Bed-Material Load Computations for Nonuniform Sediments

Baosheng Wu, M.ASCE1; Albert Molinas, M.ASCE2; and Pierre Y. Julien, M.ASCE3

Abstract: The nonuniformity of bed material affects the bed-material load calculations. A size gradation correction factor $K_d$ is developed to account for the lognormal distribution of bed material. The use of $K_d$ in conjunction with bed-material load equations originally developed for single particle sizes improves the accuracy of transport calculations for sediment mixtures. This method is applicable to laboratory flumes and natural rivers with median diameter $d_{50}$ of bed material in the sand size ranges. The improvement on transport rate by $K_d$ factor is significant for data with standard deviation $\sigma_g$ of bed material greater than 2, while the correction is negligible for data with $\sigma_g$ less than 1.5. Sediment in transport also follows a lognormal distribution with a median diameter $d_{50}$ generally finer than the corresponding $d_{50}$. As the size gradation increases, $d_{50}$ becomes much finer than the corresponding value of $d_{50}$. The relationship between $d_{50}$ and $d_{35}$ is defined as a function of $\sigma_g$ and agrees well with field data. The previously recommended use of $d_{35}$ as representative size of the bed material is found not to be generally applicable.

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Introduction

Riverbeds are usually composed of nonuniform sediment mixtures and the corresponding particle size distribution of sediment in transport is generally finer than the distribution of bed material because of selective transport. This makes the prediction of sediment load for natural rivers more difficult than for uniform sediment in laboratory flumes. To consider the effect of nonuniformity of bed material on sediment transport, various representative bed material sizes have been used for the computation of sediment transport rates. The commonly used representative sizes include: (1) the median diameter of bed material, $d_{50}$; (2) the diameter of bed material, $d_{35}$, for which 35% is finer as proposed by Einstein (1944) and Ackers and White (1973); (3) the mean diameter defined by Meyer-Peter and Müller (1948) as $d_m = \sum \Delta P_{db}d_i$, where $\Delta P_{db}$ is the fraction of bed material, by dry weight, corresponding to the size fraction $i$, and $d_i$ is the representative diameter of bed material corresponding to the size fraction $i$; (4) the mean fall velocity defined by Han (1973) as $\alpha_m = (\sum \Delta P_{db} \alpha_i d_i)^{1/0.4}$, where $\alpha_i$ is the fall velocity of particle of size $d_i$, and $m$ is an exponent; and (5) the effective diameter defined by Nordin (1989) as $d_e = 1/(\sum \Delta P_{db} / d_i)$.

The use of a single fixed size, such as $d_{50}$ or $d_{35}$, may not be adequate in representing the various size fractions present in sediment mixtures. As pointed out by White and Day (1982), grading curves with different shapes will certainly have different effective diameters. The effective sediment size is also expected to vary with the transport rate or flow intensity. Therefore, in addition to $d_{50}$, a sediment nonuniformity factor expressed by $d_{50}/d_{35}$ was used by Smart and Jaeggi (1983) to account for the effect of size distribution, and the size gradation coefficient defined by $G=d_{50}/d_{30}$ was used by Shen and Rao (1991), where $d_p$ is diameter for which $p$ percent of bed material is finer. The factors $d_{50}/d_{35}$, $G$, and others describing the gradation of mixtures are all believed to be significant in the transport of sediment mixtures because they represent to some extent the shape and range of particle sizes which are significantly present in the bed material.

Instead of using a single fixed size or a single fixed size with a size gradation parameter as the representative property of bed material, van Rijn (1984), Hsu and Holly (1992), Molinas and Wu (1998), and Wu (1999) suggested the use of variable representative sizes for the computation of sediment transport rates for sediment mixtures. The variable representative size is analogous to the median size or other characteristic sizes of sediments in transport. It is believed that the variable representative size is a better representation of the sediment mixture than not only a fixed particle diameter such as $d_{35}$ or $d_{50}$ of bed material, but also the simple combination of a fixed representative size and a size gradation factor.

In the development of a suspended load transport equation, van Rijn (1984) proposed an empirical equation to estimate the representative diameter $d_s$ for suspended sediment load. The equation was determined by trial and error to give the same value for the suspended load as that computed with Einstein’s method. This equation is expressed as

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where \( T \) = transport stage parameter defined by \( T = (\tau' - \tau_c)/(\gamma \tau_c) \); and \( \tau' \) and \( \tau_c \) = grain shear stress and critical shear stress, respectively.

Hsu and Holly (1992) performed an interesting study on the bedload transport for sediment mixtures. In their study, they proposed a model for the computation of the mean size, \( \bar{d}_{mt} \), of sediments in transport. Based on observation of sediment-mixture experiments, Hsu and Holly (1992) postulated that the fraction of each size class in transported material is proportional to the joint probability of the relative mobility (\( \Delta P_{mi} \)) of each particle size and the availability (\( \Delta P_{bi} \)) of each size class on the bed surface. From this concept, they expressed the size distribution of the transported bedload sediments as

\[
\Delta P_{ci} = \frac{\Delta P_{mi} \Delta P_{bi}}{\sum_{i=1}^{N} (\Delta P_{mi} \Delta P_{bi})}
\]

(2)

where

\[
\Delta P_{mi} = \frac{1}{\sigma \sqrt{2\pi}} \int_{V_{ci}/V}^{\infty} \exp \left( -\frac{x^2}{2\sigma^2} \right) dx = 0.5 - 0.5 \text{erf} \left( \frac{V_{ci}/V - 1}{\sigma \sqrt{2}} \right)
\]

(3)

where erf(\( z \)) = error function; \( V \) = cross-sectional average velocity; \( V_{ci} \) = incipient velocity for a particular size class \( i \) in a mixture; \( \sigma \) = standard deviation of \( V' \) / \( V \) distribution; and \( V' \) = absolute fluctuations of velocity.

