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# TRANSPORT OF BED SEDIMENT IN CLAY SUSPENSIONS

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### INTRODUCTION

The presence of fine sediments is known to increase the bed sediment discharge (2,3). Concentrations of suspended sediments larger than 600,000 ppm (about 40% by volume) have been reported in the United States (2) and in China (11). These hyperconcentrated flows have caused irrigation and flood control problems such as clogging and aggradation (2,6).

Several researchers (3, 7, 9, and 11) documented the effects of large concentrations of sediments on the fluid properties such as the viscosity, the density, and the reduction of fall velocity of bed sediment particles. For example, Simons, et al. (9) measured the fall velocity of sand particles in suspensions of bentonite and kaolinite. Wan (11) suggested that in clay suspensions the contact bed sediment discharge may decrease due to the increased threshold velocity. On the other hand the suspended sediment discharge could increase due to the reduction of sediment fall velocity. Colby (3) pioneered several investigations on highly concentrated flows with both fine sediments and sands. Colby's empirical method for estimating total bed sediment discharge in hyperconcentrated flows accounts for the effect of fine sediment with concentra-

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tions up to 200,000 ppm (about 9% by volume).

The purpose of this study is to test the applicability of existing sediment transport formulas for predicting the total bed sediment discharge in flows of clay suspensions. Einstein's sediment transport formula (4) is selected because the parameters are related to the fluid viscosity and density. Colby's graphical method (3) includes the effect of large concentrations of fine sediments. The flume data collected by Simons, Richardson, and Haushild (9) with high concentrations of both fine sediments and sands are used to test the applicability of these two methods.

## EINSTEIN'S METHOD

Einstein's sediment transport formula (4) predicts separately the bed sediment discharges in contact with the bed and in suspension, the sum of which gives the total bed sediment discharge. The contact sediment discharge depends on the shear intensity factor obtained after considering the stochastic motion of the sediment particles on the bed. The suspended bed sediment discharge, a portion of the total bed sediment discharge in suspension, is predicted by using the Rouse sediment concentration profile equation and the logarithmic velocity profile. The reference concentration is obtained from the contact bed sediment discharge within the bed layer. The shear velocity related to the grain roughness is calculated using the graphical method suggested by Vanoni and Brooks (10). Pemberton's (8) correction procedure for the hiding factor  $\xi$  has been followed with Einstein and Chien's (5) correction factor  $\theta$  for the laminar flow effects on fine sediment particles.

The sediment transport data for sand particles (0.47 mm and 0.59 mm) in bentonite clay suspensions collected by Simons, et al. (9) has been used. The data set includes a total of 92 runs with uniformly dispersed concentrations of fine sediments  $C_f$  less than 59,000 ppm and bed sediment discharge  $q_T$  smaller than 182 N/m/s. The 22 runs where  $C_f$  exceeds 10,000 ppm and  $q_T$  exceeds 1.9 N/m/s were used for this study. The largest apparent kinematic viscosity in the experiment was about  $2.8 \times 10^{-6}$  m<sup>2</sup>/s, which is three times larger than that of clear water at 20° C. The bed forms changed from dunes to standing waves as the average channel velocity increased from 0.6 m/s to 1.83 m/s. The channel depth varied from 11.9 cm to 26.5 cm, and the channel slope varied from 0.0018 to 0.019.

Contact Bed Sediment Discharge.—The increase in viscosity and density of the suspension alters the parameters of the Einstein bed load function. These parameters include the hiding factor, the pressure reduction effect, the laminar flow effect, and the shear velocity. As shown in Fig. 1, the ratio  $q_{bc}/q_{bu}$  of the calculated contact bed sediment discharges (corrected,  $q_{bc}$ ; and uncorrected,  $q_{bu}$ , for the viscosity and density of the suspension) is not significantly affected by the concentration of fine sediments. This implies that the increase in the total bed sediment discharge at large concentrations of fine sediments is not caused by the contact bed sediment discharge.

