PLANFORM GEOMETRY OF MEANDERING ALLUVIAL CHANNELS

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TABLE OF CONTENTS

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LIST OF FIGURES

LIST OF SYMBOLS

a	exponent of the flow resistance relationship
$a_o, a_1, \ldots a_m$	coefficients of the Fourier series
A	cross-sectional surface area of the stream
A_m	amplitude of the meander
d_b	sediment size of the bank material
d_s	sediment size of the bed material
E	energy integral equation
G	relative density of solid particles
g	gravitational acceleration
h	average flow depth
ΔH	energy loss in a single meander wavelength
ℓ	Prandtl mixing length
q_ℓ	sediment transport capacity in the longitudinal direction
q_t	sediment transport capacity in the transversal direction
$\bar{q}_\ell$	longitudinal sediment transport
$\bar{q}_t$	transversal sediment transport
r	radius of curvature
r_m	minimum radius of curvature
S	total path distance in a single wavelength
s	curvilinear coordinate or unit distance along the path
s^*	nondimensional curvilinear distance
S_f	friction slope
S_w	water surface slope in the transversal direction
t	time
u	depth-integrated downstream velocity in a curved channel

$\bar{u}$	mean velocity
v	Depth-integrated velocity profile in a straight channel
w	transversal coordinate
W	channel width
x	rectilinear longitudinal coordinate or downvalley coordinate
y	rectilinear transversal coordinate
z	vertical coordinate
β	coefficient
γ	specific weight of water
η	dynamic eddy viscosity
Θ	direction angle between path and downvalley direction
Θ_m	maximum direction angle
ϕ_ℓ	longitudinal Shields number
ϕ_t	transversal Shields number
λ	downvalley wavelength
μ	dynamic viscosity of water
ρ	density of water
τ	shear stress
τ_s	side shear stress in the longitudinal direction
τ_o	bed shear stress in the longitudinal direction
τ_t	shear stress in the transversal direction
τ_{to}	bed shear stress in the transversal direction

I. INTRODUCTION

Streams are very broadly classified as meandering, straight or transitional, and braided. Braided and meandering patterns represent extremes in a continuum of channel patterns. The planform geometry of a stream is determined by the interaction of numerous variables and one should anticipate to observe a complete range of channel patterns in most river systems (Simons and Julien, 1984). Various planform properties of meandering rivers were classified by Brice (1984).

This report is to emphasize analysis of meandering channels. Equilibrium planform geometry rarely develops when natural flow and sediment supply variations are considered. In this report, the analysis of a pseudo-equilibrium state provides insight into fundamental geometric shape, shear stresses distribution and sediment transport. A theoretical analysis based on the principal variables affecting the geometry of alluvial streams is presented to yield a discussion on the planform geometry of meanders, the variation of the radius of curvature, the sinuosity, the longitudinal and transversal shear stresses, the energy grade line and the longitudinal and transversal components of sediment transport.

II. LITERATURE REVIEW

Previous studies on meandering streams have been summarized by Rozovskii (1957), Graf (1971), Chitale (1973), Engelund and Skovgaard (1973), Callander (1978), and Engelund and Fredsøe (1982). Besides the formation of meanders in alluvial channels, evidences of meandering in ice, bedrock, density currents, hydrophobic surfaces and flow of the Gulf Stream were reported by Leopold and Wolman (1960), Leopold et al.

(1964), Dury (1965), Zeller (1967), Gorycki (1973), Parker (1975), and Nakagawa and Scott (1984).

Numerous hypothesis have been suggested to explain the origin of meandering, namely: Coriolis force resulting from earth rotation, transverse flow oscillations caused by local flow disturbances, secondary flow, turbulence, wave motion, bank erosion and bar formation, energy dissipation and minimum variance. Previous studies can be broadly classified under one of the following categories: a) regime approach, b) minimum stream power, c) statistical theory and spectral analysis, d) secondary currents and e) stability analysis.

2.1 Regime Approach and Morphology

After replacing the word "equilibrium" with "regime," the regime approach was developed by Kennedy (1895), Lindley (1919), Lacey (1929), Lane (1937), and Blench (1969, 1972). Several empirical relationships supported by field observations were derived to define the geometry of alluvial channels. Simons and Albertson (1963) differentiated several channel conditions and their graphical relationships were supported analytically by Henderson (1966). From dimensional analysis and physical reasoning, several authors, Chien (1957), Henderson (1961), Stebbins (1963), and Gill (1968) have presented some physical support to the regime equations. Brice (1984) recently proposed a classification of planform properties of meandering rivers. Hey (1978, 1984) discussed the geometry of river meanders and Schumm (1963, 1967, 1977, 1984) offers a geomorphologic approach to the meandering problem. An analysis of downstream hydraulic geometry relationship based on similitude in alluvial channels is presented by Julien and Simons (1984).

2.2 Minimum Stream Power

The principle of minimum variance was first exposed by Langbein and Leopold (1966). Accordingly, meandering is the most probable form of a channel. Its geometry is more stable than a nonmeandering alignment. Though it does not explain the physical processes, the net behavior of a river can be described. The minimization involves the adjustment of the planimetric geometry and the hydraulic factors of depth, velocity, and local slope. Yang (1971a, 1976) stated that the time rate of energy expenditure explains the formation of meandering streams. He also describes alluvial processes in terms of minimum stream power. Other studies by Maddock (1970), Chang and Hill (1977), and Chang (1979b, 1980, 1984) use the principle of minimum stream power. Chang (1979a) concluded that a meandering river is more stable than a straight one as it expends less stream power per unit channel length for the system. Onishi et al. (1976) also suggest that meandering channels can be more efficient than straight ones because for a given water discharge a smaller energy gradient is required to transport a larger sediment load. As summarized by Cherkauer (1973), streams adjust their flow so as to minimize total power expenditure, and to minimize the sums of variances of power and of the dependent variables.

2.3 Statistical Theory and Spectral Analysis

Thakur and Scheidegger (1968) analyzed the probability for a stream to deviate by an angle $d\theta$ in progressing an elemental distance ds along its course. Their statistical study confirms the probabilistic view of meander development suggested by Langbein and Leopold (1966). Further developments were provided by Surkan and Van Kan (1969) showing that neither the directions, curvatures, nor their changes in natural meanders are Gaussian independent. Spectral analysis of meanders by Speight (1965), Ferguson (1975), Dozier (1976) and Sinnock and Rao (1983) indicate that the characteristic meander wavelength is a poor indicator of the dominant frequencies of oscillation. As pointed out by Thakur and Scheidegger (1970), there seems to be more than one characteristic wavelength in a meander system.

2.4 Secondary Currents

According to Quick (1974), the meander mechanism is basically a fluic mechanics problem in which vorticity plays a leading role. Flow in a meander bend has been studied in detail by Rozovskii (1957), Yen (1972) and others. The problem is extremely complex and the Navier-Stokes Equation must be simplified to obtain theoretical approximation. Rouse (1965) recognized that the energy gradient of flow in a meandering channel is Froude number dependent. Einstein and Li (1958) made a theoretical investigation of secondary currents under laminar and turbulent conditions. Einstein and Shen (1964) recognized two types of meander patterns in straight alluvial channels with nonerodible banks: 1) those when the flow is nearly critical; and 2) alternating scour holes flow between rough banks.

The motion of fluid in a curved channel is generally based on the equations of motion. Different approaches are suggested by Rozovskii (1957), Yen (1972, 1984), Dietrich and Smith (1983, 1984), DeVriend (1977), Engelund (1974), Chiu and Hsiung (1981), Falcon and Kennedy (1983), Odgaard (1981, 1983), Olesen (1984) and others.

2.5 Stability Analysis

Several attempts have been made to explain the origin of meandering. Callander (1969) pointed out that straight bank channels with loose boundaries are unstable with the possible exception of channels just beyond the threshold of grain movement. The stability of the sediment-water interface was presented by Exner (1925). Einstein (1926) described the effect of earth rotation and Coriolis forces to induce circulation. An analytical approach to local disturbances was presented by Werner (1951). A similar relationship for meander length was also derived from the concept of transverse oscillations by Anderson (1967). He concluded that meander length is related to the Froude number and that no unique relationship exists between meander length and discharge.