From the size distribution computed utilizing Eq. (2), the mean size \( d_{mt} \) can be determined. Hsu and Holly argued that if \( d_{mt} \) is visualized as the representative property of a uniform sediment, the bedload discharge could be evaluated using any appropriate bedload equations.

Molinas and Wu (1998) developed a size gradation compensation factor to incorporate the effect that the size distribution has on the transport of sediment mixtures. The resulting equivalent representative diameter, \( d_r \), can be expressed as

\[
d_r = \frac{1.8d_{50}}{1 + 0.8(V_{ci}/V_{so})^{0.5}(\sigma_y - 1)^{2.2}}
\]

(4)

where \( V_{ci} \) = shear velocity; \( \sigma_y \) = dimensionless standard deviation of bed material, which is equal to \( \sqrt{d_{42}/d_{50}} \) and \( V_{so} \) = fall velocity of sediment corresponding to particle size \( d_{50} \). This equivalent representative diameter was proposed for existing sediment transport formulas to produce more accurate prediction of transport rates for nonuniform mixtures.

In the proceeding approaches, the ultimate goal in defining a variable representative size is to improve the prediction of sediment transport rates for nonuniform mixtures. Unfortunately, the representative size of Eq. (1) is developed based on the results computed with Einstein’s method; and it is limited to suspended load. The representative size based on Eq. (2) proposed by Hsu and Holly is for bed load; and although representing a promising approach it is not verified with measurements. The equivalent diameter given by Eq. (4) was mainly developed to compensate for sediment nonuniformity effects for existing transport formulas in bed-material load computations, so it lacks generality.

In this paper, the effect of bed material nonuniformity on the transport of sediment mixtures in sand-bed channels is studied. A size gradation correction factor is derived based on the lognormal distribution of bed material. The median diameter \( d_{50} \) of sediment in transport and the variable representative size for the computation of bed-material load are discussed.

**Lognormal Size Distribution of Bed Material**

The particle size distribution of bed material is generally skewed (Mahmood 1973a, b). Particle size distributions can often be converted into symmetrical, nearly Gaussian (normal) distribution by a logarithmic transformation. The corresponding particle size distribution in this case is called a lognormal particle size distribution.

Two examples of the lognormal particle size distribution are presented in Fig. 1. The data shown in Fig. 1 were obtained in Rio Grande near Bernalillo, New Mexico on June 1, 1953 and June 18, 1958, respectively (Nordin and Beverage 1965). The frequency distributions displayed in Figs. 1(a and c) are obviously skewed. However, when the particle diameters are plotted on a logarithmic scale against the frequency of occurrence, bell-shaped curves or lognormal curves as shown in Figs. 1(b and d) are generated. Fig. 2 shows the normalized log-probability plot of a large number of bed materials from Rio Grande. It can be seen that the size distribution from 10th to 90th percentile is closely approximated by lognormal distribution. This type of lognormal bed material size distributions is often encountered in most alluvial rivers with sand sediments.

If two variables \( x \) and \( y \) are related such that \( y = \ln(x) \), where \( 0 < x < \infty \), and if \( y \) follows a Gaussian distribution with mean \( \mu_y \) and standard deviation \( \sigma_y \), given by

\[
F_y(u) = \int_{-\infty}^{u} \frac{1}{\sigma_y \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{u - \mu_y}{\sigma_y} \right)^2 \right] \text{du}
\]

(5)

then, variable \( x \) is lognormally distributed as

\[
F_x(v) = \int_{0}^{v} \frac{1}{\sigma_v \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{\ln(v) - \mu_y}{\sigma_y} \right)^2 \right] \text{dv}
\]

(6)

where \( u \) and \( v \) = dummy variables of integration.

The bed material size distributions shown in Figs. 1 and 2 can be best described by the cumulative distribution function (CDF) of lognormal distribution expressed by Eq. (5) or (6) given that \( x \) is the particle size. The lognormal distribution is a skewed distribution and the two parameters required to define this distribution are \( \mu_y \) and \( \sigma_y \). In defining sediment mixtures, the median diameter \( d_{50} \) and the geometric standard deviation \( \sigma_y \) of the bed material are commonly reported. For this two-parameter lognormal distribution, it can be shown that \( d_{50} \) and \( \sigma_y \) are related to \( \mu_y \) and \( \sigma_y \) as

\[
\mu_y = \ln(d_{50})
\]

(7)

and

\[
\sigma_y = \ln(\sigma_y)
\]

(8)

In other words for naturally occurring sediment mixtures, the lognormal distribution is defined by \( d_{50} \) and \( \sigma_y \).

By a simple transformation, the distribution expressed by Eq. (5) can be written as a standard normal distribution \( N(0, 1) \). Thus when \( z = (y - \mu_y)/\sigma_y \), \( dz = \sigma_y \text{dz} \), the probability density function becomes

\[
f_z(z) = \frac{1}{\sigma_y \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{z}{\sigma_y} \right)^2 \right]
\]
The variable $z$ is called the standard unit, which is normally distributed with zero mean and unit standard deviation.

### Effects of Nonuniformity of Bed Material

There are two types of methods commonly used to compute the transport rates for nonuniform mixtures. The first type of method is based on the computation of transport rates for each size fraction present in the nonuniform mixture. After knowing the transport rates corresponding to each size fraction, the total bed-material transport rate is determined by summation of the fractional transport rates. The classical Einstein method (Einstein 1950) is an excellent example in this category. This type of method was generally found unsatisfactory in predictions of total bed-material transport rate for sediment mixtures due to the complexity of transport of sediment mixtures and the lack of knowledge concerning the motion of individual size and its effect on other sizes (Misri et al. 1984; Samaga et al. 1986b; Swamee and Ojha 1991).

