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Solve computer problem 3.1 on p. 63 of the text *Erosion and Sedimentation.*

Consider steady flow ($q = 3.72\ m^2/s$) in the impervious rigid boundary channel. Assume a very wide channel and $f = 0.03$. Determine the distribution of the following parameters along the 25km channel reach when the water surface elevation at the dam is 10m above the bed elevation.

After considering that a 15 km reach at a bed slope of 70 cm/km has been added to the upstream portion of the channel sketched on p. 49. The width of the added segment has the same width as the lower reach. Assume a constant value of the Manning coefficient $n = 0.025$ throughout the entire reach. It is fine to assume a wide-rectangular channel for the calculations. Solve the problem in English units and discuss the results of the three main graphs.

a) Flow depth in ft
b) Mean flow velocity in ft/s
c) Bed shear stress in psf

## Available Data

- Steady flow (q) = $3.72\ m^2/s = 40.02\ ft^2/s$
- Friction factor (f) = 0.03
- Water surface elevation at the dam = 10m = 32.81 ft
- Bed slope of most upstream slope (15km) = 70 cm/km = 0.0007
- Manning coefficient n = 0.025
- $L_1$ =15 km = 49213 ft – Reach 1
- $L_2$ = 10 km = 32808 ft – Reach 2
- $L_3$ = 15 km = 49213 ft – Reach 3

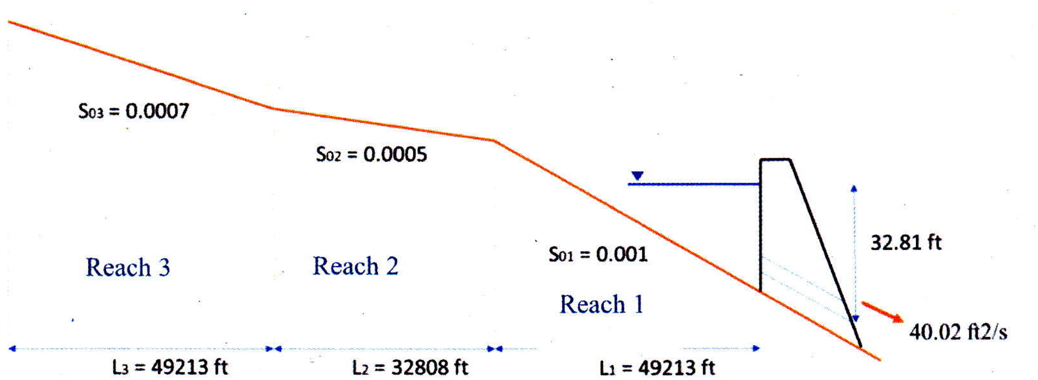


*Figure 1: Sketch of the channel*

## Assumption

- The channel is very wide, hence.
  Hydraulic radius ($R_h$) = h
- Gravitational acceleration = 9.81 m/s$^2$ =32.2 ft/s$^2$
- Specific weight of water = 9810 N/m$^3$ = 62.4 lb/ft$^3$

## Theory

- Direct step method

  Boundary conditions, x = 0 ft, h = 32.81 ft
  Distance increment Δx = 30 ft
  If $h_1$ is a known depth

$$h_2 = h_1 - \Delta x \frac{dh}{dx}$$

- Gradually varied flow

  We used the equation (backwater curve equation) to determine the water surface profile.

$$\frac{dh}{dx} = \frac{S_0 - S_f}{1 - F_r^2}$$

If we consider wide rectangular channel,

$$\frac{dh}{dx} = \frac{S_0\left[1 - \left(\frac{h_n}{h}\right)^{\frac{10}{3}}\right]}{\left[1 - \left(\frac{h_c}{h}\right)^{3}\right]}$$

Where,
h = Flow depth
x = distance from the boundary (Dam)
$S_0$ = Bed slope
$S_f$ = Friction slope
$F_r$ = Froude Number
$h_c$ = Critical flow
$h_n$ = Normal depth

Nice.

