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The ratio of measured to total sediment discharge

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ABSTRACT

It is important to examine the ratio of measured to total sediment discharge to determine the error in measured sediment transport rates from depth-integrated samplers. The ratio of measured to total sediment discharge as well as the ratio of suspended to total sediment discharge are examined based on the Modified Einstein Procedure. Both ratios reduce to a function of the ratio of shear velocity, u_* , to the fall velocity, ω , of suspended material, u_*/ω , and the ratio, h/d_{50} , of flow depth, h , to the median grain size of bed material, d_{50} . In rivers transporting fine material (such as silt or clay), the ratio of suspended to total load is a function of the ratio, h/d_{50} . In this study, it is found that the ratio of measured to total load becomes a simple function of flow depth. For fine sediment transport, with a Rouse number (Ro) < 0.3 , at least 80% of sediment load is in suspension when $h/d_{50} > 15$, and at least 90% of sediment load is measured from depth integrating samples when $h > 1$ m. Detailed measurements from 35 river stations in South Korea demonstrate that sand sizes and finer fractions predominantly are transported in suspension. Also, at least 90% of sand and finer fractions are transported in suspension in gravel and sand bed rivers when the discharge is larger than the mean annual discharge.

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1. Introduction

The quantification of the total sediment transport rate from the combination of suspended and bed loads is still one of the difficult topics in river engineering. Bedload is naturally difficult to measure and the total sediment load is often difficult to accurately determine (Turowski et al., 2010; Wohl et al., 2015). Einstein (1950) proposed a bedload function to calculate bed material load in sand bed rivers. The method was a breakthrough in the study of sediment transport. Several methods to calculate the total sediment load were developed from it. Colby and Hembree (1955) and Toffaleti (1977) are two of the examples. The current study focuses on Colby and Hembree (1955) and its derivatives. Colby and Hembree (1955) compared the total sediment load computed by the Einstein Procedure to sediment measurement in Niobrara River and found that the computed size distributions of sediment discharge compared poorly to the measurements. They modified the Einstein Procedure to compute the total sediment load based on depth-integrated sediment samples. The method is known as the Modified Einstein Procedure (MEP). The MEP is generally applicable to sand-bed rivers because depth integrating samplers

measure much of the sediment discharge for fine sediment. However, the MEP has been subjected to several empirical adjustments over time. Colby and Hubbell (1961) introduced four nomographs to simplify the computation of the MEP.

The Rouse number, Ro , describes a concentration profile of suspended sediment and is defined as $Ro = \omega/\beta_s \kappa u_*$, where ω is the settling velocity of suspended material, β_s is the ratio of the turbulent mixing coefficient of sediment to the momentum exchange coefficient, κ is the von Karman constant, and u_* is the shear velocity (Rouse, 1937). Lara (1966) pointed out that the following assumption by Colby and Hembree (1955) was not valid: the value of Ro is not always 0.7. Lara (1966) suggested Ro values should be determined based on the power relation between Ro and fall velocity, $Ro = \alpha \omega^\beta$ (where α and β are determined by calibration), for the size fractions having significant percentage quantities in both suspended and bed sediment materials.

Two major revisions of the MEP were proposed by Burkham and Dawdy (1980): (1) they replaced the effective roughness k_s from d_{65} to $5.5d_{65}$ (where d_{65} is the diameter for which 65% of the sediment particles are finer); and (2) they developed a procedure to compute the bedload transport intensity, ϕ_* , from shear intensity, ψ_* , instead of arbitrarily dividing ϕ_* by 2. The method by Burkham and Dawdy (1980) is known as the Revised Modified Einstein procedure. Shen and Hung (1983) reaffirmed the use of

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least squares fitting of the power relation between Ro and ω . In addition, they proposed an optimization procedure for the calculation of suspended sediment discharge.

Computer programs have been developed to facilitate the MEP calculations (Burkham & Dawdy, 1980; Holmquist-Johnson & Raff, 2006; Stevens, 1985). The latest development is the Bureau of Reclamation Automated Modified Einstein Procedure (BORAMEP) by Holmquist-Johnson and Raff (2006). Shah-Fairbank (2006) did a thorough testing of BORAMEP. In 2009, Shah-Fairbank proposed the Series Expansion of the Modified Einstein Procedure (SEMEP) to obtain the total sediment transport rate for depth-integrating (Shah-Fairbank, 2009; Shah-Fairbank et al., 2011), and point samplers (Shah-Fairbank & Julien, 2015). The SEMEP assumes a Ro value evaluated by Rouse's (1937) equation, so there is no need for a minimum of two overlapping size fractions for both the bed material and suspended material. In addition, the bedload is estimated directly from measured load, so there is no need to arbitrarily divide the bedload transport intensity by two. Lastly, unlike the MEP, the calculated total sediment discharge is always larger than the suspended sediment discharge (Julien, 2010). A recent comparison of SEMEP and BORAMEP was done on the Low Conveyance Channel in New Mexico by Baird and Varyu (2011). They concluded that the results from both methods were comparable except that 37% of the samples where BORAMEP failed to predict the total load could be predicted by SEMEP.

The objective of the current study is to derive the ratio of measured to the total sediment discharge as well as the ratio of suspended to the total sediment load as functions of water depth and grain size. A case study in South Korea was used for a comparison with BORAMEP.

2. Method

2.1. Basics

The following defines the components for the derivation of the sediment transport function. The settling velocity of the median suspended particle, d_{50ss} :

$$\omega = \frac{8\nu}{d_{50ss}} \left[\left(1 + 0.0139d_*^3 \right)^{0.5} - 1 \right] \quad (1)$$

$$d_* = d_{50ss} \left[\frac{(G-1)g}{\nu^2} \right]^{\frac{1}{3}} \quad (2)$$

where ω = the settling velocity, d_* = dimensionless grain size, G = the specific weight of sediment, ν = the kinematic viscosity of water, g = gravitational acceleration, and d_{50ss} = the median size of suspended material.