The second type of methods computes the total bed-material transport rate based on a single representative size for graded sediment mixtures. They usually can produce more reliable predictions and have been widely used in practice. The formula of Engelund and Hansen (1967) developed based on the median bed material size $d_{50}$ is well known in this category. It can be expressed as

$$f' \Phi = 0.16^{2.5}$$

where

$$\theta = \frac{\tau}{(\gamma_s - \gamma)d_{50}}$$

**Fig. 1.** Frequency histogram of bed-material size distribution for samples obtained in Rio Grande near Bernalillo, New Mexico: (a) and (b) Data observed on June 1, 1953, $d_{50}=0.33$ mm, $\sigma_s=1.62$; (c) and (d) data observed on June 18, 1958, $d_{50}=0.25$ mm, $\sigma_s=1.4$

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Where the variable $z$ is normally distributed with zero mean and unit standard deviation.

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$

and the CDF

$$F_z(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-u^2/2} du$$

**Fig. 2.** Bed-material size distribution plotted on log-probability paper for the 112 samples measured over 1952–1962 in Rio Grande at Albuquerque and Bernalillo, New Mexico ($d_{50}=0.18–0.43$ mm, $\sigma_s=1.36–2.78$, actual sizes normalized to yield $\sigma_s=1.8$)
\[ \Phi = \frac{q_t}{\gamma (s_z - 1)gd_{50}} \]  
(13)

where \( f' \) = friction factor defined by Engelund and Hansen; \( \theta \) = dimensionless shear parameter; \( \Phi \) = dimensionless sediment transport function; \( \tau \) = shear stress along the bed; \( g \) = gravitational acceleration; \( q_t \) = total bed-material sediment discharge by weight per unit width; \( s_z \) = specific gravity given by \( \gamma_t/\gamma \); and \( \gamma \) and \( \gamma_t \) = specific weight of water and sediment, respectively.

Conceptually the Engelund and Hansen method can be applied to compute the fractional transport rates for nonuniform sediment mixtures by replacing \( d_{50} \) with the average (or geometric mean) diameter \( d_i \) of the corresponding size fraction. This concept assumes that a channel bed can be considered as a hypothetical mixture of sediment particles; the mixture can be formed into class intervals by size, and a potential transport capacity can be calculated for each class interval, whether or not particles are physically present. Subsequently, particle availability can be evaluated and expressed as \( \Delta P_{bi} \). Availability and potential transport capacity can then be combined to give transport capacity as follows:

\[ Q_i = \sum_{i=1}^{N} Q_{bi} = \sum_{i=1}^{N} \Delta P_{bi}Q_{ij} \text{ or } Q_i = \int_{-\infty}^{\infty} Q_{j}f(u)du \]  
(14)

where \( Q_i \) = total bed-material transport rate; \( Q_{bi} \) = fractional bed-material transport rate; \( Q_{ij} \) = potential bed-material transport rate for size fraction \( i \) assuming uniform sediment of size \( d_i \) under identical hydraulic conditions; \( i \) denotes the size fraction number in a mixture; \( N \) = number of size fractions present in the sediment mixture; and \( f(u) \) = density function of lognormal size distribution expressed by Eq. (9). The concept expressed by Eq. (14) neglects the sheltering-exposure effects in rivers with mixed sizes. Fortunately, this phenomenon is not significant in sand-bed rivers since the nonuniform sediment is commonly under full motion. Keep this in mind, further justifications are needed if this concept is to be extended to gravel-bed rivers.

According to the Engelund and Hansen equation, the sediment transport rate is inversely proportional to particle diameter \( d \), i.e. \( Q_i \propto d^{-1} \). If another form of the Engelund and Hansen equation \( f'\Phi=0.30\sqrt{\theta^2+0.15} \) is considered, then we get \( f'\Phi=2 \) for small \( \theta \) and \( f'\Phi=0 \) for large \( \theta \) (Chien and Wan 1999), resulting in \( Q_i \propto d^{-(0.5-1.5)} \). In addition, the methods by Bagnold (1966), Velikanov (1954), and Dou (1974) show that \( Q_i \) is inversely proportional to \( \omega \), while Zhang (1959) and Zhang and Xie (1993) indicates \( Q_i \propto \omega^{-0.5-1.5} \) and Molinas and Wu (2001) gives \( Q_i \propto \omega^{-1.5} \), where \( \omega \) is the fall velocity of sediment. Considering \( \omega \propto d^{0.5-2} (\propto d^2 \text{ for } d<0.1 \text{ mm and } \propto d^{0.5} \text{ for } d>1.0 \text{ mm}) \), it is more general to assume that

\[ Q_i \propto Cd^{-b} \]  
(15)

where \( C \) = integrated coefficient; and \( b \) = exponent.