Adachi (1967) and Hayashi (1970) used small amplitude oscillation techniques to explain the origin of meandering. Engelund and Skovgaard (1973) developed a three-dimensional model to analyze the hydrodynamic stability of a straight alluvial channel. Parker (1976) used a perturbation technique involving the ratio of sediment transport to water transport in a straight reach. He concluded that existence of sediment transport and friction are necessary for occurrence of instability. He also suggested that in absence of sediment load the origin of sinuosity is purely hydrodynamic. The theoretical work by Parker et al. (1976, 1982, 1983) and Ikeda et al. (1981, 1984) prepared the simulation models developed by Beck (1984), Parker (1984), Howard (1984) and Ferguson (1984). The data collected by Nanson and Hickin play an important role in these models.

Local disturbances, earth rotation, excessive energy and hydrodynamic stability figure among the best explanations available so far. What causes meandering is still a question without a complete answer, although the explanation based on dynamic stability is promising.

III. VARIABLES AND GEOMETRY RELATIONSHIPS

This chapter deals with the description of the most probable planform geometry of alluvial meandering streams. In the first section the principal variables are introduced while mechanical concepts, energy dissipation and variational principles are discussed in the following three sections. The last section introduces three important variables in meander geometry, namely the radius of curvature, the wavelength and the sinuosity.

3.1 Variables

A reach of meandering alluvial stream is schematized in Fig. 1 to illustrate the principal variables describing sinuous patterns. Two systems of coordinates are defined: one rectilinear and one curvilinear. The principal axis x in the rectilinear system defines the center line of the meandering pattern downstream the valley slope. In the curvilinear system, the sinuous axis s follows the center line of the meandering path of the stream in the longitudinal direction. The radius of curvature r in the transversal direction w remains orthogonal to the principal curvilinear axis. Both the magnitude and direction of the radius of curvature vary along the path of the channel of width W and mean velocity $\bar{u}$. These variables are used in the following illustration of the concept of separation of the boundary layer near the river bank.

3.2 Flow Separation and Planform Geometry

The concept of flow separation near the channel boundary plays a significant role in the rate of energy expenditure in a bend and also exerts some control on the planform geometry of the channel. Flow separation occurs when an adverse pressure gradient deflects the

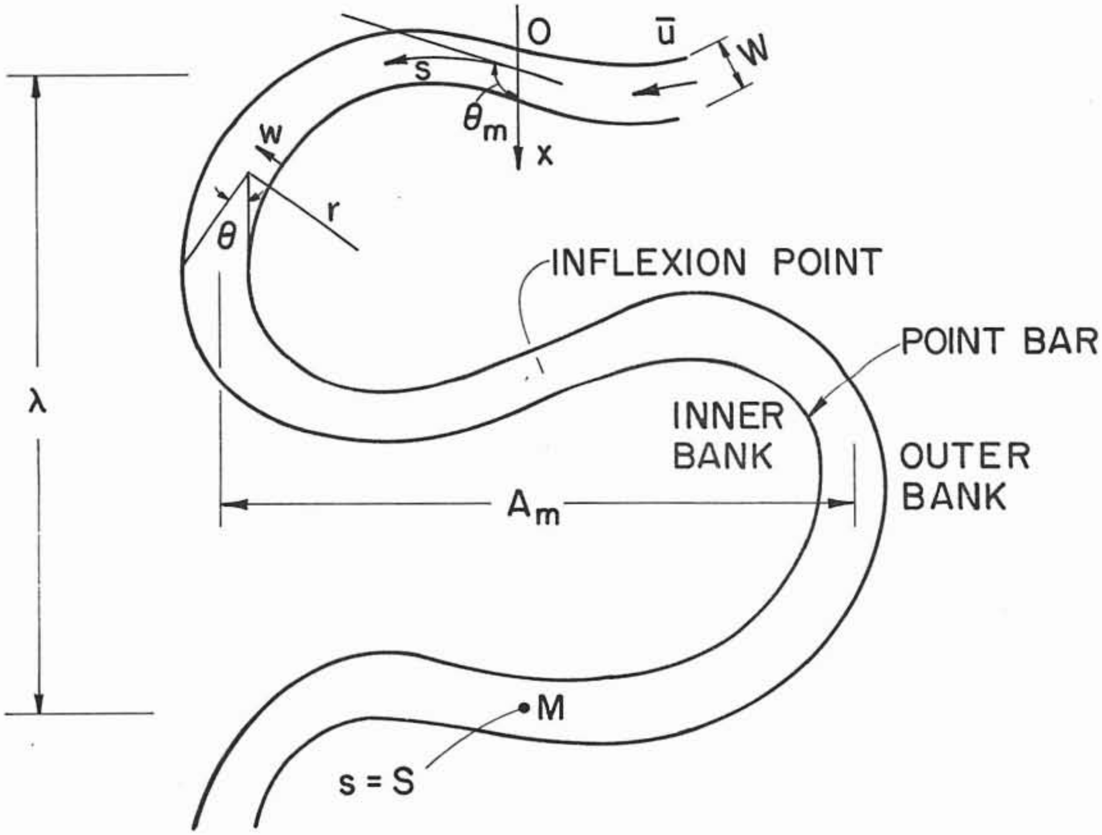


Figure 1. Definition sketch of a meandering alluvial stream.

boundary layer sideways into the main stream. The fluid particles generally move upstream in the zone of adverse pressure behind the point of separation. Therefore, the equations describing the boundary layer are only valid upstream the point of separation and the energy losses in the eddies generated by the separation of the boundary layer are significant. In a sediment laden alluvial stream, the phenomenon of separation induced by a local perturbation of the flow near the boundary is alleviated by the deposition of sediments in the separation zone. The geometry profile of the inner bank is gradually modified until new equilibrium conditions are reached without separation of the boundary layer. The point of separation near a boundary is mathematically defined as the limit between the downstream and the upstream flows as:

$$\left.\frac{du}{dw}\right|_{w=o} = 0 \tag{1}$$

in which u is the time-averaged velocity near the boundary and w is the transversal coordinate.

In the case of a meandering alluvial channel, the direction of the mean velocity follows the sinuous path given by the planform geometry illustrated in Fig. 2. After a given time increment dt along the bend, the downstream velocity profile between points A and B has rotated by an angle $d\Theta$ giving $d\Theta/dt$ for the corresponding angular velocity. The criterion to describe similar conditions of flow separation for similar streams is:

$$\frac{du}{dw} \sim \frac{dv}{dw} + \frac{d\Theta}{dt} \quad . \tag{2}$$

in which, dv/dw refers to the velocity profile in a straight channel. This velocity gradient is assumed to be invariant in the downstream