The Rouse number (Ro) reflects the ratio of the sediment settling properties to the hydraulic characteristics of the flow, and it is defined as

$$Ro = \frac{\omega}{\beta_s \kappa u_*} \quad (3)$$

where β_s = the ratio of the turbulent mixing coefficient of sediment to the momentum exchange coefficient which has been found equal to 1 for most practical applications; κ = von Karman constant usually close to 0.4, and u_* = shear velocity,

The concentration profile is as defined by Rouse (1937):

$$C = C_a \left(\frac{h-z}{z} \frac{a}{h-a} \right)^{Ro} \quad (4)$$

where $C_a = q_b / 11.6u_*a$ according to Einstein (1950), q_b = unit

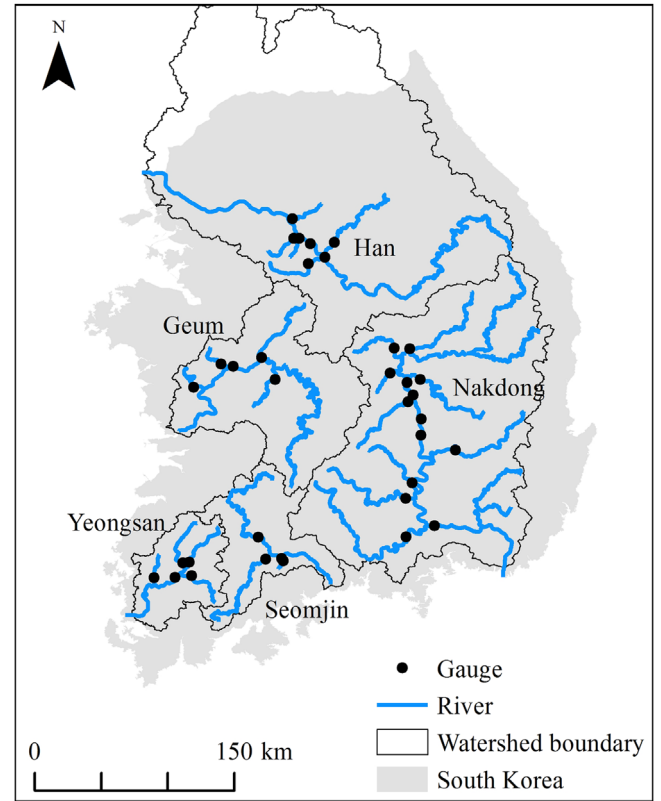


Fig. 1. Study area.

bedload discharge, h = flow depth, z = vertical elevation above the channel bed, and a = the thickness of the bed layer and it is assumed $a = 2d_s$, where d_s is the median bed material size, d_{50} .

The turbulent velocity profile can be expressed using the logarithmic velocity profile (Keulegan, 1938), :

$$v = \frac{u_*}{\kappa} \ln \left(\frac{30z}{k_s} \right) \quad (5)$$

where v = velocity measured at z from the river bed, and k_s = boundary roughness.

2.2. Measured and suspended sediment discharge

The measured unit sediment discharge can be evaluated by integrating the product of flow velocity and volumetric sediment concentration from the nozzle height of suspended discharge sampler, z_n , to the free surface at $z = h$:

$$q_m = \int_{z_n}^h C v dz \quad (6)$$

where q_m = measured unit sediment discharge, and C is the sediment concentration by volume.

By inserting Eqs. (5) and (4) into Eq. (6), the measured load is written as:

$$q_m = \int_{z_n}^h C_a \left(\frac{h-z}{z} \frac{a}{h-a} \right)^{Ro} \frac{u_*}{\kappa} \ln \left(\frac{30z}{k_s} \right) dz \quad (7)$$

The solution of Eq. (7) is given as:

$$q_m = 0.216q_b \frac{E^{Ro-1}}{(1-E)^{Ro}} \left\{ \ln \left(\frac{30h}{d_s} \right) J'_1 + J'_2 \right\} \quad (8)$$