It is expected that differences exist between the total bed-material transport rate \( Q_i \) obtained from Eq. (14) and from equation like Eq. (11) based on \( d_{50} \). Let’s denote \( K_d \) the ratio of \( Q_i \) obtained by these two different methods, i.e.

\[ K_d = \frac{Q_i}{Q_{bi}} \text{ by size fractions for lognormal distribution} \]

\[ = \frac{\int_{-\infty}^{\infty} Q_{j}f(u)du}{Q_{s}d_{50}} \]  
(16)

Considering that \( Q_{j}=Cd^{-b} \) and \( Q_{s}d_{50}=Cd_{s0}^{-b} \), Eq. (16) can be expressed as

\[ K_d = \int_{-\infty}^{\infty} \left( \frac{d}{d_{50}} \right)^{-b} f(u)du \]  
(17)

From the definition of \( z \) we have

\[ z = \frac{y - u}{\sigma_g} = \frac{\ln d - \ln d_{50}}{\ln \sigma_g} \text{ or } \frac{d}{d_{50}} = -\sigma_g^z \]  
(18)

Thus

\[ K_d = \int_{-\infty}^{\int_{-\infty}^{\infty} \left( \frac{d}{d_{50}} \right)^{-b} f(u)du \]  
\[ = \int_{-\infty}^{\infty} \left( \frac{d}{d_{50}} \right)^{-b} e^{-0.5d^2} du \]  
\[ = \int_{-\infty}^{\infty} e^{-b\mu} \ln \sigma_g \left( \frac{1}{\sqrt{2\pi}} \right) e^{-0.5\sigma_g^2} du \]  
\[ = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-0.5\sigma_g^2} e^{-b\mu} \ln \sigma_g du \]  
\[ = e^{0.5(b \ln \sigma_g)^2} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-0.5(\alpha + \beta \ln \sigma_g)^2} du \]  
(19)

Finally, we get

\[ K_d = e^{0.5(b \ln \sigma_g)^2} \]  
(20)

The earlier equation shows that \( K_d \) increase with the increase in \( \sigma_g \) values, having a minimum value of 1 corresponding to uniform distribution or \( \sigma_g=1 \). This means that a sediment transport equation developed for uniform sediments based \( d_{50} \) usually underpredicts the transport rate for nonuniform mixtures. As such, \( K_d \) can be used as a correction factor to obtain the correct prediction for nonuniform sediment mixtures in conjunction with a sediment transport equation, such as the Engelund and Hansen equation, originally developed for uniform sediment.

**Characteristic Particle Sizes**

The size distribution of sediment in transport is different from that of bed material. Consequently, the median diameter of sediment in transport is different from that of the bed material. Similar to Eq. (16), the CDF of sediment in transport can be obtained by
F_0(z) = \frac{\int\int_{x,y} Q_{spi} f(u) du}{\int\int_{x,y} Q_{spi} f(u) du} 
= (Cd_{50}^{b}) e^{0.5(\ln \sigma_{g})^{2}} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\xi} e^{-0.5u^{2} - bu \ln \sigma_{g}} du 
= (Cd_{50}^{b}) e^{0.5(\ln \sigma_{g})^{2}} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\xi} e^{-0.5u^{2} - bu \ln \sigma_{g}} du 
\tag{21}

or

F_0(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\xi} e^{-0.5(u + b \ln \sigma_{g})^{2}} du 
\tag{22}

Eq. (22) indicates that the sediment in transport also has a lognormal distribution. The 50 percentile of the particle size distribution of transported sediment d_{50} corresponds to the value of z in Eq. (22) that gives F_0(z)=0.5, i.e.

0.5 = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\xi} e^{-0.5u^{2} \ln \sigma_{g}} du 
\tag{23}

Denoting \xi = z + b \ln \sigma_{g}, then dz=d\xi, Eq. (23) becomes

0.5 = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\xi} e^{-0.5\xi^{2}} du 
\tag{24}

Eq. (24) is a standard normal distribution. It can hold only when \xi=0 or z + b \ln \sigma_{g}=0. Thus substituting z=-b \ln \sigma_{g} into Eq. (18) results in

\frac{d_{50}}{d_{50}} = \sigma_{g}^{-b \ln \sigma_{g}} 
\tag{25}

Eq. (25) describes the relationship between d_{50} and d_{50}. There exists a bed material size which matches d_{50}. In order to determine the bed material size corresponding to d_{50}, a bed material size is set equal to d_{50} and the corresponding percentage is computed.

According to Eq. (18), the pth percentile of the sediment size distribution d_{p} can be determined by

d_{p} = d_{50} \sigma_{g}^{\xi_{p}} 
\tag{26}

where \xi_{p}=p^{th} percentile of standard normal distribution \[-\mathcal{N}(0,1)]

For P<50\%, the value of \xi_{p} in Eq. (26) is negative which results in a value of d_{p} smaller than the corresponding value of d_{50}. For P>50\%, the value of \xi_{p} is positive which gives d_{p} greater than d_{50}. Assuming d_{p}=d_{50} and combining Eqs. (25) and (26) yields

\sigma_{g}^{\xi_{p}} = \sigma_{g}^{-b \ln \sigma_{g}} 
\tag{27}

and

\xi_{p} = -b \ln \sigma_{g} 
\tag{28}

Using the value of \xi_{p} given by Eq. (28) as the upper boundary for the standard normal distribution, the percentage for which the diameter of bed material corresponds to d_{50} for a given \sigma_{g} can be determined by

\frac{d_{p}}{d_{50}} = \sigma_{g}^{-0.385} 
\tag{29}

Substituting Eq. (28) into Eq. (29) yields

\frac{d_{55}}{d_{50}} = \sigma_{g}^{-0.385} 
\tag{30}

The relationship between P and \sigma_{g} given by Eq. (30) is shown in Fig. 3. For uniform sediment, P=50\% which means that the median size, d_{50}, of sediment in transport is equal to the median diameter d_{50} of bed material. As the value of \sigma_{g} increases, the P value decreases, resulting in a smaller bed material diameter that equals to d_{50}.