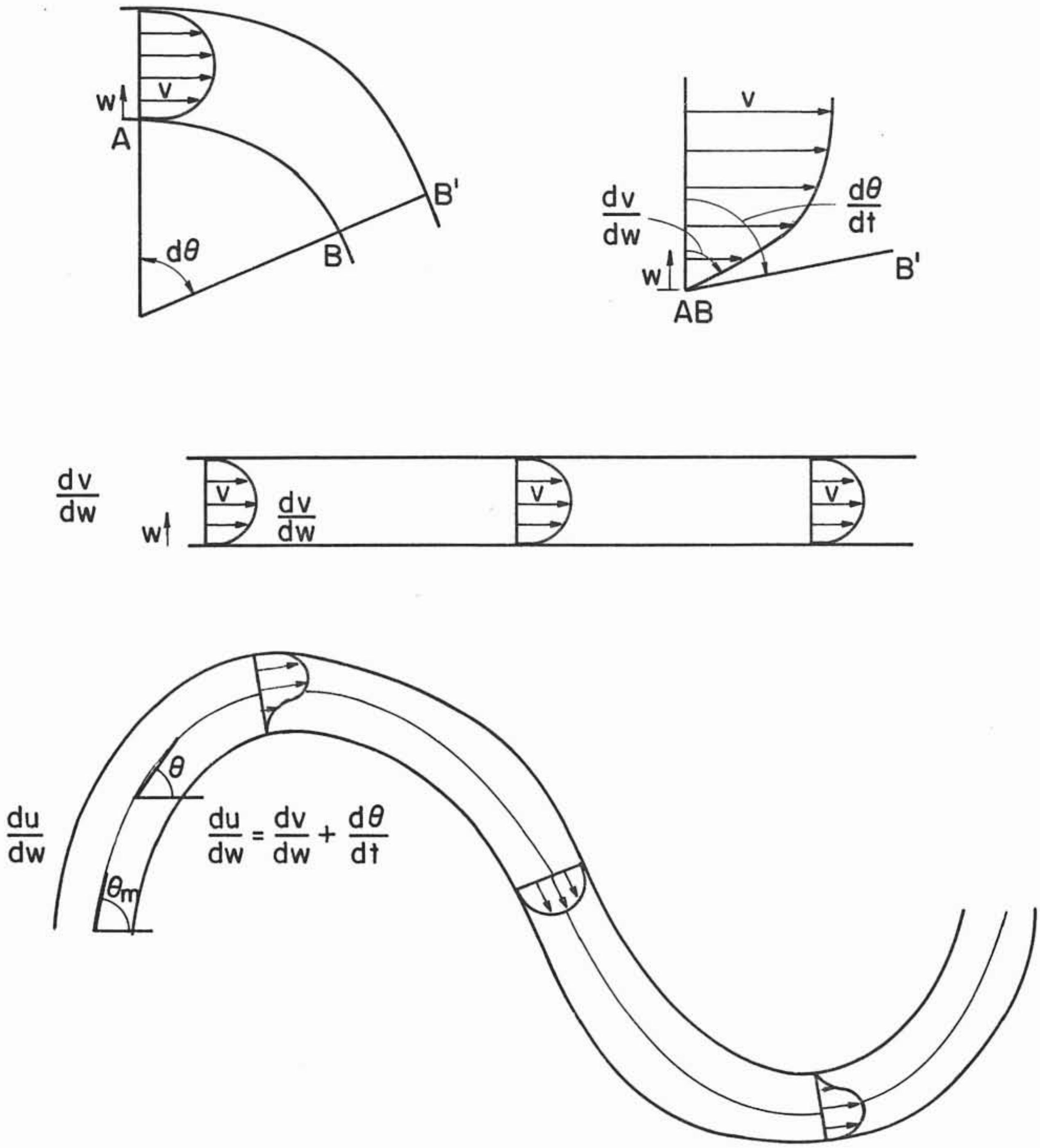


Figure 2. Concept of boundary layer separation near the inner bank.

direction while the variation of $d\Theta/dt$ is defined from the following two fundamental geometry relationships:

$$ds = \bar{u}dt \quad , \tag{3}$$

$$r = \frac{ds}{d\Theta} \quad . \tag{4}$$

After combining Eqs. 3 and 4, the angular velocity transforms to:

$$\frac{d\Theta}{dt} \sim \frac{\bar{u}}{r} \tag{5}$$

From Eqs. 2 and 5, the resulting shape of the velocity profiles is therefore skewed toward the outer bank a shown in Fig. 2.

3.3 Energy Dissipation and Planform Geometry

In a meandering alluvial channel the bed topography and the complex nature of secondary circulation obscures the evaluation of bed shear stress and rate of energy dissipation along the meander. A first-order approximation of the rate of energy dissipation in a meandering channel is obtained using the following approach. It is assumed that the channel geometry can be simplified such that the bed shear stress component is differentiated from the shear stress acting on the bank in the downstream direction. The energy-integral along the bed should remain more or less similar to the energy integral for a straight channel. The component due to shear stress exerted on the sides of a channel is then considered. In a meandering turbulent stream the longitudinal shear stress exerted on the sides of the channel τ_s can be approximated by a function of the velocity gradient as follows:

$$\tau_s \simeq \mu \frac{du}{dw} - \rho u'v' \tag{6}$$

in which μ is the dynamic viscosity of water and $-\rho u'v'$ is the turbulent shear stress component.

The component of energy per unit weight and unit time which is transformed into heat due to bank shear stress in a meandering stream takes the form of the energy integral equation E for turbulent flows (Schlichting, 1968):

$$E \sim \frac{1}{WSg} \int_{o}^{S} \int_{o}^{W} \frac{\tau_s}{\rho} \frac{du}{dw} \, dw \, ds \quad . \tag{7}$$

Taking a constant channel width W, we assume the angular velocity $d\Theta/dt$ to be independent of w for a given curvilinear meandering wavelength S. After combining Eqs. 2, 6 and 7, it is shown that the function dv/dw is positive on one side of the channel and negative on the other side. Therefore, the integration of functions with even exponents of this function vanishes except when r/w is very small because of the symmetry of the planform geometry profile. The integration of dv/dw across the channel width and $d\Theta/dt$ along the channel length should be small compared to the main component of the energy integral due to the symmetry of the planform geometry. The energy integral equation therefore transforms to the first-order approximation:

$$E \sim \int_{s} \left(\frac{d\Theta}{dt}\right)^2 ds + C \quad . \tag{8}$$

After selecting the origin at point 0 in Fig. 1 any periodic function describing the meandering path of a stream by the spatial variation of $d\Theta/dt$ can be written in terms of a Fourier series of the form:

$$\frac{d\Theta}{dt} = \frac{a_o}{2} - \sum_{m=1}^{\infty} a_m \sin \frac{2m\pi s}{S} \tag{9}$$

in which $a_o, a_1, \ldots, a_n$ are the coefficients of the Fourier series. For any planform geometry the meandering energy dissipation component is obtained after substituting Eq. 9 into Eq. 8. The trivial solution to the minimization of the energy dissipation component corresponds to zero values for all the coefficients of the Fourier series. The coefficient a_o gives a constant rate of energy dissipation and the most interesting situation arises when only one coefficient, a_1 for example, differs from zero. The rate of energy dissipation is then less than any other function for which any of the coefficients besides a_1 would be non-zero. In other words, among all possible meandering planform geometry except straight channels, the simple sine-generated function minimizes the meandering component of the rate of energy dissipation. The corresponding meandering pattern can be written:

$$\frac{d\Theta}{dt} = - a_1 \sin \frac{2\pi s}{S} \quad . \tag{10}$$

This equation can be integrated for Θ, assuming that the velocity given by Eq. 3 remains constant along the meander length:

$$\Theta = - \frac{a_1}{\bar{u}} \int_o^s \sin \frac{2\pi s}{S} \, ds = \frac{Sa_1}{2\pi\bar{u}} \cos \frac{2\pi s}{S} + C \tag{11}$$

in which the maximum angle Θ_m corresponds to:

$$\Theta_m = \frac{Sa_1}{2\pi\bar{u}} \quad . \tag{12}$$

These results are similar to those obtained by Langbein and Leopold (1966) using a different approach based on the principle of minimum variance.

3.4 Meandering as a Variational Problem

The problem of meandering can also be treated as a variational problem for which we seek the extremum of a functional. Taking the energy-integral given by Eq. 8 as the functional of the variational problem, the derivation found in Elsgolts (1977) yield the solution

$d\Theta/dt = 0$, which is similar to the trivial solution given by the Fourier series. On the other hand, after the sine function (Eq. 10) is combined with Eq. 8, integration along the distance s gives:

$$E(s) = \int_o^s a_1^{\,2} \sin^2 \frac{2\pi s}{S}\, ds + C \quad , \tag{13}$$

$$E(s) = a_1^{\,2}\, s - a_1^{\,2} \int_o^s \cos^2 \left(\frac{2\pi s}{S}\right) ds + C \quad , \tag{14}$$

$$E(s) = \frac{a_1^{\,2}\, s}{2} - \frac{S a_1^{\,2}}{8\pi} \sin \frac{4\pi\, s}{S} + C \ . \tag{15}$$

This relationship indicates that the rate of energy dissipation is not uniformly distributed along the meandering distance s, though the sine function in Eq. 15 vanishes at four points along a meander wavelength, respectively when $\Theta = 0$ and $d\Theta/dt = 0$. From Eq. 14, a modified energy function $E^*(s)$ is defined as follows:

$$E^*(s) = E(s) + a_1^{\,2} \int_o^s \cos^2 \left(\frac{2\pi s}{S}\right) ds = a_1^{\,2}\, s + C \tag{16}$$

