}{3}\gamma R^3 - p_o R^2 \frac{\pi}{2} + p_o R^2 \frac{\pi}{2} + 0$$

$$\boldsymbol{F_{B_z} = \frac{\pi}{3}\gamma R^3}$$

/

X Direction:

Surface of the quarter-sphere:

$\frac{\partial x}{\partial n} = \sin(\theta)\sin(\Psi)$

$p = p_o - \rho_m g R cos(\theta) = p_o - \gamma R cos(\theta)$

$dA = (Rd\theta)(Rsin(\theta)d\Psi) = R^2 \sin(\theta)\, d\Psi d\theta$

$$F_{B_x} = \int_A -p\left(\frac{\partial x}{\partial n}\right) dA$$

$$F_{B_x} = \int_A -(p_o - \gamma R cos(\theta))(\sin(\theta)\sin(\Psi))dA$$

$$F_{B_x} = \iint -(p_o - \gamma R cos(\theta))(\sin(\theta)\sin(\Psi))(R^2 \sin(\theta)\, d\Psi d\theta)$$

$$F_{B_x} = -p_o R^2 \int_0^{\frac{\pi}{2}} \sin^2(\theta) d\theta \int_0^{\pi} \sin(\Psi)\, d\Psi + \gamma R^3 \int_0^{\frac{\pi}{2}} \sin^2(\theta)\cos(\theta)\, d\theta \int_0^{\pi} \sin(\Psi)\, d\Psi$$

$$F_{B_x} = \frac{2}{3}\gamma R^3 - p_o R^2 \frac{\pi}{2}$$

Base of the quarter-sphere:

$\frac{\partial x}{\partial n} = 0$, because the normal force is parallel to the base.

Since the normal force is zero:

$$F_{B_x} = 0$$

Flat side of the quarter-sphere:

$\frac{\partial x}{\partial n} = -1$, because the normal force is perpendicular to the surface.

$p = p_o - \rho_m g r cos(\theta) = p_o - \gamma r cos(\theta)$

$dA = (rd\theta)(dr)$

$$F_{B_x} = \int_A -p\left(\frac{\partial x}{\partial n}\right) dA$$

$$F_{B_x} = \int_A -(p_o - \gamma r cos(\theta))(-1) dA$$

$$F_{B_x} = \iint -(p_o - \gamma r cos(\theta)(-1)\,(rd\theta)(dr)$$

$$F_{B_x} = p_o \int_0^R \mathrm{r\,dr} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta - \gamma \int_0^R \mathrm{r^2\,dr} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(\theta)\, d\theta$$

$$F_{B_x} = p_o R^2 \frac{\pi}{2} - \frac{2}{3}\gamma R^3$$

Summation of forces in the x-direction:

Cool!

$$F_{B_x} = \frac{2}{3}\gamma R^3 - p_o R^2 \frac{\pi}{2} + 0 + p_o R^2 \frac{\pi}{2} - \frac{2}{3}\gamma R^3$$

$$\boldsymbol{F_{B_x} = 0}$$

Question 4:

A spherical ball containing 0.1m3 of air at a density of 2 kg/m3 is held submerged at the bottom of a pool, 5 m below the water surface. What is the force required to hold it in place? Also, determine its acceleration when it is released from rest. (Hint: notice that without added mass, this ball would reach an acceleration of about 500g!)

The first step for both parts is to visualize the forces acting on each ball with a free body diagram along with the mass of each system and the direction of acceleration (Figures 5 and 6).

Part a:

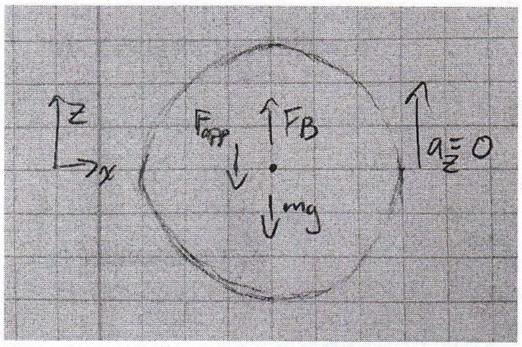


Figure 5: Free body diagram for the non-accelerating ball

$$\sum F_z = Ma_z = 0$$

$$\sum F_z = 0 = F_b - Mg - F_{applied}$$

$$= V(\gamma) - \gamma_{sphe}\ \ V - F_{applied}$$

$$F_{applied} = V(\gamma) - \gamma_{sphere}V$$

$$F_{applied} = 0.1m^3\left(9810\frac{N}{m^3}\right) - 2\frac{kg}{m^3}9.81\frac{m}{s^2}0.1m^3$$

$$F_{applied} = -979N$$

✓

Part b:

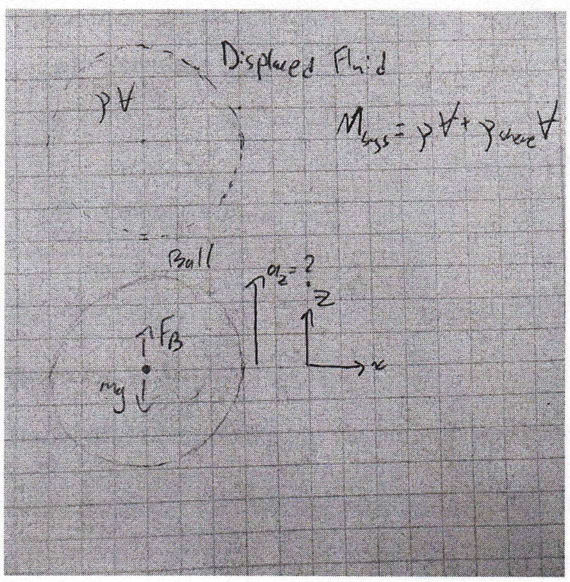


Figure 6: Free body diagram for the accelerating ball

$$\sum F_z = Ma_z$$

$$a_z = \sum F_z \,/\, M$$

$$a_z = \underbrace{(V(\gamma) - \gamma_{sphere}V)}_{\sum F_z}/\underbrace{(\rho_{spher}\ * V + \rho_{water}V)}_{M}$$

$$a_z = (0.1m^3\left(9810\frac{N}{m^3}\right) - 2\frac{kg}{m^3} * 9.81\frac{m}{s^2} * 0.1m^3)/(2\frac{kg}{m^3} * 0.1m^3 + 1000\frac{kg}{m^3}0.1m^3)$$

$$a_z = 9.77\frac{m}{s^2}$$

✓ up or down?