13.3.4 Advection and dispersion in rivers

The propagation of contaminants in natural rivers can be analyzed from solving the advection-dispersion equations as

$$\frac{\partial C}{\partial t} = -U\frac{\partial C}{\partial x} + K\frac{\partial^2 C}{\partial x^2} - kC \qquad (13.5)$$

where U in m/s is the mean river flow velocity, K in m^2/s is the dispersion coefficient ($K \cong 12Uh$ where h is the flow depth) and k in s^{-1} is the settling rate of sediment or the contaminant decay rate [k = ω / h where ω is the settling velocity from Eq. (2.42)]. Two types of spills are considered: (1) an instantaneous spill; and (2) a continuous spill.

For the instantaneous spill, the mass of contaminant M in grams is tracked at a distance x downstream from the spill as a function of time t in a river of width W and depth h and mean flow velocity U . Once the sediment is fully mixed over the entire cross section area, i.e. $x > 10W^2 / h$, the contaminant concentration is estimated from

$$C(x,t) = \frac{M}{2Wh\sqrt{\pi K t}} e^{-\frac{(x-Ut)^2}{4Kt} - kt} \qquad (13.6)$$

The maximum concentration can be defined as a function of downstream distance x as

$$C_{\max} = \frac{M}{2Wh\sqrt{\pi K t}} e^{-kt} \qquad (13.7)$$

As an example, six metric tons of contaminants are instantaneously spilled into a river 20 m wide, 1 m deep and flowing at 1.5 m/s. Dispersion starts after lateral mixing is complete, i.e. $x > 10 \times 20^2 / 1 = 4$ km, with $K \cong 12Uh = 12 \times 1.5 \times 1 = 18$ m^2/s. Assuming settling of silt-size particles at $\omega = 1 \times 10^{-4}$ m/s and $k = \omega / h = 1 \times 10^{-4}$ s^{-1}, the maximum concentration 10 km from the spill is obtained at time

$t = x/U = 10,000/1.5 = 6,667$ s from Eq. (13.7) $C_{max} = \dfrac{6\times10^6}{2\times20\times1\sqrt{\pi\times18\times6,667}} e^{-6,667\times10^{-4}} = 125$ mg/l.

In the case of a continuous spill at a concentration C_{spill} and flow rate Q_{spill} in a river at a flow rate Q_{river} , the initial concentration is $C_0 = C_{spill}Q_{spill}/(Q_{spill}+Q_{river})$. A dimensionless settling parameter is defined as $\Gamma = \sqrt{1+4kK/U^2}$. The concentration for a constant contaminant release of duration *T* varies with distance *x* and time *t* as

$$C(x,t) = \frac{C_0}{2}\left\{ \begin{aligned} & e^{\frac{Ux(1-\Gamma)}{2K}}\left[erfc\left(\frac{x-Ut\Gamma}{2\sqrt{Kt}}\right) - erfc\left(\frac{x-U(t-T)\Gamma}{2\sqrt{K(t-T)}}\right)\right] \\ & + e^{\frac{Ux(1+\Gamma)}{2K}}\left[erfc\left(\frac{x+Ut\Gamma}{2\sqrt{Kt}}\right) - erfc\left(\frac{x+U(t-T)\Gamma}{2\sqrt{K(t-T)}}\right)\right] \end{aligned} \right\} \quad (13.8)$$

where erfc is the complementary error function $erfc(x) = 1 - \frac{2}{\sqrt{\pi}}\int_0^x e^{-a^2}da$ which is easily calculated with any mathematical package, or from the values given in Table 4.1. An example is shown in Fig. 13.12. For complex cases, the numerical simulation with the Leonard scheme in Chapter 7 is also very helpful in practice.