Using Eqs. 11 and 8, Eq. 16 transforms to:

$$E^*(s) = \int_o^s \left(\frac{d\Theta}{dt}\right)^2 ds + \frac{4\pi^2\, \bar{u}^{-2}}{S^2} \int_o^s \Theta^2\, ds + C \tag{17}$$

The modified energy function is linear with s, while Eq. 17 is equivalent to the energy equation of a mass-spring system. Indeed, the system transforms to the following equation for free oscillations:

$$\frac{d^2\Theta}{dt^2} + \frac{4\pi^2\, \bar{u}^{-2}}{S^2}\, \Theta = 0 \quad . \tag{18}$$

After selecting the origin as shown in Fig. 1, the solution is identical to Eq. 11. It can also be demonstrated (Elsgolts, 1977; Goldstein, 1981; Sokolnikoff and Redheffer, 1966) that Eq. 18 represents the Euler-Lagrange differential equation of the following functional:

$$F[\Theta(s)] = \int_{o}^{s} [(\frac{d\Theta}{dt})^2 - \frac{4\pi^2 \bar{u}^{-2}}{S^2} \Theta^2] \, ds \tag{19}$$

with the boundary conditions: $\Theta(0) = \Theta_m$; $\Theta(S/4) = 0$.

The first term of this functional is equal to the energy-integral (Eq. 8) and represents the kinetic energy of the system while the second term refers to the potential energy. It is concluded that under this form the system optimizes the difference between kinetic and potential energy while keeping the sum of these components E^* linear with s. The result in terms of the geometry of a meandering alluvial stream is similar to the sine-generated curve provided by Eqs. 10 and 11.

The sine-generated curve suggested by Langbein and Leopold (1966) has been tested with observed meandering patterns and some conclusive results are presented in Fig. 3. The planform geometry is similar to Fargue's spiral and Von Schelling's curve and the sine-generated curve is widely accepted as the simplest and most convenient representation of symmetrical meanders. Ferguson (1973) showed that typical irregular meander patterns can be simulated by a disturbed periodic model which reduces to the sine-generated curve as the irregularity becomes vanishingly small. Carson and Lapointe (1983) suggested that meander planform is inherently asymmetrical but the author believes that asymmetrical meanders can be simulated by Fourier series (Eq. 9) with several nonzero coefficients. The so-called Kinoshita equation has been used by Parker

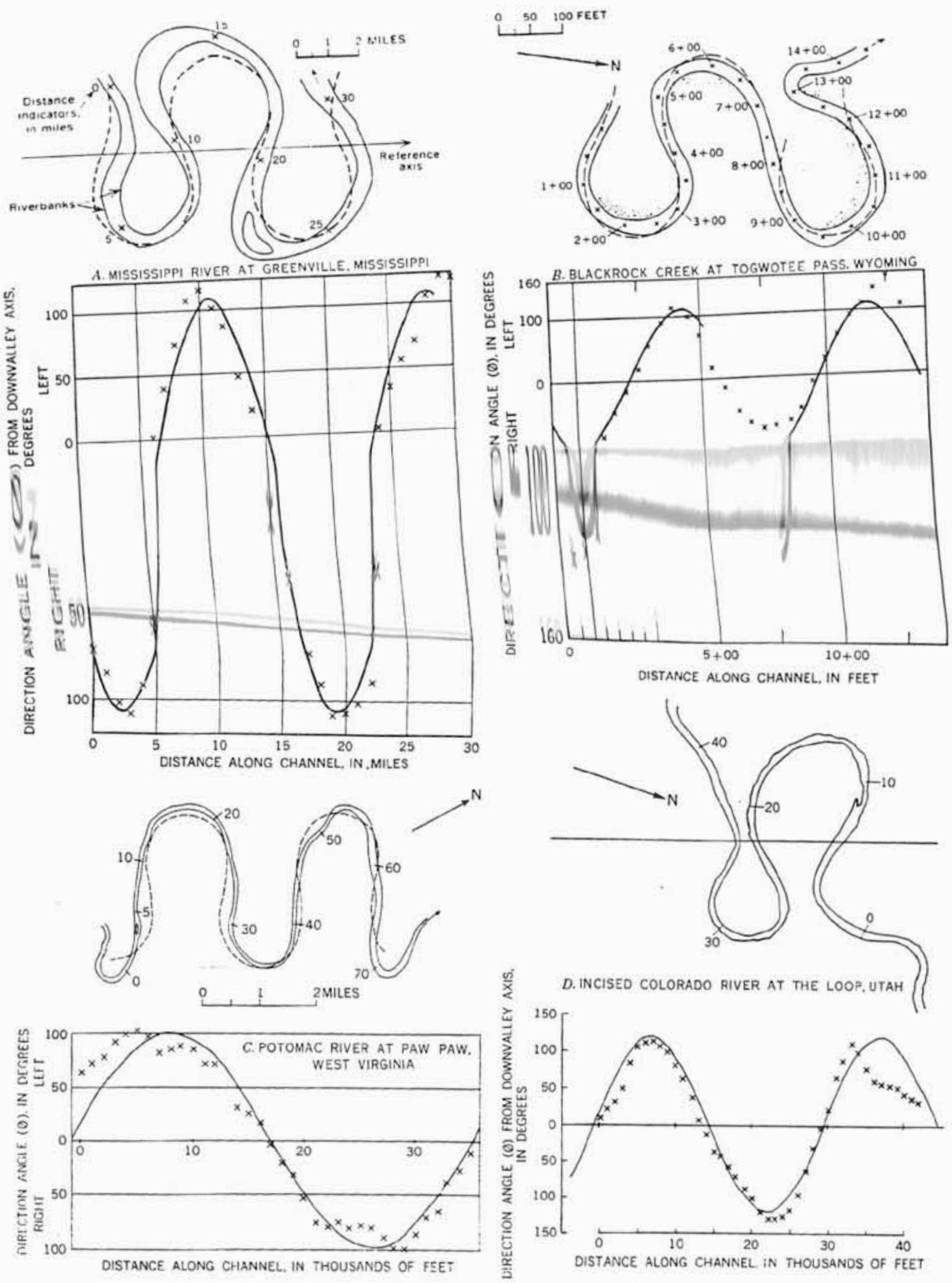
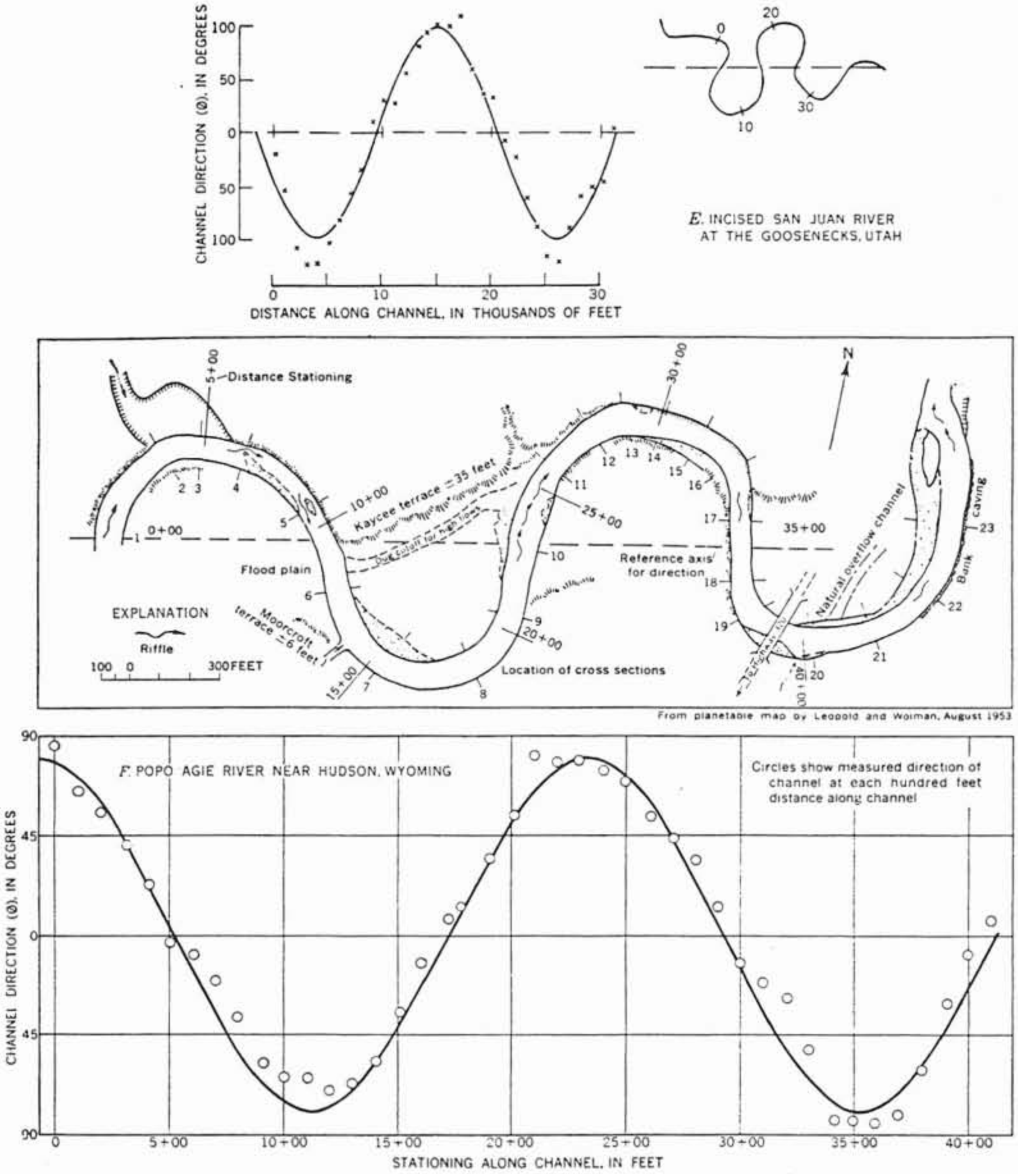