An appropriate variable representative diameter d_{e} may be used for the computation of sediment transport rates for sediment mixtures. The use of d_{e} is equivalent to the K_{d} factor to account for the effect of size gradation, resulting in K_{d}Q_{sd}=Q_{sde}. Considering that Q_{sd} \approx Cd_{50} and Q_{sd} \approx Cd_{e}^{b}, the variable representative diameter now can be expressed as

\frac{d_{e}}{d_{50}} = e^{-0.5b \ln \sigma_{g}} 
\tag{31}

It is mentioned earlier that Einstein (1944) and Ackers and White (1973) suggested the use of d_{50} as the representative size in sediment load computations for nonuniform mixtures. For this special case, P=35\% and \xi_{p}=-0.385, which results in, according to Eq. (26), the following relation

\frac{d_{35}}{d_{50}} = \sigma_{g}^{-0.385} 
\tag{32}

Test of the Correction Factor

The exponent b may be determined based on measured data for fractional transport rates for sediment mixtures since this paper focuses on effects of sediment nonuniformity. For this purpose the relative fractional transport rates/capacities of each data set in selected laboratory experiments and natural rivers are plotted in Fig. 4 to check the variation of transport capacities with sediment sizes. The procedure to find the relative transport capacity for each size is illustrated in Table 1. In this table the values of d_{e}, \Delta P_{br} Q_{pi} are direct measurements, Q_{spi} is computed by Q_{si}/\Delta P_{br}.
It can be seen from Fig. 4 that majority of the data sets show a similar trend in which the relative transport capacity decreases with the increase of particle size. A trend line may be drawn for the data shown in the figure, i.e.

$$\log\left(\frac{Q_{sp_i}}{Q_{sp_{50}}}\right) = -1.2 \log\left(\frac{d_i}{d_{50}}\right)$$

or

$$\frac{Q_{sp_i}}{Q_{sp_{50}}} = \left(\frac{d_i}{d_{50}}\right)^{-1.2}$$

This results in a value of $b$ in Eq. (20) to be 1.2. It is expected that $b$ should vary with particle size and flow intensity, showing nonlinear variation. However, for simplicity it is assumed to be a constant value in this paper. Since the Engelund and Hansen equation was developed based on relatively uniform sediments in the sand range, the validity of $K_d$ correction should be tested using nonuniform sediments in sand range and with relatively high $g$ values. It is expected that the sediment transport rate would be underestimated by Engelund and Hansen’s original equation. The use of $K_d$ would then produce better predictions by accounting for the effects of nonuniformity of bed material.

Even though a lot of laboratory and field sediment transport data for sand sizes can be found in the literature, only a few have relatively high $g$ values. After careful review, the laboratory data

![Fig. 4. Variation of fractional transport capacity with relative particle size](image)

**Table 1.** An Example to Illustrate the Computation of Relative Transport Rates for Measured Data

<table>
<thead>
<tr>
<th>Representative diameter of group I $d_i$ (mm)</th>
<th>Relative diameter $d_i/d_{50}$</th>
<th>Size fraction of group $i$ $\Delta P_{bi}$</th>
<th>Fractional transport rate $Q_{bi}$ (kg/s/m)</th>
<th>Potential transport capacity $Q_{sp}$ (kg/s/m)</th>
<th>Relative transport capacity $Q_{sp_i}/Q_{sp_{50}}$</th>
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</thead>
<tbody>
<tr>
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<td>0.181</td>
<td>0.044</td>
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<td>0.020</td>
<td>0.314</td>
<td>0.305</td>
</tr>
<tr>
<td>0.986</td>
<td>4.870</td>
<td>0.034</td>
<td>0.012</td>
<td>0.348</td>
<td>0.338</td>
</tr>
</tbody>
</table>

Note: The data is extracted from Einstein and Chien’s Laboratory Data No. 22, $d_{50}=0.135$ mm; $Q_{sp_i}=Q_{si}/\Delta P_{bi}$; and $Q_{sp_{50}}$=potential transport capacity corresponding to $d_{50}$.

**Table 2.** Summary of Laboratory and Field Data Used for Testing $K_d$ Correction

<table>
<thead>
<tr>
<th>Data source</th>
<th>Flow discharge (m$^3$/s)</th>
<th>Flow depth (m)</th>
<th>Median diameter of bed material (mm)</th>
<th>Geometric standard deviation of bed material</th>
<th>Bed-material concentration</th>
<th>Number of data sets</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Laboratory data</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Einstein and Chien (1953)</td>
<td>0.043–0.066</td>
<td>0.18–0.21</td>
<td>0.10–0.37</td>
<td>1.41–2.95</td>
<td>2,115–57,970</td>
<td>22</td>
</tr>
<tr>
<td>Samaga et al. (1986a,b)</td>
<td>0.0056–0.015</td>
<td>0.056–0.10</td>
<td>0.21–0.40</td>
<td>1.58–2.46</td>
<td>3,392–10,260</td>
<td>33</td>
</tr>
<tr>
<td>(b) River data</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Atchafalaya River (Toffaleti 1968)</td>
<td>382–14,190</td>
<td>6.10–14.75</td>
<td>0.091–0.31</td>
<td>1.50–1.93</td>
<td>0.6–567</td>
<td>72</td>
</tr>
<tr>
<td>Mississippi River at Tarbert</td>
<td>4,248–28,830</td>
<td>6.74–16.40</td>
<td>0.178–0.327</td>
<td>1.38–2.00</td>
<td>12–262</td>
<td>53</td>
</tr>
<tr>
<td>Landing (Toffaleti 1968)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rio Grande River (Toffaleti 1968)</td>
<td>35.1–286.0</td>
<td>0.33–1.46</td>
<td>0.214–0.387</td>
<td>1.62–1.88</td>
<td>463–4,530</td>
<td>38</td>
</tr>
<tr>
<td>American Canal (Simon 1957)</td>
<td>1.22–29.4</td>
<td>0.80–2.59</td>
<td>0.096–0.715</td>
<td>2.01–3.85</td>
<td>44–448</td>
<td>12</td>
</tr>
<tr>
<td>Total of laboratory and river</td>
<td>0.0056–28,830</td>
<td>0.056–16.4</td>
<td>0.091–0.715</td>
<td>1.38–3.85</td>
<td>0.6–57,970</td>
<td>230</td>
</tr>
</tbody>
</table>
Table 3. Summary of Comparison between Computed and Measured Bed-Material Concentrations for Laboratory Data