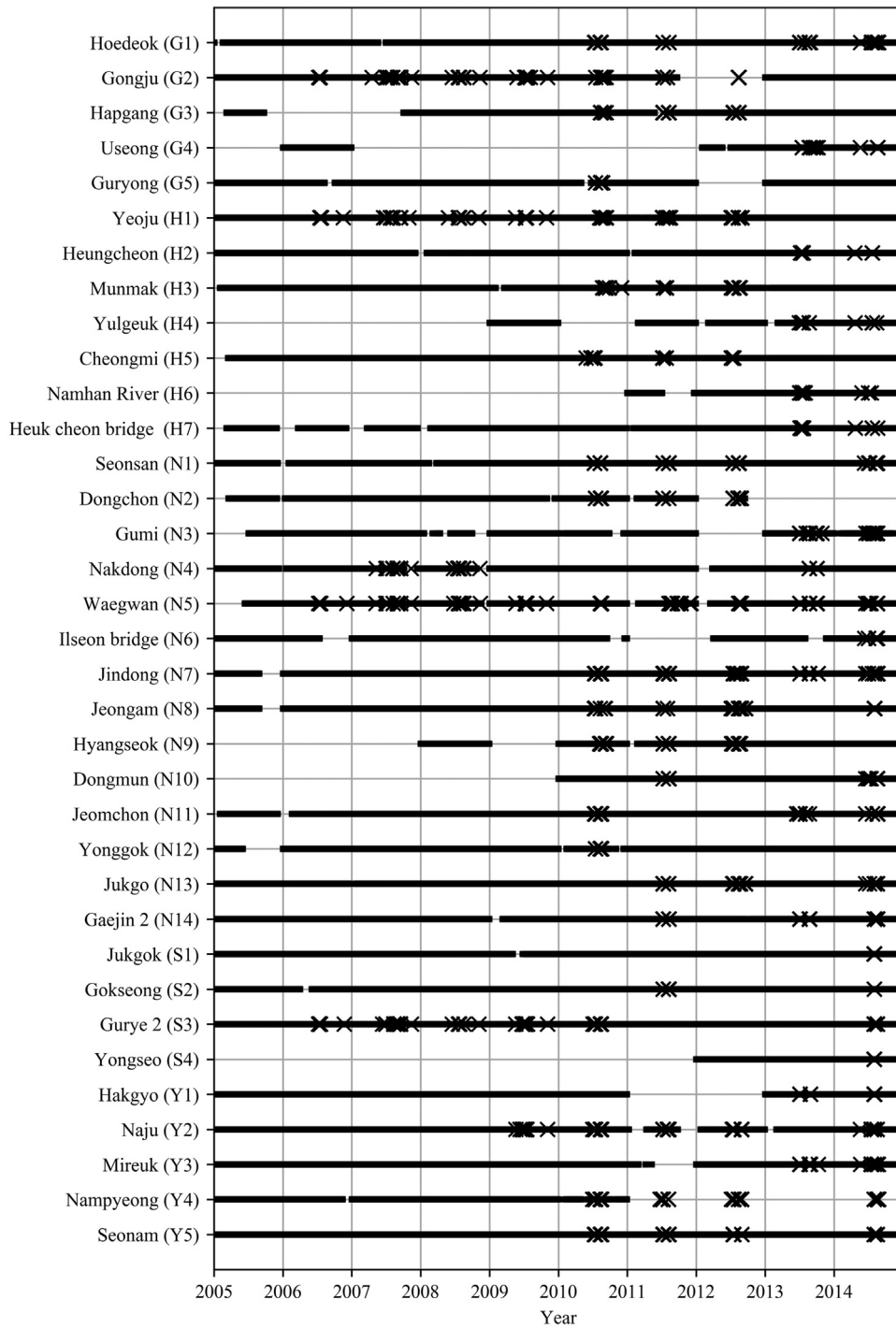


Fig. 2. Available daily discharges (line) and sediment surveys (x).

$$J'_1 = \int_A^1 \left(\frac{1-z}{z} \right)^{Ro} dz \quad (9)$$

$$J'_2 = \int_A^1 \ln z \left(\frac{1-z}{z} \right)^{Ro} dz \quad (10)$$

where $E = a/h$ is the ratio of the thickness of the bed layer to flow depth, and $a = 2d_s$ is commonly used where d_s is the median grain size (d_{50}) of the bed material, $A = z_n/h$, and z_n is the vertical elevation of the sediment sampler nozzle above the channel bed.

The unit suspended load, q_s , is given by Einstein (1950) as:

$$q_s = 0.216 q_b \frac{E^{Ro-1}}{(1-E)^{Ro}} \left\{ \ln \left(\frac{30h}{d_s} \right) J_1 + J_2 \right\} \quad (11)$$

$$J_1 = \int_E^1 \left(\frac{1-z}{z} \right)^{Ro} dz \quad (12)$$

$$J_2 = \int_E^1 \ln z \left(\frac{1-z}{z} \right)^{Ro} dz \quad (13)$$

The only difference between the measured load, q_m , and the suspended load, q_s , is found in the integration limits. For the J'_1 and J'_2 values the integrals starts from the nozzle height, z_n , to the free surface, h . On the other hand, J_1 and J_2 start from the bed layer $2d_{50}$ to the free surface. In the current study, the Einstein integrals J_1 , J_2 , J'_1 , and J'_2 are computed using the `scipy.integrate.quad` method in Python. These integrals can also be effectively solved with the method of Guo and Julien (2004) and other algorithms reviewed by Zamani et al. (2016).

2.3. Ratios of suspended to total loads, Q_s/Q_t

The total sediment load, Q_t , is obtained by multiplying the unit sediment discharge, q_t , by the channel width, W . The measured and suspended sediment discharge are obtained in similar fashion. Therefore, the ratio $Q_s/Q_t = q_s/q_t$ and $Q_m/Q_t = q_m/q_t$.

Since the unit total sediment discharge $q_t = q_b + q_s$, the ratio of suspended to total sediment discharge can be derived as follows:

$$\begin{aligned} \frac{q_s}{q_t} &= \frac{q_s}{q_b + q_s} \\ &= \frac{0.216q_b \frac{E^{Ro-1}}{(1-E)^{Ro}} \left\{ \ln \left(\frac{30h}{d_s} \right) J_1 + J_2 \right\}}{q_b + 0.216q_b \frac{E^{Ro-1}}{(1-E)^{Ro}} \left\{ \ln \left(\frac{30h}{d_s} \right) J_1 + J_2 \right\}} \\ &= \frac{0.216 \frac{E^{Ro-1}}{(1-E)^{Ro}} \left\{ \ln \left(\frac{30h}{d_s} \right) J_1 + J_2 \right\}}{1 + 0.216 \frac{E^{Ro-1}}{(1-E)^{Ro}} \left\{ \ln \left(\frac{30h}{d_s} \right) J_1 + J_2 \right\}} \end{aligned} \quad (14)$$

It is interesting to observe that the ratio of suspended to total load, q_s/q_t , only changes with two variables now, i.e., h/d_s and Ro : $q_s/q_t = f(h/d_s, Ro)$ (15)