- Froude Number ($F_r$)

$$Fr = \frac{V}{\sqrt{gh}} \quad \text{if } V = q/h$$

$$Fr = \left(\frac{h_c}{h}\right)^{\frac{3}{2}}$$

- Critical depth ($h_c$) and Normal depth ($h_n$)

$$h_c = \sqrt[3]{\frac{q^2}{g}} \qquad h_n = \left(\frac{nq}{1.49\, S_0^{1/2}}\right)^{3/5}$$

- $\frac{S_f}{S_0} = \left(\frac{h_n}{h}\right)^{10/3}$
- Bed shear stress ($\tau_0$)

$\tau_0 = \gamma\, h_n\, S_0$

table of results

**Calculation**

**Sample calculation**

- Critical depth

$$h_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{40.02^2}{32.2}} = 3.678 \text{ ft}$$

- Normal depth ($h_n$)

$$h_n = \left(\frac{nq}{1.49\, S_0^{1/2}}\right)^{3/5}$$

Reach 1 -> $h_{n\text{-}1} = \left(\frac{0.025 \times 40.02}{1.49 \times 0.001^{1/2}}\right)^{3/5} = 6.255$ ft

Reach 2 -> $h_{n\text{-}1} = \left(\frac{0.025 \times 40.02}{1.49 \times 0.0005^{1/2}}\right)^{3/5} = 7.701$ ft

Nice

Reach 3 -> $h_{n\text{-}1} = \left(\frac{0.025 \times 40.02}{1.49 \times 0.0007^{1/2}}\right)^{3/5} = 6.961$ ft

- Critical velocity ($V_c$)

$V_c = q/h_c = 40.02/3.678 = 10.881$ ft/s

- Normal velocities

$V_n = q/h_n$

Reach 1 -> $V_{n\text{-}1} = 40.02/6.255 = 6.398$ ft/s

Reach 2 -> $V_{n\text{-}2} = 40.02/7.701 = 5.197$ ft/s

Reach 3 -> $V_{n\text{-}3} = 40.02/6.961 = 5.749$ ft/s

- Bed stress ($\tau_0$)

$\tau_0 = \gamma\, h_n\, S_0$

Reach 1 -> $\tau_{0-1}$ = 62.4*6.255*0.001 = 0.390 psf

Reach 2 -> $\tau_{0-2}$ = 62.4*7.701 *0.0005 = 0.240 psf

Reach 3 -> $\tau_{0-3}$ = 62.4*6.961 *0.0007 = 0.304 psf

Calculations continued using excel and determined the water surface elevation, water depth, velocity and bed stress.

| Reach | Length -L (ft) | Cumulative length (ft) | So | hn | Vn | Final Station |
|---|---|---|---|---|---|---|
| Reach 1 | 49213 | 49213 | 0.001 | 6.255 | 6.398 | 1640 |
| Reach 2 | 32808 | 82021 | 0.0005 | 7.701 | 5.197 | 2734 |
| Reach 3 | 49213 | 131234 | 0.0007 | 6.961 | 5.749 | 4374 |

| | |
|---|---|
| Steady flow (q), ft2/s | 40.02 |
| Friction factor, (f) | 0.03 |
| g, (ft/s2) | 32.2 |
| Specific weight of water ($\gamma$), lb/ft3 | 62.4 |
| Distance increment ($\Delta$x), ft | 30 |
| Manning coefficient (n) | 0.025 |