<table>
<thead>
<tr>
<th>Method</th>
<th>Data source</th>
<th>Data in range of discrepancy ratio ( R_j(%) )</th>
<th>Average geometric deviation</th>
<th>Root mean square</th>
<th>Number of data sets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engelund and Hansen equation based on ( D_{50} )</td>
<td>Einstein &amp; Chien</td>
<td>13.6</td>
<td>10.8</td>
<td>5.0</td>
<td>2.34</td>
</tr>
<tr>
<td></td>
<td>Smaga et al.</td>
<td>27.3</td>
<td>90.9</td>
<td>90.9</td>
<td>1.52</td>
</tr>
<tr>
<td></td>
<td>All laboratory data</td>
<td>21.8</td>
<td>92.7</td>
<td>74.6</td>
<td>1.80</td>
</tr>
<tr>
<td>Engelund and Hansen equation corrected by ( K_d )</td>
<td>Einstein &amp; Chien</td>
<td>45.5</td>
<td>96.4</td>
<td>72.7</td>
<td>1.66</td>
</tr>
<tr>
<td></td>
<td>Samaga et al.</td>
<td>84.9</td>
<td>100.0</td>
<td>100.0</td>
<td>1.14</td>
</tr>
<tr>
<td></td>
<td>All laboratory data</td>
<td>69.1</td>
<td>94.6</td>
<td>89.1</td>
<td>1.33</td>
</tr>
</tbody>
</table>

by Einstein and Chien (1953) and (Samaga et al. 1986a,b) and the river data from Atchafalaya River (Tofaleta 1968), Mississippi River at Tarbert Landing (Tofaleta 1968), Rio Grande River (Tofaleta 1968), American Canal (Simons 1957) were selected, see Table 2. The \( \sigma_g \) values of laboratory data by Einstein and Chien and Samaga et al. are in the range of 1.4–3.0 and 1.6–2.5, respectively, with most data points bigger than 2.0; the \( d_{50} \) values are in the range of 0.10–0.37 and 0.21–0.40 mm, respectively. The \( \sigma_g \) values of river data from Atchafalaya River, Mississippi River, Rio Grande River, American Canal are in the range of 1.5–1.9, 1.4–2.0, 1.6–1.9, and 2.0–3.9, respectively; the \( d_{50} \) values are in the range of 0.178–0.327, 0.214–0.387, and 0.096–0.715 mm, respectively.

The expression for the root mean square between computed and measured bed-material concentrations was reduced from 2.34 and 16,580 to 1.66 and 12,810, respectively, for the Einstein and Chien data, and from 1.52 and 2,403 to 1.14 and 1,108, respectively, for the Samaga et al. data. The improvement in discrepancy ratio happened in all ranges. Taking the range of 0.5–2.0 as an example, the improvement was from 50.5 to 72.7% for the Einstein and Chien data, and from 90.9 to 100.0% for the Samaga et al. data.

Similar improvement was observed in river data as indicated in Table 4. The average geometric deviation and the root mean square between computed and measured bed-material concentrations were reduced from 2.34 and 16,580 to 1.66 and 12,810, respectively, for the Einstein and Chien data, and from 1.52 and 2,403 to 1.14 and 1,108, respectively, for the Samaga et al. data. The improvement in discrepancy ratio happened in all ranges. Taking the range of 0.5–2.0 as an example, the improvement was from 50.5 to 72.7% for the Einstein and Chien data, and from 90.9 to 100.0% for the Samaga et al. data.

Significant improvements for laboratory data by using \( K_d \) correction were demonstrated in Fig. 5. Improvements in predictions for river data can also be observed in Fig. 6. The improvement for the data from the American Canal was higher than for data from other rivers. The relatively small improvements by \( K_d \) factor for the Atchafalaya River and Mississippi River were partially resulted from Table 3 it can be seen that the average geometric deviation and the root mean square between computed and measured bed-material concentrations were reduced from 2.34 and 16,580 to 1.66 and 12,810, respectively, for the Einstein and Chien data, and from 1.52 and 2,403 to 1.14 and 1,108, respectively, for the Samaga et al. data. The improvement in discrepancy ratio happened in all ranges. Taking the range of 0.5–2.0 as an example, the improvement was from 50.5 to 72.7% for the Einstein and Chien data, and from 90.9 to 100.0% for the Samaga et al. data.

3. The root mean square

\[
\text{RMS} = \left( \frac{1}{J} \sum_{j=1}^{J} (C_{tcj} - C_{tmj})^2 \right)^{1/2}
\]

where \( J \) is the total number of data sets.