2.4. Ratios of measured to total loads, Q_m/Q_t

The unit bedload q_b can be directly calculated from Eq. (8) when q_m is known. The ratio of measured to total sediment discharge can be shown as:

$$\frac{q_m}{q_t} = \frac{0.216 \frac{E^{Ro-1}}{(1-E)^{Ro}} \left\{ \ln \left(\frac{30h}{d_s} \right) J'_1 + J'_2 \right\}}{1 + 0.216 \frac{E^{Ro-1}}{(1-E)^{Ro}} \left\{ \ln \left(\frac{30h}{d_s} \right) J_1 + J_2 \right\}} \quad (16)$$

As can be seen from Eq. (16), q_m/q_t is a function of Ro , h/d_s , and A . But since z_n is fixed with the same sampler, q_m/q_t becomes a function of Ro , h , and d_s .

$$q_m/q_t = f'(Ro, h, d_s) \quad (17)$$

2.5. Case study

The daily discharges and sediment measurements of 35 gaging stations in South Korea were analyzed, including seven gaging stations on the Han River, fourteen on the Nakdong River, five on the Geum River, five on the Yeongsan River, and four on the Seomjin River, respectively (Fig. 1). The total area of the studied watersheds covered 47000 km². Of 2084 sediment samples, 2036 were measured by depth-integrating using a D-74 sampler. The rest were measured with either a P-61 sampler or a grab sampler. We retained the 1962 samples from the D-74 sampler for which

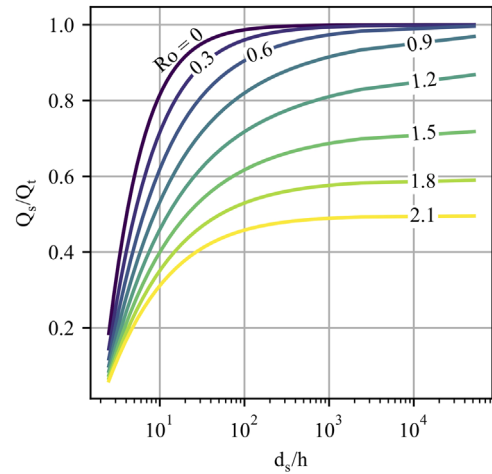


Fig. 3. Theoretical ratio of suspended to total sediment load, Q_s/Q_t , as a function of h/d_s and Ro .

the water level was higher than the nozzle height (10 cm). The grain size distributions of bed material and suspended material were provided at all stations. Bed materials were sampled using the US BM-54 bed material sampler, 60L van Veen Grab sampler, or by grid sampling. The grain size distributions of suspended material were determined by laser diffraction. The lengths of record for the daily discharge and sediment measurements of all rivers in this study are summarized in Fig. 2, and a detailed data description and analysis is available in Julien et al. (2017).

The total sediment discharge is calculated by K-water using a version of the MEP that is quite similar to the BORAMEP. In this study, the total sediment load will be recalculated using the SEMEP and these results are compared with the result of the MEP.

3. Results and discussion

3.1. Theoretical ratios of Q_s/Q_t and Q_m/Q_t

As shown in Eq. (14), the ratio of suspended to total sediment discharge, Q_s/Q_t , is a function of only h/d_s and Ro . The analytical solution of Eq. (14) is plotted in Fig. 3. Fig. 3 shows the ratio Q_s/Q_t at constant values of Ro while varying the value of h/d_s . The ratio of Q_s/Q_t increases when the value of Ro decreases.

Fig. 4 shows values of the ratio of the measured to total sediment discharge, Q_m/Q_t , by varying the value of water depth as a function of d_s . As shown in Fig. 4, when $Ro=0$, the average difference in Q_m/Q_t for $d_s = 0.625$ mm and $d_s = 2$ mm is 0.3%. When $Ro=0.3$, the average difference in Q_m/Q_t for $d_s = 0.625$ mm and $d_s = 2$ mm is 0.9%. It is important to note that when $Ro < 0.3$, the measured sediment discharge is more than 90% of the total sediment discharge at flow depths $h > 1$ m.

As shown in Eq. (14), the ratio Q_s/Q_t only changes with Ro and h/d_s . Because the materials in suspension are fine, the values of Ro are small ($Ro < 0.16$), and, therefore, the change of Ro only results in small changes in Q_s/Q_t .

3.2. MEP vs SEMEP

The total sediment discharge Q_t calculations obtained from the MEP and the SEMEP first were compared. All of the suspended load samples was able to be calculated using the SEMEP. Using the MEP, 1808 samples could be calculated. This means that 154 (out of 1962)

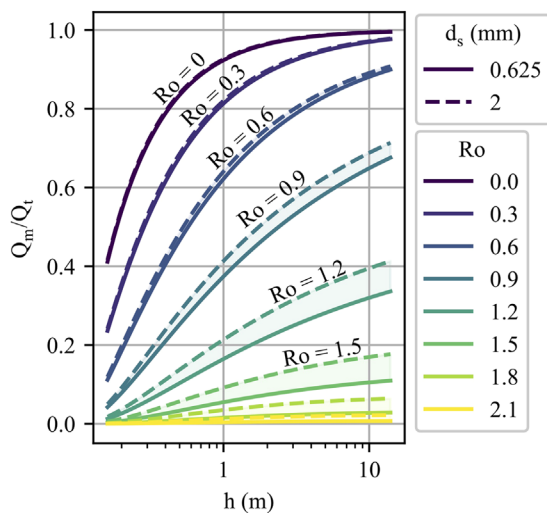


Fig. 4. Theoretical ratio of measured to total sediment load, Q_m/Q_t , as a function of h and Ro for sands.

samples could not be analyzed with the MEP. The number of calculated samples for the SEMEP is greater because the MEP requires at least two overlapping bins between suspended material and bed material to determine Ro . In addition, Ro for the remaining bins are determined by regression analysis when overlapping bins exist, and sometimes a negative Ro exponent can be generated in the MEP, which erroneously implies that the sediment concentration increases towards the free surface (Shah-Fairbank, 2009). On the other hand, Ro for the SEMEP is directly calculated based on d_{50} in suspension by using Eq. (3). This avoids the possibility of erratic results of Ro with the MEP method.