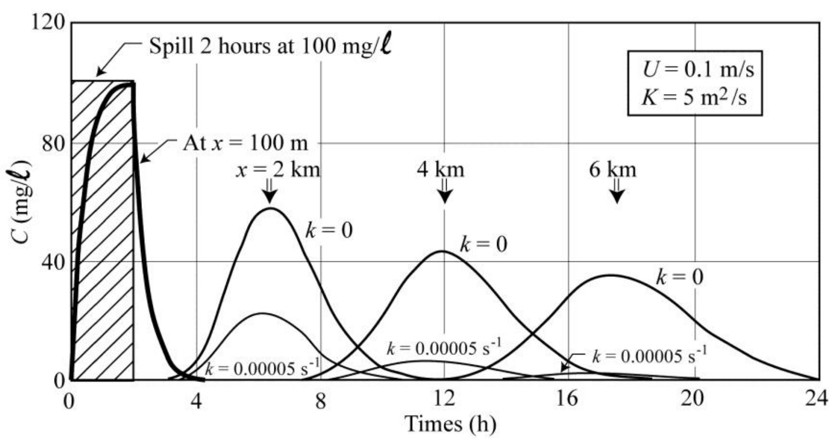

Figure 13.12. Example of advection-dispersion of contaminant

x	erf x	erf -x	erfc x	erfc -x
-2	-0.99532	0.995322	1.995322	0.004678
-1.9	-0.99279	0.99279	1.99279	0.00721
-1.8	-0.98909	0.989091	1.989091	0.010909
-1.7	-0.98379	0.98379	1.98379	0.01621
-1.6	-0.97635	0.976348	1.976348	0.023652
-1.5	-0.96611	0.966105	1.966105	0.033895
-1.4	-0.95229	0.952285	1.952285	0.047715
-1.3	-0.93401	0.934008	1.934008	0.065992
-1.2	-0.91031	0.910314	1.910314	0.089686
-1.1	-0.88021	0.880205	1.880205	0.119795
-1	-0.8427	0.842701	1.842701	0.157299
-0.9	-0.79691	0.796908	1.796908	0.203092
-0.8	-0.7421	0.742101	1.742101	0.257899
-0.7	-0.6778	0.677801	1.677801	0.322199
-0.6	-0.60386	0.603856	1.603856	0.396144
-0.5	-0.5205	0.5205	1.5205	0.4795
-0.4	-0.42839	0.428392	1.428392	0.571608
-0.3	-0.32863	0.328627	1.328627	0.671373
-0.2	-0.2227	0.222703	1.222703	0.777297
-0.1	-0.11246	0.112463	1.112463	0.887537
6.38E-16	7.2E-16	-7.2E-16	1	1
0.1	0.112463	-0.11246	0.887537	1.112463
0.2	0.222703	-0.2227	0.777297	1.222703
0.3	0.328627	-0.32863	0.671373	1.328627
0.4	0.428392	-0.42839	0.571608	1.428392
0.5	0.5205	-0.5205	0.4795	1.5205
0.6	0.603856	-0.60386	0.396144	1.603856
0.7	0.677801	-0.6778	0.322199	1.677801
0.8	0.742101	-0.7421	0.257899	1.742101
0.9	0.796908	-0.79691	0.203092	1.796908
1	0.842701	-0.8427	0.157299	1.842701
1.1	0.880205	-0.88021	0.119795	1.880205
1.2	0.910314	-0.91031	0.089686	1.910314
1.3	0.934008	-0.93401	0.065992	1.934008
1.4	0.952285	-0.95229	0.047715	1.952285
1.5	0.966105	-0.96611	0.033895	1.966105
1.6	0.976348	-0.97635	0.023652	1.976348
1.7	0.98379	-0.98379	0.01621	1.98379
1.8	0.989091	-0.98909	0.010909	1.989091
1.9	0.99279	-0.99279	0.00721	1.99279

♦♦Problem 13.10

Marcos Palu (pers. Comm.) reported that after the collapse of Fundao Dam in Brazil, the sediment concentration in the Doce River reached a value of 580g/l for about 6 hours. Consider the following river characteristics: river width 130m, flow depth 3.5 m, flow velocity 1.1 m/s, bed slope 0.0005, and shear velocity 0.13 m/s. Use Eq. 13.8 with the following dispersion coefficient K_d = 150 m^2/s and the sediment settling rate k = 0.0000036 s^{-1} to estimate the sediment concentration as a function of time at Oculos station located 94 km downstream. Compare the results with k =0.

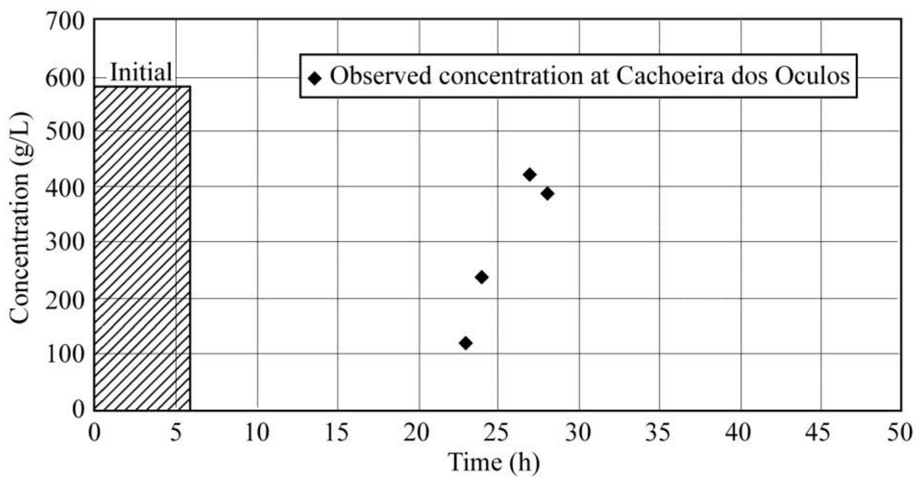

Figure P.13.10. Dispersion of sediment in the Doce River