| | Δx*[Col 1] | [Col 3]+Δx*So | [Col 4]-Δx*[Col 7] | So*(hn/[Col 4])^(10/3) | (hc/[Col 4])^3 | (So -[Col 5)/(1- [Col 6]) | [Col 3]+[Col 4] | q/[Col 4] | ([Col 9]^2)/(2*g) | [Col 8]+[Col10] | γ*[Col 4]*[Col 5] |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| Station | Distance from Dam, x ft | Bed elevation, Z (ft) | Flow depth, h (ft) | Sf | Fr^2 | dh/dx | HGL | V | V^2/2g | EGL | Bed Shear (τ0) |
| 0 | 0 | 0 | **32.810** | 3.98756E-06 | 0.00140825 | 0.000997417 | 32.810 | 1.2198 | 0.023 | 32.833 | 0.008163913 |
| 1 | 30 | 0.03 | 32.780 | 3.99971E-06 | 0.00141211 | 0.000997409 | 32.810 | 1.2209 | 0.023 | 32.833 | 0.008181312 |
| 2 | 60 | 0.06 | 32.750 | 4.0119E-06 | 0.00141598 | 0.0009974 | 32.810 | 1.222 | 0.023 | 32.833 | 0.008198764 |
| 3 | 90 | 0.09 | 32.720 | 4.02415E-06 | 0.00141987 | 0.000997392 | 32.810 | 1.2231 | 0.023 | 32.833 | 0.008216269 |
| 4 | 120 | 0.12 | 32.690 | 4.03644E-06 | 0.00142377 | 0.000997384 | 32.810 | 1.2242 | 0.023 | 32.834 | 0.008233828 |
| 5 | 150 | 0.15 | 32.660 | 4.04878E-06 | 0.00142769 | 0.000997375 | 32.810 | 1.2253 | 0.023 | 32.834 | 0.008251439 |
| 6 | 180 | 0.18 | 32.630 | 4.06116E-06 | 0.00143162 | 0.000997367 | 32.810 | 1.2265 | 0.023 | 32.834 | 0.008269105 |
| 7 | 210 | 0.21 | 32.601 | 4.0736E-06 | 0.00143557 | 0.000997358 | 32.811 | 1.2276 | 0.023 | 32.834 | 0.008286825 |
| 8 | 240 | 0.24 | 32.571 | 4.08609E-06 | 0.00143953 | 0.00099735 | 32.811 | 1.2287 | 0.023 | 32.834 | 0.008304598 |
| 9 | 270 | 0.27 | 32.541 | 4.09863E-06 | 0.0014435 | 0.000997341 | 32.811 | 1.2298 | 0.023 | 32.834 | 0.008322426 |
| 10 | 300 | 0.30 | 32.511 | 4.11121E-06 | 0.00144749 | 0.000997332 | 32.811 | 1.231 | 0.024 | 32.834 | 0.008340309 |
| 11 | 330 | 0.33 | 32.481 | 4.12385E-06 | 0.00145149 | 0.000997324 | 32.811 | 1.2321 | 0.024 | 32.834 | 0.008358246 |
| 12 | 360 | 0.36 | 32.451 | 4.13654E-06 | 0.00145551 | 0.000997315 | 32.811 | 1.2332 | 0.024 | 32.835 | 0.008376239 |
| 13 | 390 | 0.39 | 32.421 | 4.14928E-06 | 0.00145955 | 0.000997306 | 32.811 | 1.2344 | 0.024 | 32.835 | 0.008394286 |
| 14 | 420 | 0.42 | 32.391 | 4.16207E-06 | 0.00146359 | 0.000997298 | 32.811 | 1.2355 | 0.024 | 32.835 | 0.008412389 |
| 15 | 450 | 0.45 | 32.361 | 4.17491E-06 | 0.00146766 | 0.000997289 | 32.811 | 1.2367 | 0.024 | 32.835 | 0.008430548 |
| 16 | 480 | 0.48 | 32.331 | 4.1878E-06 | 0.00147174 | 0.00099728 | 32.811 | 1.2378 | 0.024 | 32.835 | 0.008448763 |

Hearts

$\frac{V^2}{2g}$, τ at normal depth.

