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Sediment load calculations from point measurements in sand-bed rivers

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Abstract

Point velocity and suspended sediment concentration measurements are used to calculate the total sediment discharge in sand-bed rivers. Calculations with the Series Expansion of the Modified Einstein Point Procedure (SEMEPP) depend on grain diameter d_s and settling velocity ω, flow depth h, shear velocity u_*, and sampling depth h_p. This procedure extends the applicability of the Modified Einstein Procedure (MEP) by using point sediment concentration and velocity measurements. This procedure is tested using the laboratory data from Coleman, and field measurements from the Enoree, Middle Rio Grande and Mississippi Rivers. Based on 801 point measurements over 124 verticals at flow depths ranging from 0.17 m to 33.5 m and sediment concentrations less than 0.1 kg L^{-1}, the accuracy of the calculations depends on u_*/ω and h_p/d_s. Point measurement techniques like SEMEPP are well-suited when $u_*/\omega > 5$ where at least 60% of the total sediment load is measured when 90% of the flow depth is sampled. The determination of sediment discharge from point measurements is most accurate in deep rivers when $h_p/d_s > 10{,}000$, and $u_*/\omega > 10$. Point measurements are not well-suited for shallow rivers and laboratory flumes where $h < 0.5$ m and when $u_*/\omega < 2$.

Key Words: Point measurements, Concentration profiles, Velocity profiles, Total sediment load, Modified Einstein Procedure

1 Introduction

Difficulties in calculating the total sediment load in sand-bed rivers from the combination of suspended and bed loads continue to persist. Suspended sediment concentration profiles in open channels corroborate the theory of turbulent mixing and the logarithmic velocity profile for turbulent flows (Rouse, 1937). The Rouse number $Ro = \quad / \quad u_*$ is function of the fall velocity ω, the shear velocity u_*, the momentum correction factor for sediment β and the von Kármán constant κ. The Rouse number (Ro) is a ratio of the sediment property to the hydraulic characteristics of the flow. High values of the Ro (greater than 5) describe bed-load sediment transport, while low values of Ro (less than 0.5) result in near-uniform sediment concentration profiles and sediment transported in suspension (Dade and Friend, 1998; Julien, 2010).

Einstein (1950) developed a method to determine the total sediment load in open channels. The method uses the bed load function from Einstein (1942) and determines the suspended sediment concentration profiles from Rouse (1937). The integral of the product of the logarithmic velocity and the Rouse concentration profiles resulted in the Einstein integrals, which are analytically intractable. Recent developments through series expansions have improved the accuracy of the estimation of the Einstein integrals (Guo and Julien, 2004), which are applied in this study.

When the Einstein procedure was later applied to sand-bed rivers, inaccuracies emerged because this method could not define the wash load, which comprises the transport of the very fine sediment particles not found in the bed. The determination of the wash load concentration can only be accomplished through field measurements of suspended sediment concentrations. The subsequent analysis of the measured suspended sediment concentration led to the determination of the total sediment load using the Modified Einstein Procedure (MEP) developed by Colby and Hembree (1955). The MEP uses a combination of the Einstein bed load function (Einstein, 1942; 1950) and an extension of the velocity and concentration profiles in the unmeasured zone to estimate a total load as sketched in Fig. 1. The original MEP used depth-integrated sediment concentration measurements and the particle size distribution of the bed material to determine the total load. The MEP was first successfully tested with data from the Niobrara River

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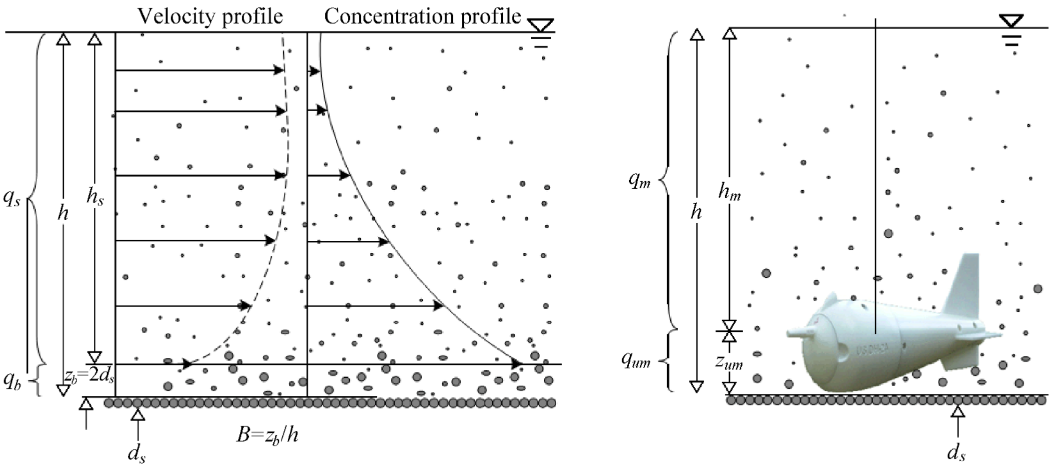
(Colby and Hembree, 1955). However, further complications did arise when there is not enough overlap between sediment particles sizes in suspension and in the bed material. For instance, the MEP could not calculate the total sediment load for gravel-bed streams with sand sizes in suspension. Several modifications have thereafter been proposed to determine the total sediment load transported in sand-bed rivers (Colby and Hubbell, 1961; Lara and Pemberton, 1963; Ackers and White, 1973; Burkham and Dawdy, 1980; Shen and Hung, 1983; O'Brien, 2000).