FIGURE 3.—MAPS OF RIVER CHANNELS AND

A, Upper, Map of channel compared to sine-generated curve (dashed); *lower*, plot of actual channel direction against distance (crosses) and a sine curve (full line).
B. Upper, Map of channel compared to sine-generated curve (dashed); *lower*, plot of actual channel direction against distance (crosses) and a sine curve (full line).
C, Upper, Map of channel compared to sine-generated curve (dashed); *lower*, plot of actual channel direction against distance (crosses) and a sine curve (full line).
D, Upper, Planimetric map of river; *lower*, plot of actual channel direction against distance (crosses) and a sine curve (full line).

Figure 3. Meandering channel geometry (after Langbein and Leopold, 1966).



PLOTS OF CHANNEL DIRECTION AGAINST DISTANCE

E, Right, Planimetric map of river; *left*, plot of actual channel direction against distance (crosses) and a sine curve (full line). *F, Upper*, Planimetric map of river; *lower*, plot of actual channel direction against distance (crosses) and a sine curve (full line).

Figure 3. Meandering channel geometry (after Langbein and Leopold, 1966) (continued).

(1984) for an analytical formulation of bend deformation. This equation reduces to the sine-generated curve at small amplitude and displays skewness at high amplitude. The Kinoshita equation and Fourier series used by Yamaoka and Hasegawa (1984) are considered as extensions of the fundamental mode described by the sine-generated curve. Perhaps the deviations from the fundamental mode are related to the complex erosion and deposition processes observed in meander bends as stage and discharge vary with time.

The properties of the fundamental mode described by the sine-generated curve are discussed in terms of geometric, dynamic and sediment transport characteristics.

3.5 Radius of Curvature, Wavelength, Amplitude and Sinuosity

The radius of curvature varies along the bend as demonstrated from combining Eqs. 5, 10, and 12:

$$r = - \frac{S}{2\pi\Theta_m} \operatorname{cosec} \frac{2\pi s}{S} \quad . \tag{20}$$

The minimum radius of curvature r_m is reached when the cosecant function equals unity or:

$$r_m = \frac{S}{2\pi\Theta_m} \quad . \tag{21}$$

This relationship demonstrates that the ratio S/r_m varies linearly with Θ_m.

The meander wavelength is computed from the following relationship:

$$\lambda = \int_o^\lambda dx = \int_o^S \cos \Theta \, ds \quad . \tag{22}$$

The nondimensional curvilinear distance $s^* = \frac{s}{S}$ is defined and Eqs. 11 and 12 are substituted for Θ and ds in Eq. 22 to give:

$$\lambda = S \int_o^1 \cos[\Theta_m \cos(2\pi s^*)]\, ds^* \quad . \tag{23}$$

The sinuosity is defined as to the ratio and S/λ increases gradually with Θ_m as illustrated in Fig. 4. The ratio λ/r_m of the wavelength to the minimum radius of curvature is obtained from Eq. 21:

$$\frac{\lambda}{r_m} = \frac{2\pi\Theta_m\lambda}{S} \quad . \tag{24}$$

For a given wavelength λ the minimum radius of curvature corresponds to the maximum value of λ/r_m which varies with Θ_m as follows:

$$\frac{\lambda}{r_m} = 2\pi\Theta_m \int_o^1 \cos\,[\Theta_m \cos\, 2\pi s^*]\, ds^* \quad . \tag{25}$$

After integration, this ratio varies with Θ_m as shown in Fig. 4 and reveals that the minimum radius of curvature for a given meander wavelength correspond to the maximum angle $\Theta_m = 75°$. Consequently, the increase in radius of curvature beyond this maximum Θ_m constitutes an extremely important feature since the radius of curvature controls the magnitude of the centrifugal force in bends. This feature will be analyzed in detail in the following section dealing with the dynamics of flow in bends based on force equilibrium.

The amplitude of the meander A_m as defined in Fig. 1 is evaluated analytically by the following integral:

$$A_m = 2 \int_o^{S/4} \sin\Theta\, ds \quad . \tag{26}$$

Using the nondimensional curvilinear distance $s^* = s/S$, and Eqs. 11 and 12 for Θ and ds, the ratio of amplitude to the distance S is:

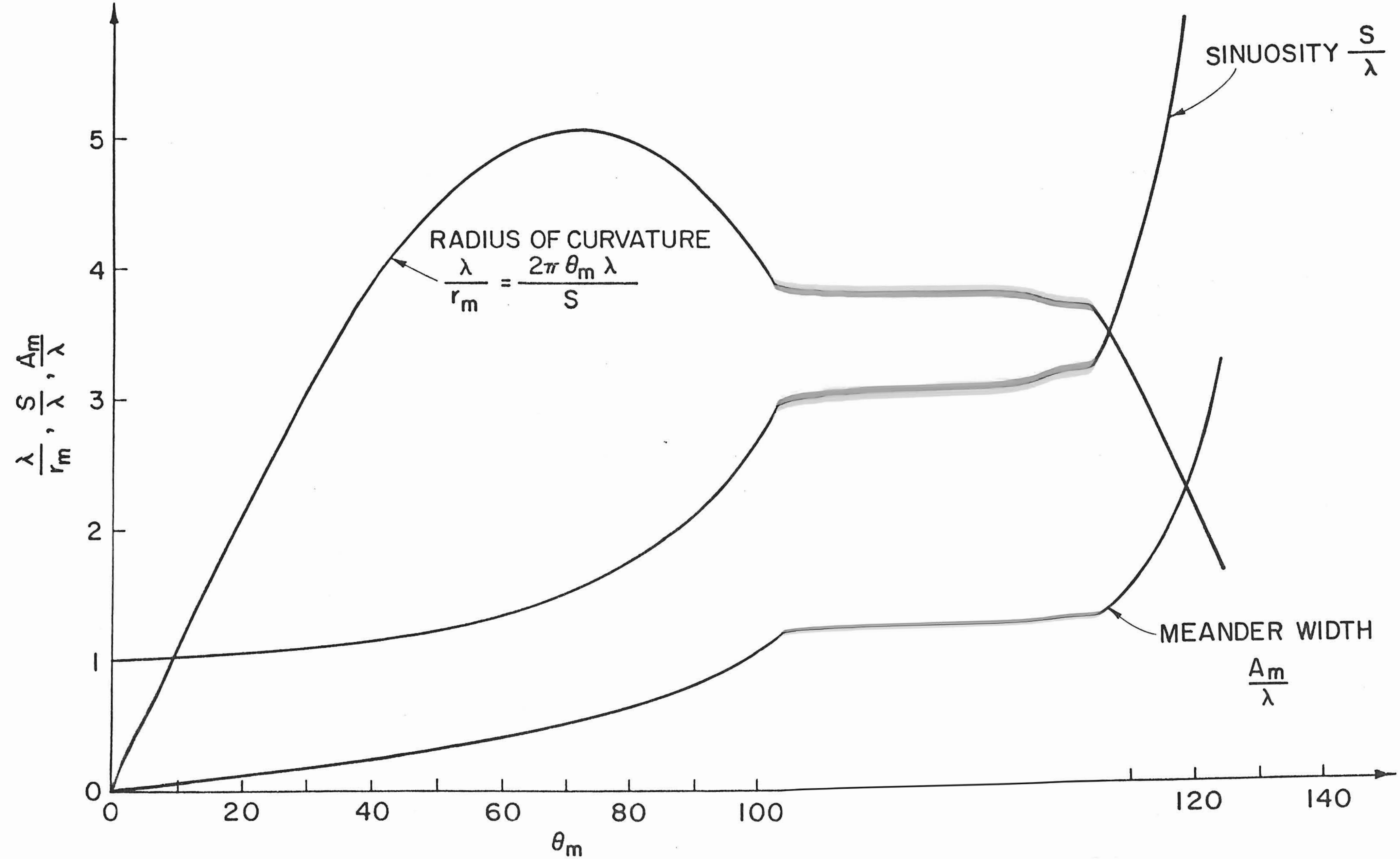


Figure 4. Sinuosity, meander width and minimum rad[illegible]us of curvature.

$$\frac{A_m}{S} = 2 \int_o^{1/4} \sin[\Theta_m \cos(2\pi s^*)]\, ds^* \quad . \tag{27}$$

The ratio of meander amplitude A_m to the wavelength λ is more convenient and can be obtained after S is cancelled between Eqs. 23 and 27:

$$\frac{A_m}{\lambda} = \frac{2 \int_o^{1/4} \sin[\Theta_m \cos(2\pi s^*)]\, ds^*}{\int_o^{1} \cos[\Theta_m \cos(2\pi s^*)]\, ds^*} \quad . \tag{28}$$

Equation 28 has been integrated numerically and the ratio A_m/λ is plotted in Fig. 4 as a function of θ_m. As expected, this ratio increases rapidly when θ_m exceeds 90°, and reaches the value 3.25 at the meander cutoff.

IV. DYNAMIC EQUILIBRIUM AND SEDIMENT TRANSPORT

The motion of water in a meandering channel can be subdivided into two components. The longitudinal component in the s direction is nearly uniform or gradually varied while the transverse component in the w direction varies significantly over a meander wavelength.

4.1 Longitudinal Equilibrium

The primary source of energy to be expended by flowing water is controlled by the energy gradient of the valley given by the ratio of the energy loss ΔH over a meander wavelength λ. The flow characteristics in the alluvial channel, however, are dependent upon the friction slope in the stream corresponding to the energy loss ΔH over the meandering path of the sinuous stream S:

$$S_f = \frac{\Delta H}{S} \quad . \tag{29}$$

The following flow resistance relationship is used for gradually varied steady flow conditions:

$$\bar{u} \sim \sqrt{g}\ (\frac{h}{d_s})^a\ h^{\frac{1}{2}}\ S_f^{\frac{1}{2}} \tag{30}$$

in which, h is the average flow depth; d_s is the bed material size; and a is the exponent of the resistance equation. As demonstrated by Julien and Simons (1984), the exponent a corresponding to the Keulegan logarithmic velocity profile is a function of the relative roughness in a turbulent flow and it can be written as:

$$a = \frac{1}{\ln\ (\frac{12.2h}{d_s})} \quad . \tag{31}$$

The Chézy equation corresponds to a=0 while a=1/6 gives the Manning-Strickler equation. For very rough channel boundaries such as gravel-bed rivers, the exponent a can be larger than 0.2. Substituting S_f from Eq. 29 into Eq. 30 gives:

$$\bar{u}^2 \sim g(\frac{h}{d_s})^{2a}\ h\ \frac{\Delta H}{\lambda}\ \frac{\lambda}{S} \quad . \tag{32}$$

For given conditions of water discharge, sediment size d_s, valley slope $\Delta H/\lambda$, and constant g, the continuity relationship $h \sim 1/W\bar{u}$ is used to demonstrate that:

$$\bar{u} \sim (\frac{\lambda}{S})^{\frac{1}{3+2a}}\ (\frac{1}{W})^{\frac{1}{(3+2a)(1+2a)}} \quad , \tag{33}$$

$$A = Wh \sim (\frac{S}{\lambda})^{\frac{1}{3+2a}}\ W^{\frac{1}{(3+2a)(1+2a)}} \quad , \tag{34}$$

$$S_f \sim \frac{\lambda}{S} \quad . \tag{35}$$

These relationships indicate that the friction slope and the velocity decrease as the sinuosity increases. The velocity and the cross-section area vary slightly with the relative roughness of the flow and when the width remains fairly constant, the flow depth increases with sinuosity.

The longitudinal stability of an alluvial channel depends on the relative magnitude of shear stress τ_o exerted on the bed as compared to the resistive stress to motion of the sediment particles. For noncohesive sediments, the ratio of these two forces defines the longitudinal Shields number ϕ_ℓ and similar ratios can be expected for similar channels:

$$\phi_\ell = \frac{\tau_o}{\gamma(G-1)d_s} \quad . \tag{36}$$

in which γ is the specific weight of water, τ_o is the bed shear stress, and G is the relative density of the solid particles. The critical value of the Shields number describes the incipient condition of motion of sediment particles. As the Shields number increases above the critical value, the sediment particles are brought to motion and the rate of sediment transport increases. Therefore, the downstream sediment load in a channel is proportional to the longitudinal Shields number. The bed shear stress in a wide channel is given by:

$$\tau_o = \gamma h S_f \quad . \tag{37}$$

The relationship between the Shields number and the sinuosity in a channel of uniform width and sediment size is derived from Eqs. 34, 35, 36, and 37:

$$\phi_\ell \sim \frac{hS_f}{d_s} \sim \left(\frac{\lambda}{S}\right)^{\frac{2+2a}{3+2a}} \quad . \tag{38}$$

Since the exponent a varies between 0 and 0.4, the decrease in longitudinal Shields number as the sinuosity increases demonstrates that sinuous channels have a reduced ability to transport sediments as compared to straight channels at any given valley slope.

4.2 Transversal Equilibrium

The fundamental equilibrium condition in the transversal direction of the flow in a bend has been described by Rozovskii (1957) and Yen (1972). After neglecting second order terms, the force equilibrium per unit mass can be written:

$$\frac{\bar{u}^2}{r} = - \frac{1}{\rho} \frac{\partial \tau_t}{\partial z} + g \, S_w \tag{39}$$

in which τ_t is the transversal shear stress component, z is the vertical coordinate, g is the gravitational acceleration and S_w is the water surface slope in the radial direction. The centrifugal acceleration term in Eq. 39 is balanced by the friction term and the transversal hydrostatic pressure gradient. After integration over the flow depth h Julien and Simons (1984) showed from moment equilibrium that the transversal shear stress is proportional to the centrifugal acceleration:

$$\tau_{to} \cong \beta \, \frac{\rho h \bar{u}^2}{r} \tag{40}$$

in which β is the proportionality factor and τ_{to} is the bed shear stress in the transversal direction. This shear stress component exerts a significant influence on the stability of the outer bank in a bend. Considering a different sediment size d_b for the bank material than for the bed material, the ratio of the bed shear stress to the resistive stress in the transversal direction defines the transversal Shields number ϕ_t as:

$$\phi_t = \frac{\tau_{to}}{\gamma(G-1)d_b} \quad . \tag{41}$$

For noncohesive sediments, the incipient condition of motion in the transversal direction for different channels is expected to occur at the same value of ϕ_t. After introducing the ratio R_d of bank to bed material sizes ($R_d = d_b/d_s$), the transversal Shields number can be written as:

$$\phi_t \sim \frac{\tau_{to}}{R_d d_s} \quad . \tag{42}$$

Combining Eqs. 20, 33, 34, 40 and 42 gives the variation of the transversal Shields number along the meandering path of a stream with constant width:

$$\phi_t \sim \frac{\beta 2\pi\Theta_m}{\lambda} \frac{1}{R_d d_s} \left(\frac{\lambda}{S}\right)^{\frac{4+2a}{3+2a}} \sin 2\pi s^* \quad . \tag{43}$$