Fig. 5 shows the ratio Q_m/Q_t for all samples computed by the MEP and the SEMEP. The measured sediment discharge, Q_m , is calculated as the product of discharge, Q , and measured concentration, C , in both cases. In Fig. 5a, the predicted total sediment load is compared to the measured load. The predictions from SEMEP are close to measured, while the predictions from the MEP tended to be slightly higher on average, with a scatter larger than 2 orders of magnitude. Fig. 5b shows that the values of u_* / ω range

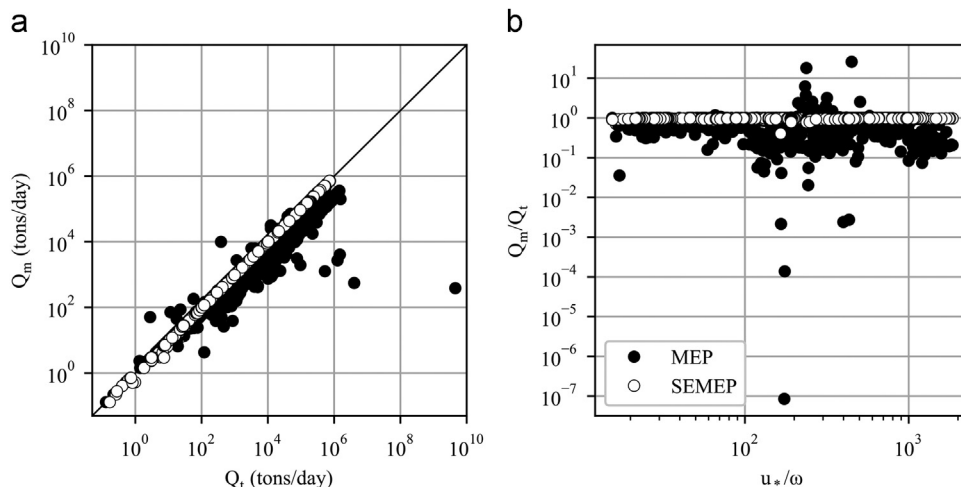


Fig. 5. Comparison of (a) measured sediment load, Q_m , to the calculated total load, Q_t , using the MEP and the SEMEP, and (b) ratio of Q_m/Q_t vs u_* / ω .

from 15 to 1825. The Q_m/Q_t of MEP range from 8×10^{-8} to 26. The Q_m/Q_t of the SEMEP range from 0.5 to 0.995. According to Julien (2010), the primary mode of transport should be suspended load when $u_* / \omega > 5$. Therefore, Q_m/Q_t is expected to be close to 1. This is true for sand-bed rivers, but deviations are noticeable for cobble and gravel-bed streams. Also Q_m/Q_t should always be lower than 1 because the total load cannot be less than measured load. The results of the MEP do not always satisfy this requirement. A total of 29 samples out of 1808 resulted in $Q_m > Q_t$ when using the MEP. It is physically impossible for the total load to be smaller than the measured load. Two reasons cause the total load sometimes smaller than the measured load by the MEP: (1) the erroneous Ro ; and (2) the MEP considers the suspended load for size classes smaller than 0.25 mm and bed load for larger size classes by default. Therefore, the SEMEP is considered more reliable and is used for the remainder of the analysis.

Furthermore, the ratio Q_s/Q_t relates to (a) u_* / ω , (b) sediment concentration, C , (c) flow discharge, Q , and (d) the ratio of flow discharge to mean annual discharge, Q/\bar{Q} , as shown in Fig. 6. Because the median grain sizes of suspended material are silt at all our stations (average $d_{50ss} = 0.023$ mm), it was found that u_* / ω generally is high. Q_s/Q_t is close to 1 and averages 0.99 in sand bed rivers (Fig. 6a). For gravel bed and cobble bed rivers, Q_s/Q_t increases as Q , C , or Q/\bar{Q} increases (Fig. 6b, c, and d). For gravel bed rivers, Q_s/Q_t varies from 0.871 to 0.999; for cobble bed rivers, Q_s/Q_t ranges from 0.232 to 0.971. Fig. 6d shows that during high flows ($Q/\bar{Q} > 1$), over 90% of the sediment is transported in suspension for gravel bed and sand bed rivers. This analysis clearly indicates that the predominant mode of sediment transport in Korean rivers is in suspension. Based on the measurements in South Korea, the key parameters with the conditions where suspended load is dominant is summarized in Table 1.

In Fig. 7, the relationships are investigated between the ratio of the measured to total sediment discharge, Q_m/Q_t , and the same variables shown in Fig. 6. In Fig. 7a, b, c, and d, the difference among various bed materials became subtle. Overall, the measurements contain over 80% of the sediment load (1936 out of 1962 samples). Most of the sediment is measured during floods in South Korean rivers (Fig. 7d).