Table 4. Summary of Comparison between Computed and Measured Bed-Material Concentrations for River Data

<table>
<thead>
<tr>
<th>Method</th>
<th>Data source</th>
<th>Data in range of discrepancy ratio ( R_j(%) )</th>
<th>Average geometric deviation</th>
<th>Root mean square (ppm)</th>
<th>Number of data sets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engelund and Hansen equation based on ( D_{50} )</td>
<td>Atchafalaya River</td>
<td>9.7</td>
<td>36.1</td>
<td>79.2</td>
<td>40.3</td>
</tr>
<tr>
<td></td>
<td>Mississippi River</td>
<td>17.0</td>
<td>64.2</td>
<td>94.3</td>
<td>66.0</td>
</tr>
<tr>
<td></td>
<td>Rio Grande River</td>
<td>44.7</td>
<td>73.9</td>
<td>100.0</td>
<td>76.3</td>
</tr>
<tr>
<td></td>
<td>American Canal</td>
<td>16.7</td>
<td>41.7</td>
<td>66.7</td>
<td>41.7</td>
</tr>
<tr>
<td></td>
<td>All river data</td>
<td>20.0</td>
<td>53.1</td>
<td>87.4</td>
<td>56.0</td>
</tr>
<tr>
<td>Engelund and Hansen equation corrected by ( K_d )</td>
<td>Atchafalaya River</td>
<td>18.1</td>
<td>48.6</td>
<td>83.3</td>
<td>51.4</td>
</tr>
<tr>
<td></td>
<td>Mississippi River</td>
<td>43.4</td>
<td>77.4</td>
<td>94.3</td>
<td>79.2</td>
</tr>
<tr>
<td></td>
<td>Rio Grande River</td>
<td>31.6</td>
<td>79.0</td>
<td>97.4</td>
<td>86.8</td>
</tr>
<tr>
<td></td>
<td>American Canal</td>
<td>16.7</td>
<td>50.0</td>
<td>75.0</td>
<td>50.0</td>
</tr>
<tr>
<td></td>
<td>All river data</td>
<td>28.6</td>
<td>64.0</td>
<td>89.1</td>
<td>67.4</td>
</tr>
</tbody>
</table>
from the small \( \sigma_g \) values in these two large rivers. Further improvement for these two large rivers may need to consider other flow parameters, which is beyond the scope of this paper.

Fig. 7 shows the variations of the relative median diameter defined by \( d_{50}/d_{50} \) with the geometric standard deviation \( \sigma_g \) of bed material. A total of 335 data values is shown in Fig. 7, including the flume data of Einstein (1978), Einstein and Chien (1953), and Guy et al. (1966), and the field data from the Niobrara River near Cody, Nebraska (Colby and Hembree 1955), and the Middle Loup River data at Dunning, Nebraska (Hubbell and Matejka 1959). This database is limited to sand sizes with median diameter in the range of 0.104–1.039 mm, to geometric standard deviations in the range of 1.245–2.968, to flow discharges in the range of 0.019–16.06 m\(^3\)/s, to velocities in the range of 0.22–1.90 m/s, to depths in the range of 0.058–0.576 m, and to slopes in the range of 0.00023–0.0193. Table 1 presents a summary of this database.

In Fig. 7, size distribution data including the unmeasured load near the bed surface evaluated by the use of indirect methods are
not included. For the laboratory data, the size distributions of transported sediments were measured directly. The transported sediment size distribution data for the Niobrara River near Cody, Nebraska, are obtained from measured values of suspended bed-material concentrations at a contracted section and are based on depth integrated samples. The size distribution of sediments in transport reported for the Middle Loup River at Dunning, Nebraska, are measured values of suspended bed-material size distribution data for the Niobrara River near Cody, Nebraska, are measured values of suspended bed-material concentrations with a turbulence flume and are also based on depth integrated samples. The size distribution of sediments in transport for the Middle Loup River at Dunning, Nebraska, are measured values of suspended bed-material concentrations with a turbulence flume and are also based on depth integrated samples.

From Fig. 7 it can be seen that the value of $d_{50}$ is finer than the corresponding value of $d_{50}$, and that as $\sigma_g$ increases the value of $d_{50}/d_{50}$ decreases. The reason is that the finer sizes in sediment mixtures are more readily transported by flow, which is commonly referred to as the selective transport of grains by flow or hydraulic sorting. It is a significant phenomenon in the transport process of nonuniform sediments.

The equation line given by Eq. (25) is plotted in Fig. 7 along with the measured data. It is seen that the measurements follow the equation line closely.

Fig. 8 is a plot showing the variation of relative diameters of $d_{35}/d_{50}$, $d_{20}/d_{50}$, and $d_{50}/d_{50}$ with $\sigma_g$. It is seen that the relative diameter of $d_{35}/d_{50}$ equals that of $d_{50}/d_{50}$ at $\sigma_g=1.38$ and $d_{20}/d_{50}$ is $d_{50}/d_{50}$ at $\sigma_g=1.9$. The $d_{35}$ and $d_{50}$ both have values smaller than $d_{50}$ so the use of $d_{35}$ or $d_{50}$ can give higher transport rate than based on $d_{50}$. However, the use of $d_{35}$ or $d_{50}$ as representative size is valid only for data with $\sigma_g$ values around 1.4 and 1.9, respectively.

The proposed size gradation correction factor $K_d$ can be applied in practice for bed-material load computation in case of nonuniform sediments. The procedure is illustrated using the data measured at Tarbert Landing, Mississippi River on April 16, 1965 ($Q=24,468$ m$^3$/s, $W=1,103$ m, $h=014.42$ m, $S=0.0000365$, $T=15.0^\circ C$, $d_{35}=0.167$ mm, $d_{50}=0.199$ mm, $\sigma_g=1.648$, $C_i=136$ ppm, and $d_{50}=0.107$ mm (from suspended load)). The detailed procedure for applying the proposed method is as follows.

Step 1. First calculate the transport rate with $d_{50}$ of the bed only. From the data given earlier, the bed-material concentration calculated by using the Engelund and Hansen equation is $C_{50}=100.7$ ppm.

Step 2. Then calculate $K_d$ and correct the calculations. According to Eq. (20) we have $K_d=\frac{d_{35}}{d_{50}}=\frac{d_{20}}{d_{50}}=1.38$ and $d_{50}/d_{50}=1.9$. Applying the $K_d$ factor gives the corrected bed-material concentration $C_{50}=1.20 \times 100.7=120.5$ ppm.