From this relationship the maximum value of Θ_t over a meander wavelength occurs at $s^* = \pi/2$. For a fixed wavelength λ, and width w:

$$\phi_t \Big|_{max} \sim \frac{\beta\ \Theta_m}{R_d d_s} \left(\frac{\lambda}{S}\right)^{\frac{4+2a}{3+2a}} \quad . \tag{44}$$

The maximum transversal Shields number has been computed as a function of Θ_m from Eq. 44 for the case of uniform sediment size distribution ($R_d d_s$ constant) and constant coefficient β. The results shown in Fig. 5 for three values of the roughness exponent: a=0 for Chézy equation, a=1/6 for Manning-Strickler relationship, and a=0.4 for very rough

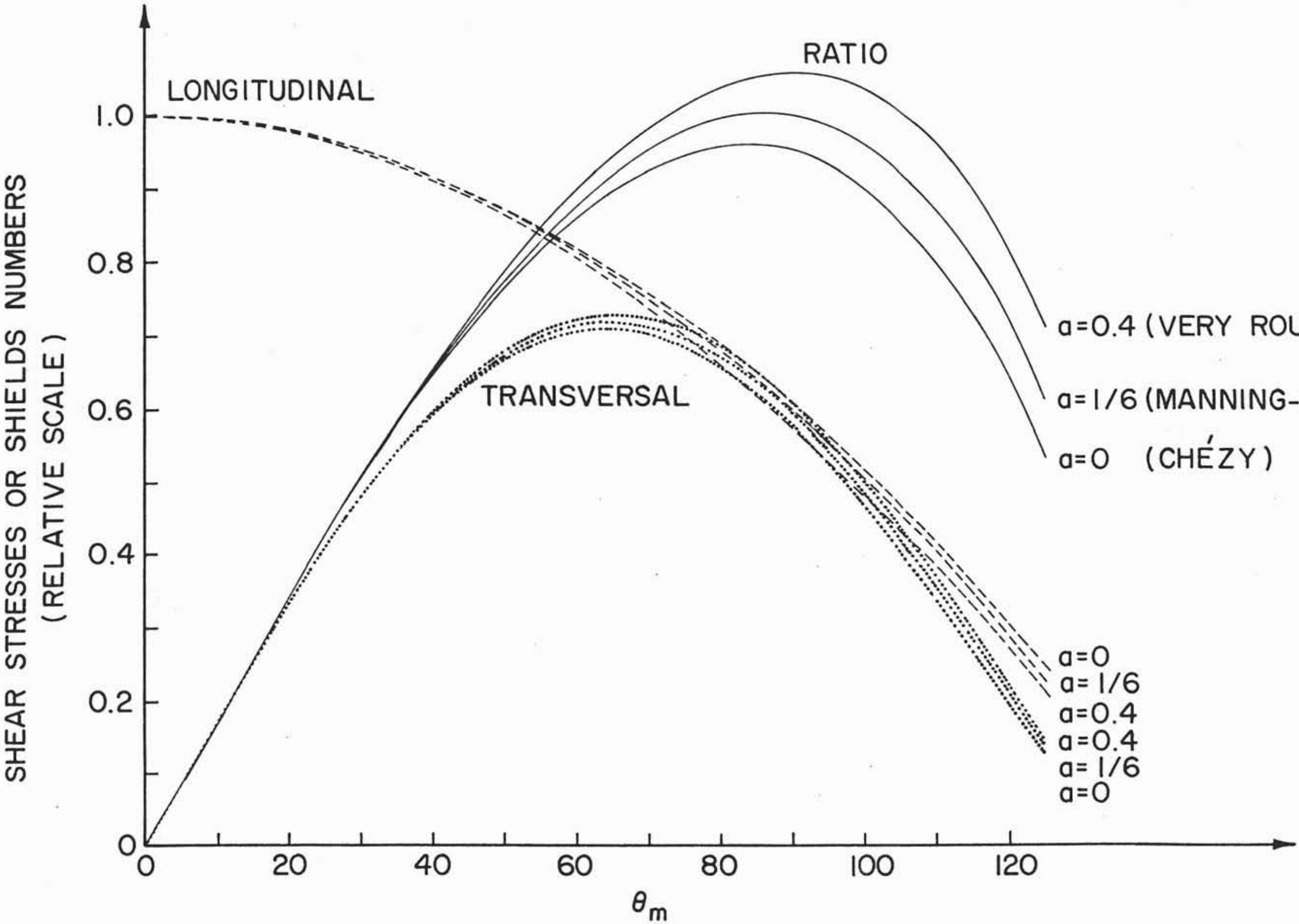


Figure 5. Shields number vs. Θ_m.

channels indicate that the influence of the sinuosity on the maximum Shields number is nearly independent of the relative roughness. As a result, is that the maximum transversal Shields number remains fairly constant for $50° < \Theta_m < 80°$. Figure 5 also shows that the magnitude of the longitudinal Shields number calculated from Eq. 38 decreases gradually with increasing Θ_m and this function is nearly independent of the relative roughness. The ratio of these two Shields numbers cancels the influence of the sediment size while keeping the ratio between bank and bed material sizes:

$$\frac{\phi_t\big|_{max}}{\phi_\ell} \sim \frac{\beta\Theta_m}{R_d} \left(\frac{\lambda}{S}\right)^{\frac{2}{3+2a}} \quad . \tag{45}$$

For a constant value of β and R_d, this ratio has been plotted on Fig. 5 using a relative scale. Starting from $\Theta_m = 0$ the ratio of transversal to longitudinal shear stresses increases gradually with Θ_m and peaks for Θ_m around 90°. The ratio of shear stresses remains fairly constant for $70° < \Theta_m < 100°$ and decreases rapidly for $\Theta_m > 100°$. These curves using relative scales indicate clearly that for nearly straight channels (small Θ_m), any increase in Θ_m increases both the transversal Shields number and the ratio of transversal to longitudinal shear stresses. This discussion leads us to the important conclusions that any discontinuity or perturbation in the planform geometry that induces a shear stress in excess of the critical value will scour the outer bank and initiate the formation of a meandering pattern since the transversal shear stress increases with Θ_m until it reaches an angle Θ_m near 90°. Any increase of Θ_m beyond this point will reduce the transversal shear stress component and therefore gradually stabilize the meandering profile. Another important factor controlling the geometry of meandering channels is the ability to transport sediments which is discussed in the next section.