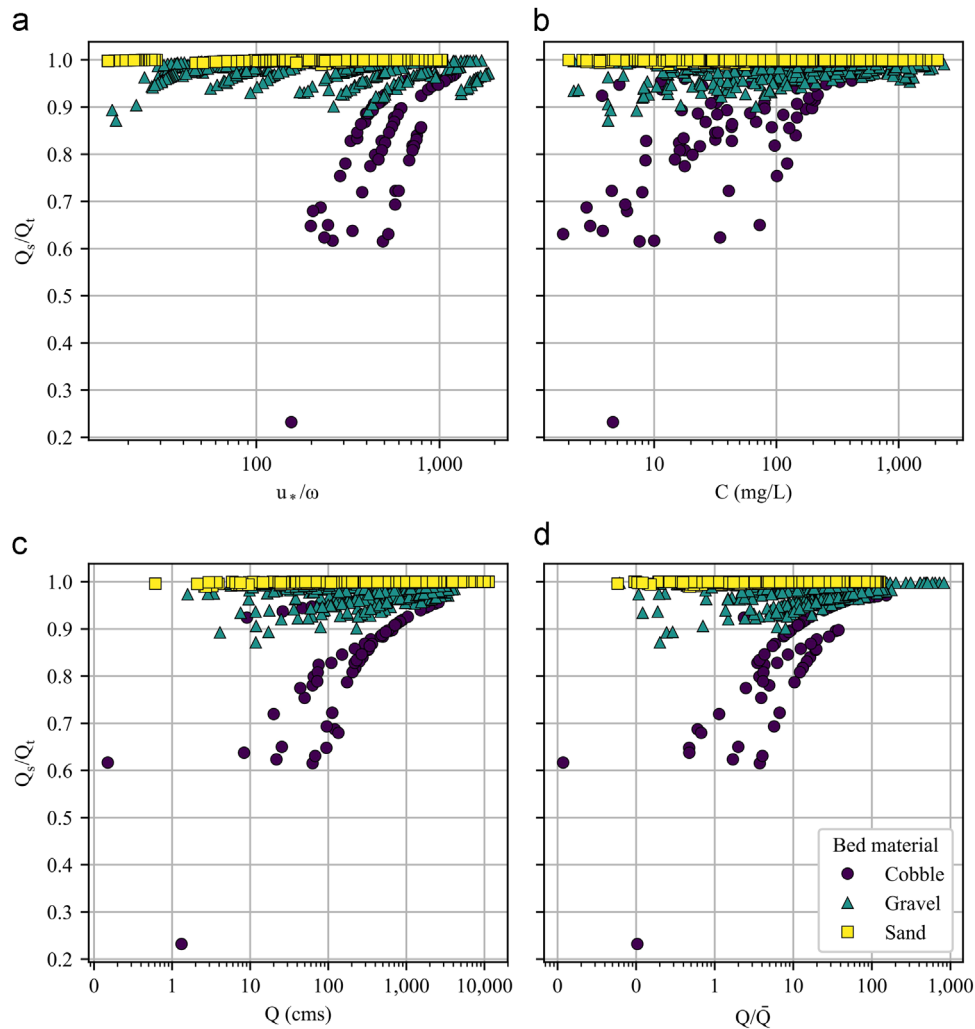


Fig. 6. Relations between Q_s/Q_t and (a) u_*/ω , (b) concentration, C , (c) discharge, Q , and (d) Q/\bar{Q} .

3.3. Ratios of Q_s/Q_t and Q_m/Q_t as functions of h and d_s

By using the methods proposed in Sections 2.3 and 2.4, the ratios Q_s/Q_t and Q_m/Q_t can be presented as functions of a single variable. As shown in Fig. 8, Q_s/Q_t becomes only a function of h/d_{50} . All of the measurements are within the theoretical solution $Ro=0$ and $Ro=0.3$. Eq. (16) shows that Q_m/Q_t is a function of u_*/ω , h/d_s , and z_n/h . When Ro and d_s are small and the nozzle height is fixed, Q_m/Q_t then simply becomes a function of water depth, h , as shown in Fig. 9. Also, Q_m/Q_t decreases when Ro increases.

4. Conclusions

This study compares the total sediment transport rates calculated by the MEP and the SEMEP with the sediment samples collected using a the depth-integrating sampler D-74 in South Korea. It is concluded that the SEMEP outperformed the MEP in terms of stability, consistency, and accuracy. There are 154 samples could not be analyzed with the MEP. The original method of the MEP requires at least two overlapping bins between suspended materials and bed materials. Errors sometimes occur when creating the

Table 1

The conditions when suspended load is dominant based on Korean measurements.

Bed materials	C (mg/l)	u_*/ω^a	Q/\bar{Q}	Q_s/Q_t
Sand	> 1	> 15	> 0.05	99 - 100
Gravel	> 500	> 500	> 10	95 - 100
Cobble	> 150	> 700	> 2	80 - 100

^a ω is calculated from d_{50} of the suspended material

power relation between the Rouse number and fall velocity. The ratios of the suspended load to total load calculated by the MEP vary from 10^{-7} to 20. The large variation in total sediment load calculation with the MEP occur because of errors in the determination of Ro and the threshold grain size of suspended and bed materials. Additionally, the ratio should never be greater than 1, and this raises suspicion regarding the accuracy of the MEP method. The ratios between suspended load and total load range from 0.2 to 1, and 97% of them are greater than 0.9. For this reason, the SEMEP calculations are considered better and more reliable than those from the MEP.

Instead of using the overlapping bins, the Rouse number for the SEMEP is determined using the median grain size of the