Nice

## Results and Discussion

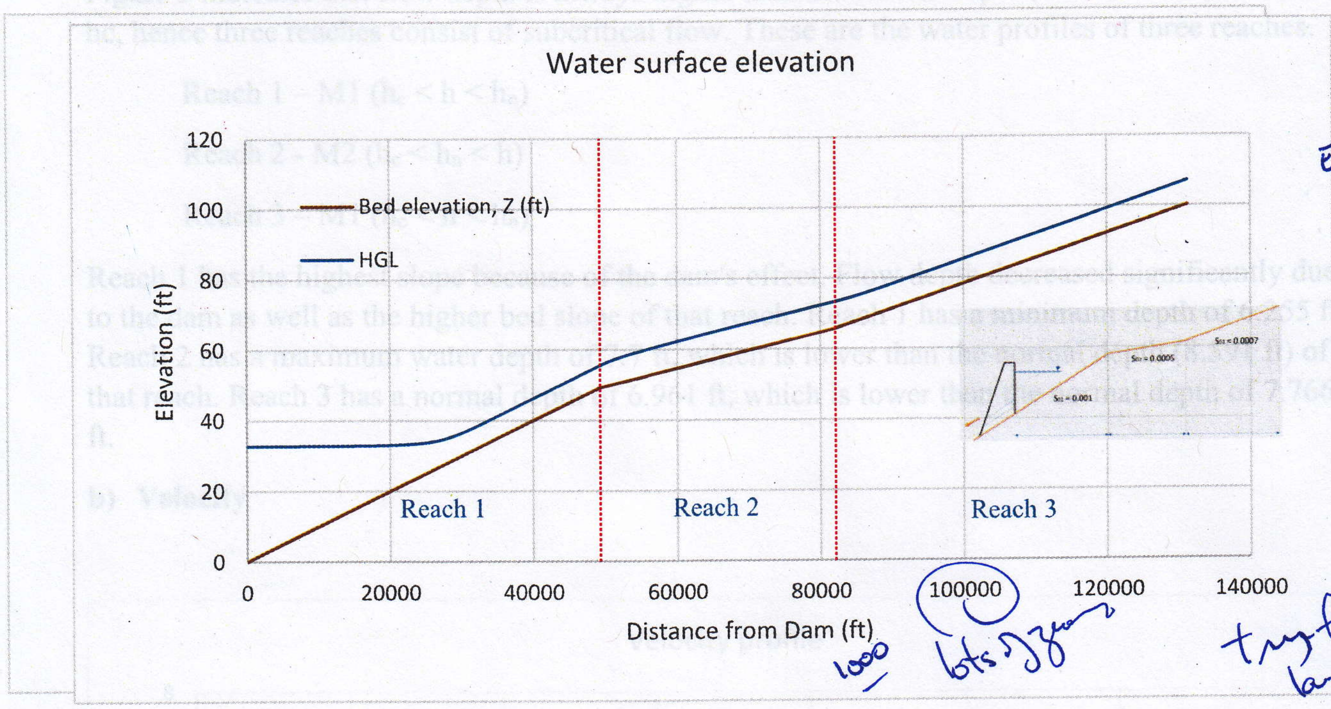


*Figure 2: Water surface elevation*

### a) Flow depth

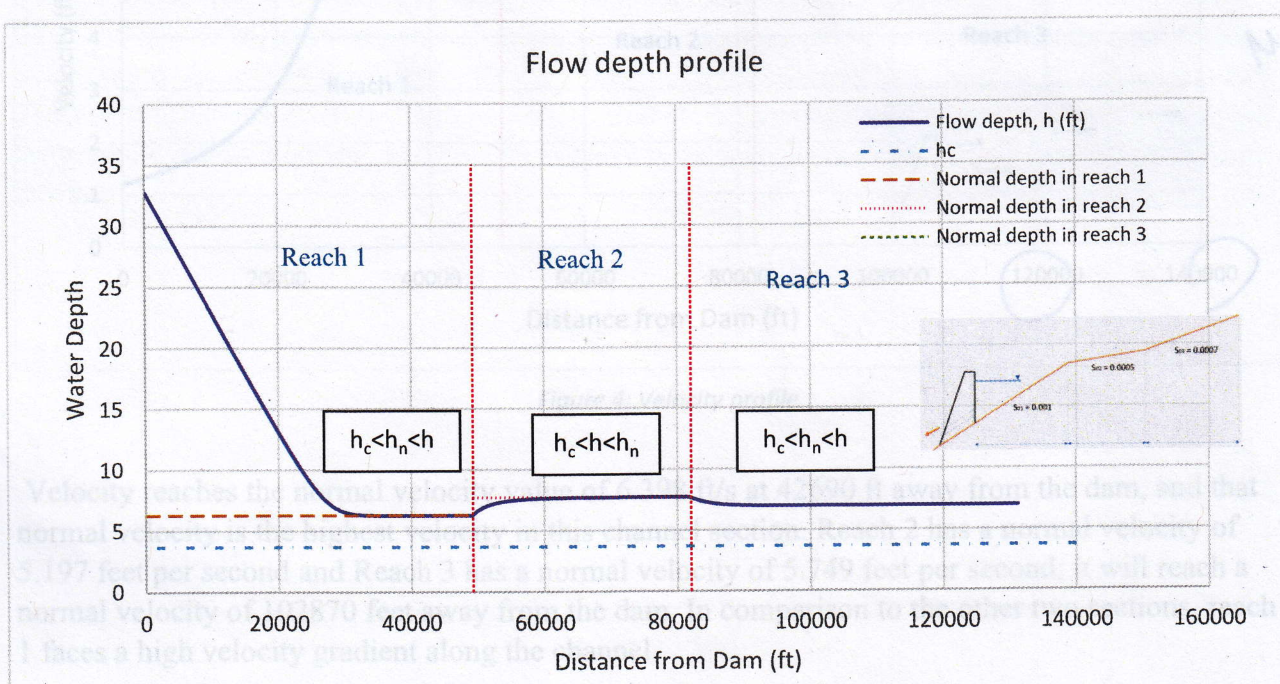


*Figure 3: Flow depth profile*

Figure 3 indicates that flow depth is always higher than the critical depth (hc = 3.678 ft) and hn > hc, hence three reaches consist of subcritical flow. These are the water profiles of three reaches.

Reach 1 – M1 ($h_c < h < h_n$)

Reach 2 - M2 ($h_c < h_n < h$)

Reach 3 – M1 ($h_c < h < h_n$)

Reach 1 has the highest slope because of the dam's effect. Flow depth decreased significantly due to the dam as well as the higher bed slope of that reach. Reach 1 has a minimum depth of 6.255 ft. Reach 2 has a