Total Bed Sediment Discharge.—The reduction in fall velocity of sand particles in a clay suspension decreases the exponent Z of the Rouse equation. This increases the concentration of suspended bed sediment

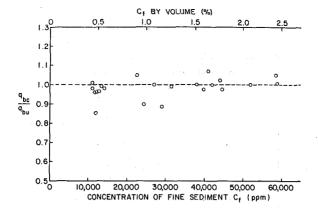


FIG. 1.—Ratio of Contact Bed Sediment Discharges Calculated from Einstein's Formula Corrected,  $q_{bc}$ , to Uncorrected,  $q_{bu}$ , for Viscosity and Density of Suspension

particles and thus the suspended bed sediment discharge in the flow. The total bed sediment discharge  $q_t$  has been calculated by Einstein's formula both with  $(q_{tc})$  and without  $(q_{tu})$  correction for the viscosity and density of the suspension due to fine sediments. Rubey's equation using the measured apparent viscosity and density gave the fall velocity of

density of the suspension due to fine sediments. Rubey's equation using the measured apparent viscosity and density gave the fall velocity of sand particles in suspension. In Bingham fluids such as a clay suspension, however, the fall velocity can be best estimated by the method of Ansley and Smith (1) after the viscosity is corrected for the yield stress of the suspension. The results obtained from Einstein's formula are shown in Fig. 2. All the calculated values fall within a factor 0.25 to 4 times the

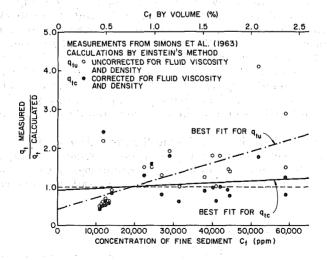


FIG. 2.—Ratio of Total Bed Sediment Discharges Measured and Calculated by Einstein's Formula

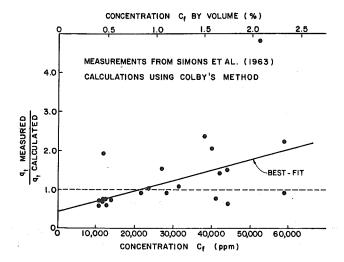


FIG. 3.—Ratio of Total Bed Sediment Discharges Measured and Calculated by Colby's Method

measured values. Nevertheless, the best fit lines show that the correction yields improved results, particularly at large concentrations. It is, therefore, concluded that the viscosity of clay suspensions increases the suspended load through fall velocity reduction while bed load remains basically constant.

#### COLBY'S GRAPHICAL RELATIONS

The merit of Colby's method for predicting sediment discharge of large concentrations of sand particles lies in its simplicity. The empirical correction factor  $k_2$  at concentrations of fine sediments up to 50,000 ppm was obtained after assuming that both the temperature and concentration of fine sediments affect the relationship between bed sediment discharge and average flow velocity only through changes in fluid viscosity. The non-Newtonian behavior of the fluid and the change in the density of the suspension are neglected in Colby's method. For concentrations of fine sediments ranging from 50,000 ppm up to 200,000 ppm, the correction factor  $k_2$  is based on the limited data from the Rio Puerco, New Mexico.

Colby's method has been applied to the same data set discussed previously. The results shown in Fig. 3 do not provide better agreement than Einstein's uncorrected formula ( $q_{tu}$  shown in Fig. 2). Colby's method underestimates sediment transport as the concentration of fine sediment increases.

#### SUMMARY AND CONCLUSIONS

Large concentrations of fine sediments increase the viscosity and density of the suspension. Colby's method and Einstein's formulation of sediment discharge have been tested with the flume data of Simons, et

al. (9). The following conclusions are drawn from this analysis: (1) According to calculations based on Einstein's method, the sediment discharge in contact with the bed remains practically unchanged in presence of fine sediments; (2) the increase in total bed sediment discharge is mainly attributable to the increase in suspended sediment discharge; and (3) Colby's graphical relations yield results quite similar to those of Einstein without correction and do not provide better prediction of the total bed sediment discharge than Einstein's formula.

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#### APPENDIX II.—NOTATION

The following symbols are used in this paper:

- $C_f$  = concentration of fine sediments (part per million);
- $k_2$  = Colby's correction factor for concentration of fine sediments;
- $q_b$  = contact bed sediment discharge per unit width of channel;
- $q_{bc}$  = contact bed sediment discharge per unit width, corrected for viscosity;
- $q_{bu}$  = contact bed sediment discharge per unit width, uncorrected for viscosity:
- $q_t$  = total bed sediment discharge per unit width of channel;

- to total bed sediment discharge per unit width, corrected for vis-
- $q_{tu}$  = total bed sediment discharge per unit width, uncorrected for viscosity;
- Z = Rouse's exponent for sediment concentration distribution;
- $\theta$  = Einstein and Chien's correction factor for laminar flow effect; and
- $\xi$  = Einstein's hiding factor.