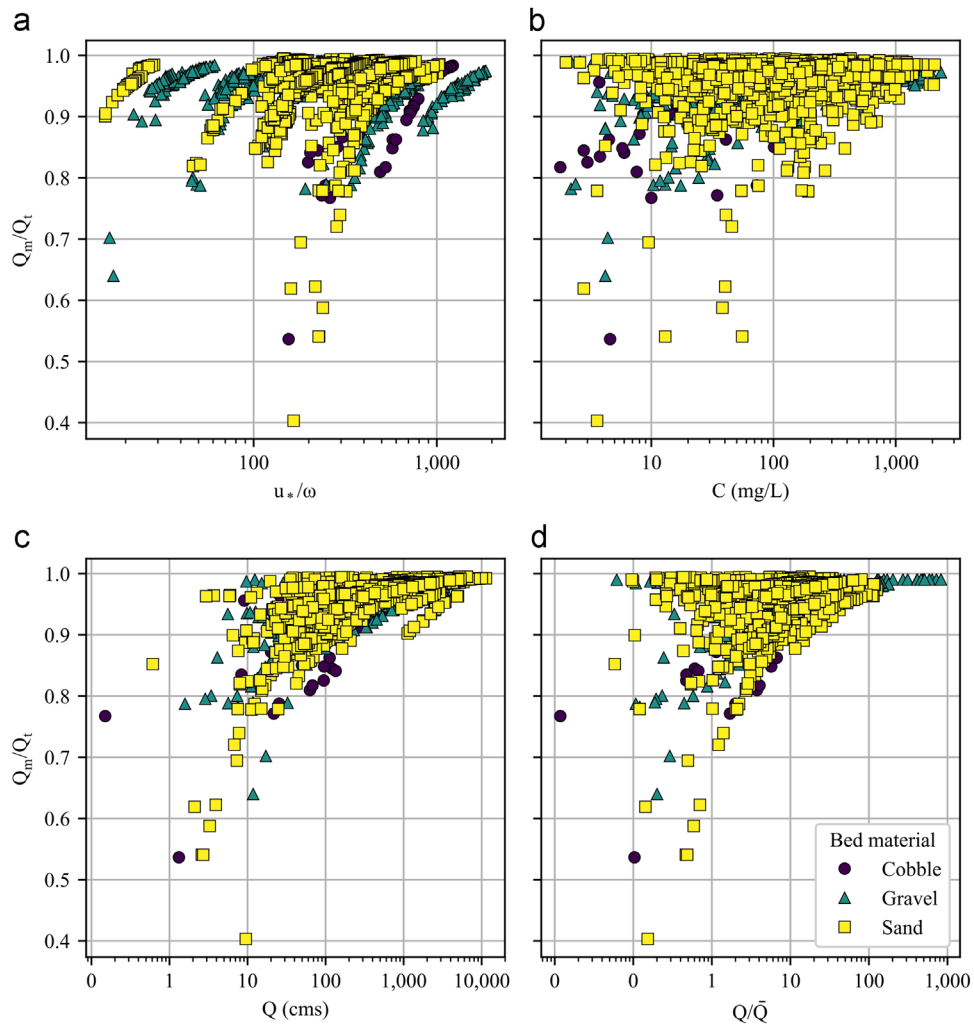


Fig. 7. Relations between Q_m/Q_t and (a) u_*/ω , (b) concentration, C , (c) discharge, Q , and (d) Q/\bar{Q} .

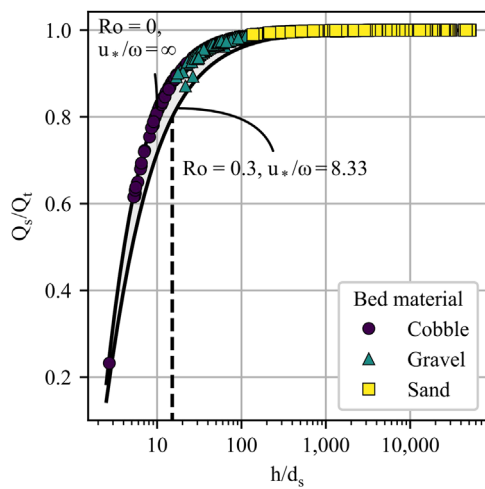


Fig. 8. Comparison of Korean measurements (1962 points) with the theoretical solution of Q_s/Q_t with $Ro=0$ and $Ro=0.3$.

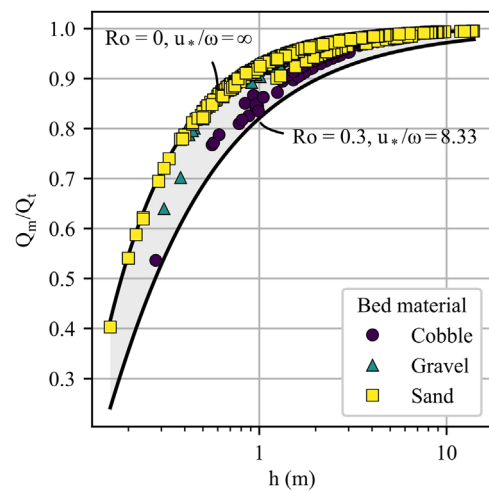


Fig. 9. Comparison of Korean measurements (1962 points) with the theoretical solution of Q_m/Q_t with $Ro=0$ and $Ro=0.3$ for $d_s = 2$ mm.

suspended material. The SEMEP calculated the total sediment load from 1962 samples. The SEMEP shows that the depth-integrated samples measured over 80% of the sediment load when the flow is larger than the mean discharge, and over 90% when the flow is 10 times larger than the mean. For sand bed rivers in South Korea, sediment is predominantly transported in suspension. The ratio of bedload increases when the grain size increases. When the discharge is larger than the mean discharge, at least 90% of sediment is transported in suspension for gravel and sand bed rivers.

The analytical solutions of Q_m/Q_t and Q_s/Q_t bring insight for practical applications. First, for rivers with fine suspended materials, such as the streams in South Korea, the ratio Q_m/Q_t depends only on water depth. When the depth is higher than 1 m, the depth-integrating sampler measures over 80% of the total sediment discharge. Second, when $Ro < 0.3$, Q_s/Q_t becomes a function of h/d_s and the measurements exceed 90% of the total sediment load when $h/d_s > 15$. For 2 mm sands, this corresponds to $h > 3$ cm.