maximum water depth of 7.7 ft, which is lower than the normal depth (8.591 ft) of that reach. Reach 3 has a normal depth of 6.961 ft, which is lower than the normal depth of 7.766 ft.

**b) Velocity**

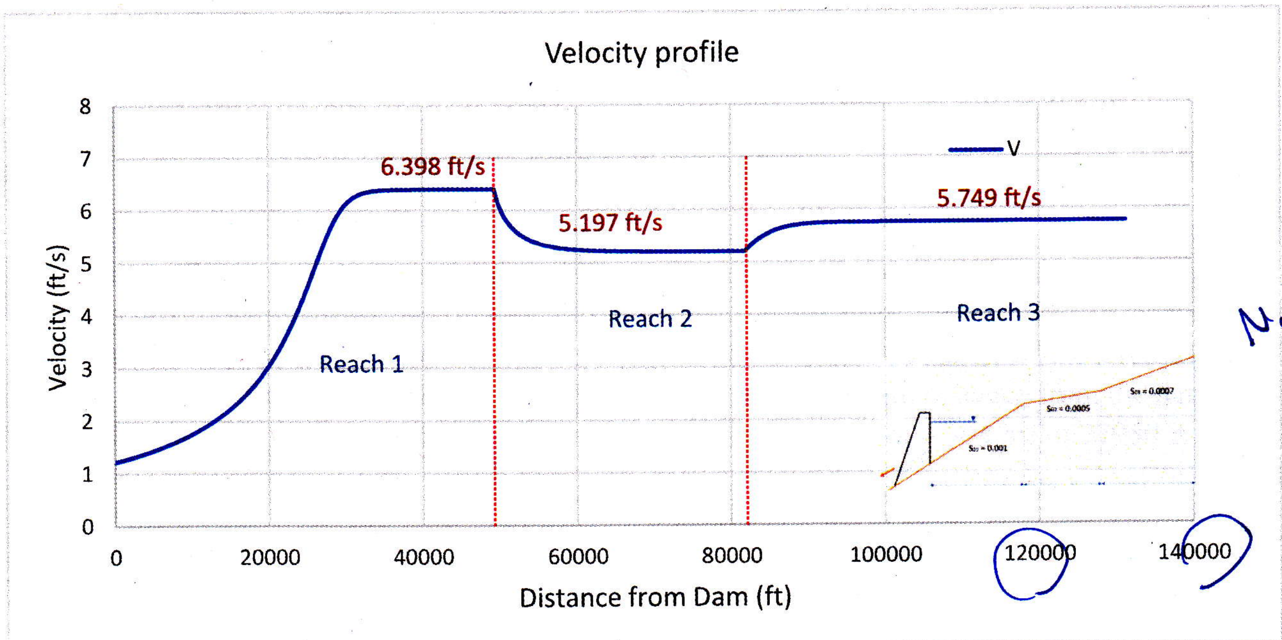


*Figure 4: Velocity profile*

Velocity reaches the normal velocity value of 6.398 ft/s at 42690 ft away from the dam, and that normal velocity is the highest velocity in this channel section. Reach 2 has a normal velocity of 5.197 feet per second and Reach 3 has a normal velocity of 5.749 feet per second; it will reach a normal velocity of 102870 feet away from the dam. In comparison to the other two sections, reach 1 faces a high velocity gradient along the channel.