4.3 Sediment Transport

The rate of sediment transport being proportional to the Shields number, two components of sediment transport can be defined from the previous analysis of longitudinal and transversal Shields numbers. In this analysis, the rate of sediment transport is computed with the Meyer-Peter and Müller relationship. Though another bed-load equation might be used as well, the characteristics to be pointed out in this analysis are expected to be similar for any sediment transport equation based on shear stress. The longitudinal rate of sediment transport can be written as:

$$\bar{q}_\ell \sim \left(\frac{\tau_o}{\gamma(G-1)d_s}\right)^{1.5} \left(1 - \frac{\tau_c}{\tau_o}\right)^{1.5} \qquad (46)$$

in which τ_c is the critical longitudinal shear stress. The first term in parentheses of Eq. 46 represents the sediment transport capacity while the second term is the loss in transport capacity due to the critical shear stress condition at the boundary. This second term simply indicates that the threshold shear stress must be exceeded in order to initiate any sediment transport and subsequent erosion of the bed material. A similar relationship is obtained for the rate of sediment transport in the transversal direction after replacing τ_o by τ_t and d_s by $R_d d_s$ into Eq. 46:

$$\bar{q}_t \sim \left(\frac{\tau_t}{\gamma(G-1)R_d d_s}\right)^{1.5} \left(1 - \frac{\tau_c}{\tau_t}\right)^{1.5} \quad . \qquad (47)$$

The first term in parentheses is the transversal Shields number. The sediment transport capacities q_ℓ and q_t for both components can be written in terms of Shields number as follows:

$$q_\ell \sim \phi_\ell^{1.5} \tag{48}$$

$$q_t \sim \phi_t^{1.5} \quad . \tag{49}$$

These components and their ratio are plotted on relative scales in Fig. 6 under the assumption that β, R_d and d_s are constant. These curves reveal that the longitudinal sediment transport capacity decreases gradually as Θ_m increases without a significant influence of the relative roughness. When a=0, the rate of sediment transport is inversely proportional to the sinuosity. The transversal sediment transport capacity increases until $\Theta_m = 65°$ and then decreases rapidly with increasing Θ_m. The magnitude of the transversal sediment transport capacity depends on the values of β and R_d and therefore the sum of these two sediment transport components must reach a maximum value between $0° < \Theta_m < 65°$. Since most of the meandering streams have angles $\Theta_m > 65°$ it must be concluded that streams do not meander in order to maximize the sediment load. However, the ratio of transversal to longitudinal rates of sediment transport peaks at Θ_m near 90° and remains fairly constant for $70° < \Theta_m < 105°$. Since this range corresponds to most of the observed values, meandering streams seem more likely to optimize the ratio of transversal to longitudinal rates of sediment transport rather than the sum of the two sediment transport components. For small angles the longitudinal sediment load is very large compared to the transversal sediment load. Under equilibrium conditions, streams with high sediment load are most likely to have a straight planform geometry, whereas large Θ_m values correspond to small sediment loads.

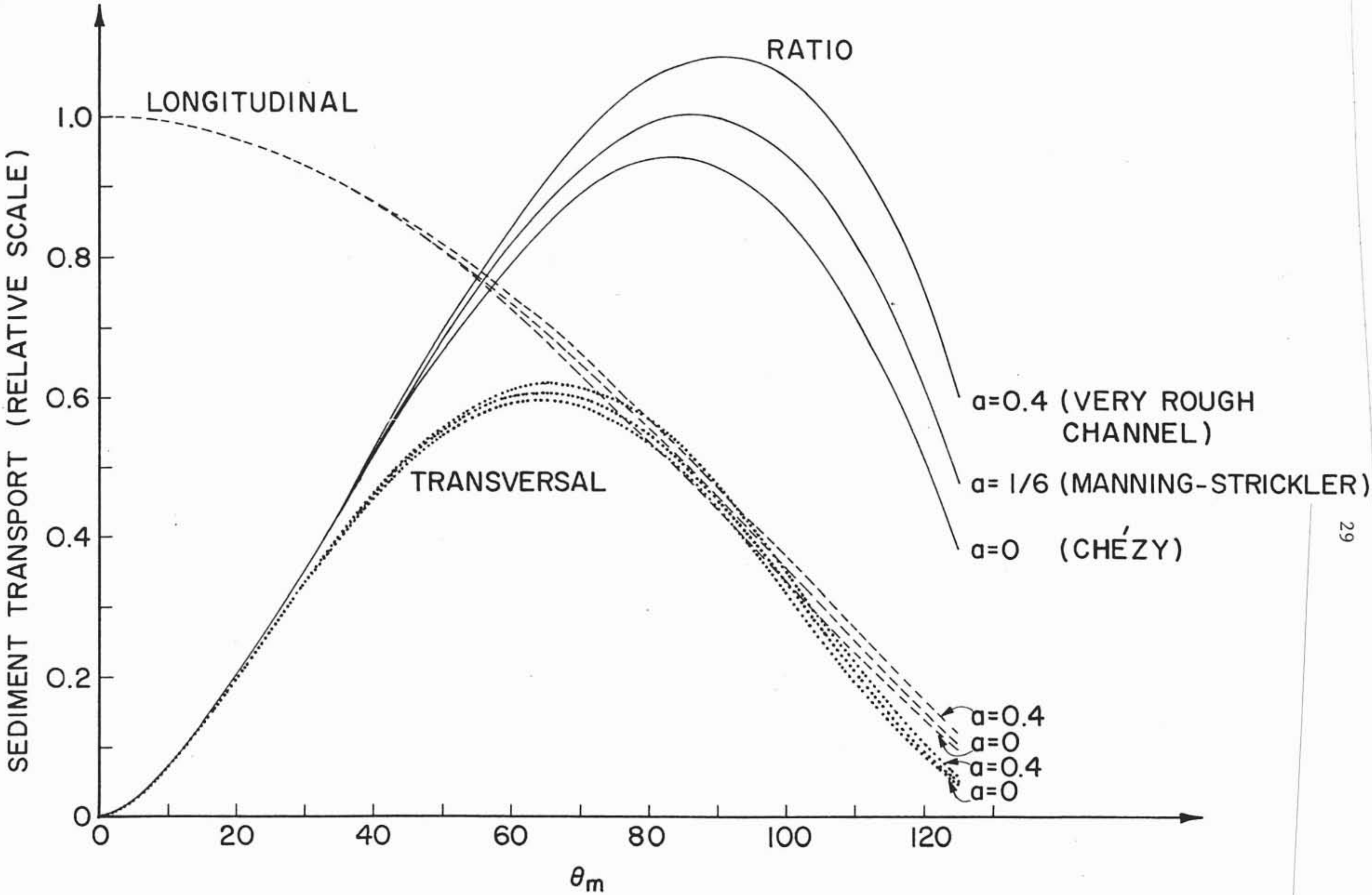


Figure 6. Sediment transport vs. Θ_m.

The response of meandering streams to changes in the principal hydraulic characteristics of a stream can be assessed from this qualitative analysis. Assuming that the response described by aggradation or degradation is proportional to the rate of sediment transport, the curves plotted in Fig. 6 indicate that the time response to changes in the longitudinal channel geometry decreases as Θ_m increases. Therefore, straight channels should adjust their channel geometry much faster than meandering channels. The time of response to changes of the planform geometry in the transversal direction (lateral migration) is expected to reach a maximum at Θ_m around 65°. In the case of nearly straight channels (Θ_m<20°) the transversal sediment transport capacity is very small and these streams require a longer period of time to adjust in the lateral direction unless there is a significant change in the longitudinal component causing severe degradation or aggradation.

V. CONCLUSION

An explanation for the fundamental shape of meandering planforms based on the separation of the boundary layer near the inner bank and the rate of energy dissipation is proposed. When written in terms of a variational problem, the functional corresponding to a sine-generated curve is composed of a potential energy component and a term describing the rate of energy dissipation. The resulting sine-generated curve has been verified extensively with field data by Langbein and Leopold (1966). In the case of asymmetrical channels, the sine-generated curve remains the fundamental mode on which the perturbations can be analyzed as random variables or in terms of Fourier series. The corresponding radius of curvature is shown to be a cosecant function of the meandering path and the sinuosity is computed from Eq. 23. The friction slope, the velocity and the longitudinal shear stress decrease as the sinuosity increases. It is shown from the equilibrium condition in the transversal direction that the transversal shear stress or Shields number varies as a sine function along the path of the meandering channel. The maximum transversal Shields number is fairly constant for $50° < \Theta_m < 80°$ and decreases for $\Theta_m > 80°$. The longitudinal sediment transport capacity is maximum for straight channels and is inversely proportional to the sinuosity. The sediment transport capacity in the transversal direction peaks at Θ_m near 65°.

Meandering streams are likely to adjust their maximum angle Θ_m to reach equilibrium between the longitudinal sediment transport capacity and the upstream sediment load, the angle Θ_m being inversely proportional to the sediment transport capacity. Very sinuous channels and straight channels have a reduced ability to reach new equilibrium conditions while meandering channels with $40° < \Theta_m < 80°$ have the maximum potential for lateral changes in the planform geometry.

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