Notation

a	L	thickness of bed layer, assumed $a = 2d_s$ in this study
A		ratio of nozzle height to flow depth z_n/h
C		sediment concentration by volume
C_a		sediment concentration at a distance a above the bed
d_{50ss}	L	median grain size of material in suspension
d_s, d_{50}	L	median grain size of bed material
d_{65}	L	grain size with 65% of the bed material finer
d_*		dimensionless particle diameter
E		ratio of bed layer thickness to flow depth, $E = a/h = 2d_s/h$
g	LT^{-2}	gravitational acceleration
G		specific weight of sediment
h	L	flow depth
J_1, J_2		Eqs. (12) and (13) defining the Einstein integrals for the suspended load
J'_1, J'_2		Eqs. (9) and (10) defining the Einstein integrals for the measured load
k_s	L	effective roughness
Q	M^3T^{-1}	flow discharge
\overline{Q}	M^3T^{-1}	mean annual discharge
q_b	$ML^{-1}T^{-1}$	unit bedload
q_m	$ML^{-1}T^{-1}$	unit measured load
Q_m	MT^{-1}	measured load
q_s	$ML^{-1}T^{-1}$	unit suspended load
Q_s	MT^{-1}	suspended load
q_t	$ML^{-1}T^{-1}$	unit total sediment load
Q_t	MT^{-1}	total sediment load
Ro		Rouse number
u_*	LT^{-1}	shear velocity
v	LT^{-1}	flow velocity
W	L	channel width
z	L	vertical elevation above the channel bed
z_n	L	vertical elevation of the sediment sampler nozzle above the channel bed
α, β		coefficients of the relationship of Ro and ω
β_s		ratio of the turbulent mixing coefficient of sediment to the momentum exchange coefficient

κ		von Karman constant, assumed equal to 0.4
ω	LT^{-1}	particle settling velocity
ν	L^2T^{-1}	kinematic viscosity
ϕ_*		Einstein's transport intensity
ψ_*		shear intensity

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References

- Baird, D. C., & Varyu, D. (2011). *Initial evaluation of the Series Expansion of the Modified Einstein Procedure (SEMEP) for computing total sediment load*. Denver, CO: Bureau of Reclamation, Technical Service Center.
- Burkham, D., & Dawdy, D. R. (1980). General study of the modified Einstein method of computing total sediment discharge (U.S. Geological Survey Water 265 Supply Paper 2066). Washington, DC.
- Colby, B. R., & Hembree, C. (1955). Computations of total sediment discharge, Niobrara River near Cody, Nebraska (U.S. Geological Survey Water Supply Paper 1357). Washington, DC.
- Colby, B. R., & Hubbell, D. W. (1961). Simplified methods for computing total sediment discharge with the Modified Einstein Procedure (U.S. Geological Survey Water Supply Paper 1593). Washington, DC.
- Einstein, H. A. (1950). The bed-load function for sediment transportation in open channel flows (Technical Bulletin No. 1026). Washington, DC: U.S. Department of Agriculture.
- Guo, J., & Julien, P. Y. (2004). Efficient algorithm for computing Einstein integrals. *Journal of Hydraulic Engineering*, 130, 1198–1201.
- Holmquist-Johnson, C., & Raff, D. (2006). Bureau of Reclamation Automated Modified Einstein Procedure (BORAMEP) Program for computing total sediment. In *Proceedings of the federal interagency sedimentation conference*. Reno, Nevada.
- Julien, P. Y. (2010). *Erosion and sedimentation* (2nd ed.). Cambridge, UK: Cambridge University Press.
- Julien, P. Y., Kang, W., Yang, C.-Y., Lee, J. H., Byeon, S., & Lee, J. (2017). Multivariate regression analysis and model development for the estimation of sediment yield from ungauged watershed in the Republic of Korea - final report presented to K-water. Fort Collins, CO.
- Keulegan, G. H. (1938). Laws of turbulent flow in open channels. *Journal of Research of the United States National Bureau of Standards*, 21, 707–741.
- Lara, J. M. (1966). Computation of Z's for use in the Modified Einstein Procedure. Denver, CO: Bureau of Reclamation.
- Rouse, H. (1937). Modern conceptions of the mechanics of fluid turbulence. *Transactions of the American Society of Civil Engineers*, 102, 463–505.
- Shah-Fairbank, S. C. (2006). *Variability in total sediment load using BORAMEP on the Rio Grande Low Flow Conveyance Channel* (Master's thesis). Fort Collins, CO: Colorado State University.
- Shah-Fairbank, S. C. (2009). *Series expansion of the Modified Einstein procedure* (Doctoral dissertation). Fort Collins, CO: Colorado State University.
- Shah-Fairbank, S. C., & Julien, P. Y. (2015). Sediment load calculations from point measurements in sand-bed rivers. *International Journal of Sediment Research*, 30, 1–12.
- Shah-Fairbank, S. C., Julien, P. Y., & Baird, D. C. (2011). Total sediment load from SEMEP using depth-integrated concentration measurements. *Journal of Hydraulic Engineering*, 137, 1606–1614.
- Shen, H. W., & Hung, C. S. (1983). Remodified Einstein Procedure for sediment load. *Journal of Hydraulic Engineering*, 109, 565–578.
- Stevens, H. H. (1985). Computer program for the computation of total sediment discharge by the modified Einstein procedure (U.S. Geological Survey Water Resources Investigations Report 85-4047). Reston, VA.
- Toffaletti, F. B. (1977). A procedure for computation of the total river sand discharge and detailed distribution bed to surface (Technical Report No. 5). Vicksburg, MS: Committee on Channel Stabilization, Corps of Engineers, U. S. Army.
- Turowski, J. M., Rickenmann, D., & Dadson, S. J. (2010). The partitioning of the total sediment load of a river into suspended load and bedload: A review of empirical data. *Sedimentology*, 57, 1126–1146.
- Wohl, E., Lane, S. N., & Wilcox, A. C. (2015). The science and practice of river restoration. *Water Resources Research*, 51, 5974–5997.
- Zamani, K., Bombardelli, F. A., & Kamrani-Moghaddam, B. (2016). Comparison of current methods for the evaluation of Einstein's integrals. *Journal of Hydraulic Engineering*, 143, 06016026.