**c) Shear stress**

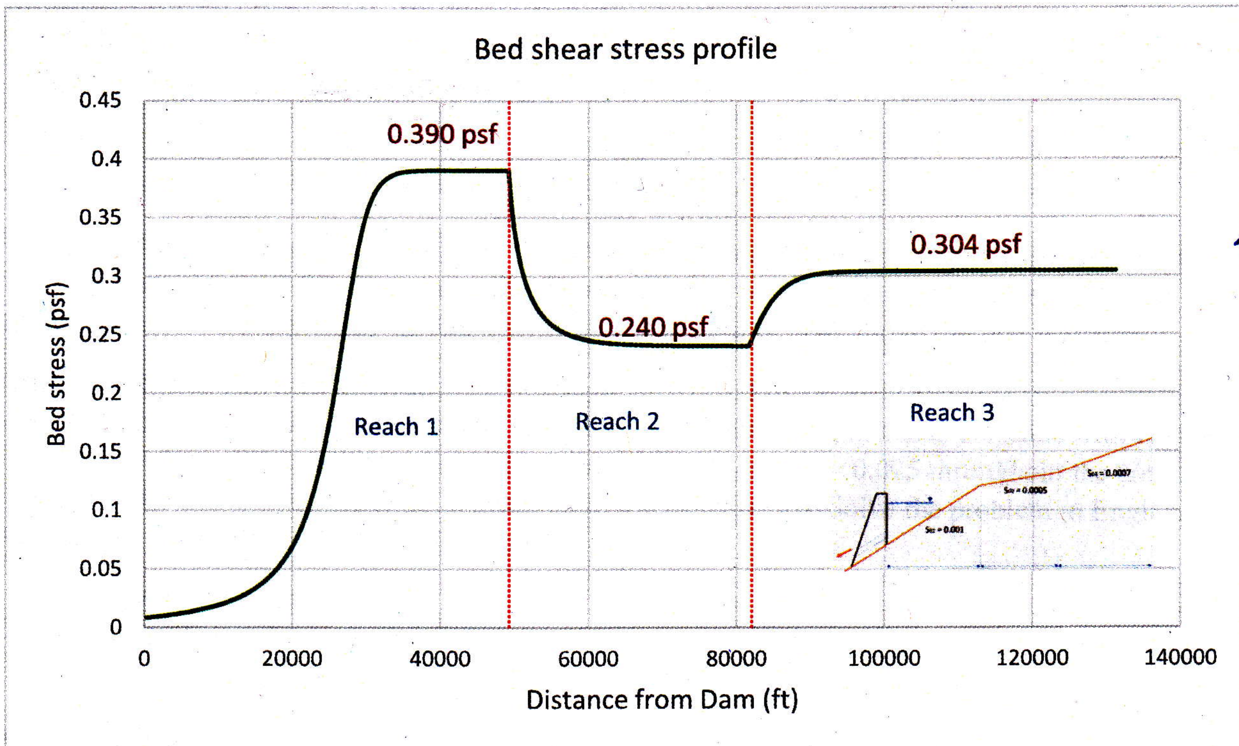


*Figure 5: Bed shear stress profile*

According to figure 5, beds experience the highest stress at reach 1 which is 37440 ft from the dam, and bed stress increases with distance from the dam. Shear stress rapidly varies near the dam due to rapid changes in flow depth. Reach 2 had the lowest normal bed shear, with decreasing bed stress as the distance from the dam increased. Bed shear increases with distance from the dam, but the gradient is less than one, and bed stress reaches its normal stress of 0.304 psf at 97950 ft from the dam.