Step 3. Calculate the $d_{35}$, $d_{20}$, and $d_{50}$.

These three characteristic sizes can be calculated from Eqs. (32), (31), and (25), respectively, and giving $d_{35}=0.164$ mm, $d_{20}=0.171$ mm, and $d_{50}=0.147$ mm.

Step 4. Compare the computed results with field measurements.

It is obvious that the corrected bed-material concentration 120.5 ppm, comparing with the value of 100.7 ppm calculated by the Engelund and Hansen equation, is more close to the measured value of 136 ppm. As expected, the variable representative diameter of bed material $d_{50}=0.171$ mm is finer than the measured $d_{50}=0.199$ mm of bed material. The calculated value of $d_{50}=0.147$ mm is much coarser than measurement, which may be explained by the fact that $d_{50}=0.107$ mm is obtained from only measured suspended load and the measured value of $d_{50}$ for total bed-material load is not available.

Summary and Conclusions

The effects of nonuniformity of bed material on the transport of sediment mixtures are studied extensively. From the analysis, the following conclusions can be reached.

1. Sediment transport equations based on $d_{50}$ for uniform sediments usually underestimate the transport rates for nonuniform sediment mixtures. The size gradation correction factor $K_d$ expressed by Eq. (20) is a function of the geometric standard deviation of bed material. It is theoretically derived from the fractional transport concept based on a lognormal particle size distribution of the bed material. The use of $K_d$ in conjunction with a sediment transport equation based on a single representative size for uniform sediments can produce more accurate predictions for nonuniform sediment mixtures.

2. Similar to the bed material size distribution, the sediments in transport follow a lognormal size distribution. The median diameter of sediment in transport is generally finer than the median diameter of bed material, due to the selective transport of grains by flow. The relative median size of sediment in transport, $d_{50}/d_{50}$, decreases as size gradation increases, and the relationship between them can be represented by Eq. (25).

3. A variable representative diameter $d_{v}$ expressed by Eq. (31) is theoretically derived for bed materials with a lognormal distribution. The representative diameter $d_{v}$ decreases as $\sigma_g$ increases, resulting in a higher transport rate for nonuniform sediment mixtures. The use of $d_{35}$ as a representative size of bed material suggested by Einstein (1944) and Ackers and White (1973) is not a generally valid value.

Acknowledgments

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Notation

The following symbols are used in this paper:

AGD = average geometric deviation between computed and measured bed-material concentrations;

\( b \) = exponent;

\( C \) = coefficient;

\( C_{isc}, C_{im} \) = computed and measured bed-material concentrations, respectively;

\( d \) = particle size of bed material;

\( d_e \) = equivalent representative diameter defined by Nordin;

\( d_i \) = representative diameter of bed material corresponding to the size fraction \( i \);

\( d_{50} \) = median diameter of sediment in transport;

\( f' \) = friction factor defined by Engelund and Hansen;

\( G \) = size gradation coefficient of bed material;

\( g \) = gravitational acceleration;

\( h \) = flow depth;

\( J \) = total number of data sets;

\( K_d \) = size gradation correction factor;

\( m \) = exponent;

\( N \) = total number of size fractions present in a sediment mixture;

\( P \) = percentage for which the diameter of bed material is corresponding to \( d_{50} \) for a given \( \sigma \);

\( Q_i \) = total bed-material transport rate;

\( Q_{dp} \) = total bed-material transport rate obtained based on \( d_i \);

\( Q_{sp} \) = potential bed-material transport rate;

\( Q_{50} \) = total bed-material transport rate obtained based on \( d_{50} \);

\( q_i \) = total bed-material sediment discharge by weight per unit width;

\( R_f \) = discrepancy ratio between computed and measured bed-material concentration;

\( \text{RMS} \) = root mean square;

\( S \) = slope;

\( s_g \) = specific gravity;

\( T \) = transport stage parameter temperature;

\( u \) = dummy variable;

\( V \) = average flow velocity;

\( V_{ci} \) = the incipient velocity for a particular size class \( i \) in a mixture;

\( V_s \) = shear velocity;

\( V' \) = the absolute fluctuations of velocity;

\( u \) = dummy variable;

\( W \) = width;

\( x \) = general variable;

\( y \) = general variable;

\( \Delta P_{bi} \) = fraction of bed material, by dry weight, corresponding to the size fraction \( i \);

\( \Delta P_{oi} \) = fraction of transported bedload sediments, by dry weight, corresponding to the size fraction \( i \);

\( \Delta P_{mi} \) = relative mobility of bed material corresponding to size fraction \( i \);

\( \gamma_s, \gamma_w \) = specific weight of sediment and water, respectively;

\( \zeta \) = general variable;

\( \theta \) = dimensionless shear parameter;

\( \mu_s \) = mean value of Gaussian distribution;

\( \xi_p \) = \( p \)th percentile of standard normal distribution;

\( \sigma \) = the standard deviation of \( V'/V \) distribution;

\( \sigma_d \) = standard deviation of bed material size;

\( \sigma_{f} \) = standard deviation of Gaussian distribution;

\( \tau \) = shear stress along the bed;

\( \tau' \) = grain shear stress;

\( \tau_c \) = critical shear stress;

\( \Phi \) = dimensionless sediment transport function;

\( \omega \) = fall velocity corresponding to particle size \( d \);

\( \omega_c \) = fall velocity corresponding to particle size \( d_{50} \);

\( \omega_{50} \) = fall velocity of sediment corresponding particle size \( d_{50} \);

Subscripts

\( i \) = size fraction number in a data set;

\( j \) = data set number; and

\( i \) = transport material.

References