A. Bed load and suspended sediment load B. Measured and unmeasured load

Fig. 1. Definition sketch for sediment measurements in suspension

Point measurements of sediment concentration and flow velocity are useful in sand-bed rivers. When compared with depth-integrated measurements, greater accuracy can be obtained with point measurements because of the larger sampling volumes for the determination of sediment concentration as well as particle size distribution of suspended sediment. Thus point measurements present definite advantages in the determination of the sediment load in deep sand-bed channels.

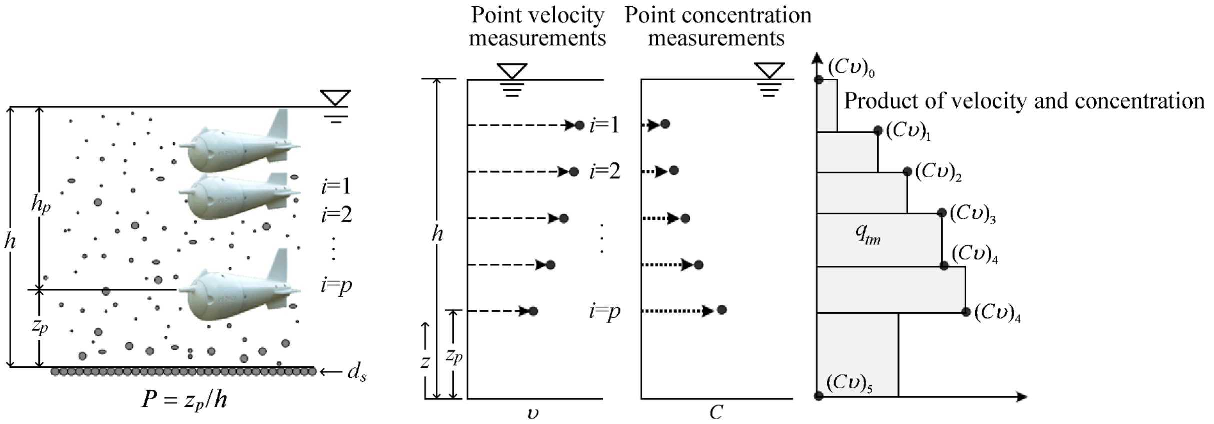
The main purpose of this study is to determine the accuracy of total sediment load calculations based on point velocity and sediment concentration measurements. In this article, a new procedure referred to as the Series Expansion of the Modified Einstein Point Procedure (SEMEPP) is developed for the analysis of the total sediment load with point measurements. This article employs the use of the series expansion to solve the Einstein integrals (Guo and Julien, 2004). The study provides an extension of the MEP to point samples by fitting the Rouse suspended sediment concentration profiles and logarithmic velocity profiles to the measured point data. The new procedure is tested with several data sets covering a wide range of flow and sediment conditions ranging from shallow laboratory experiments to deep sand-bed rivers. The analysis aims at providing guidelines on the conditions required for an accurate determination of the total sediment load. The most relevant questions addressed in this article include: (1) under what conditions can total sediment load be accurately determined using point measurements of flow velocity and sediment concentration; and (2) what percentage of the water column should be sampled for an accurate determination of the total sediment load. The analysis of field measurements will also identify the minimum sampling depth needed to obtain accurate total load calculations.

2 Total sediment load from point measurements

2.1 Measured total load

Point velocity and concentration measurements are collected at fixed locations along a vertical water column, as sketched in Fig. 2a. At each point i, flow velocity v_i and sediment concentration measurement C_i are taken. The data from the point measurements allow for fitting the sediment concentration profile directly to the data set. In some cases, the particle size distribution of each sample is collected at each point, which allows the determination of the suspended sediment concentration for each size fraction. In general, suspended sediment concentration measurements cannot be obtained at the bed or free surface (Edwards and Glysson, 1999). Sediment concentration is assumed to be zero at the water surface and flow velocity is assumed zero at the bed. On the other hand the concentration at the bed and the velocity at the free surface are assumed to be equal to the nearest measurement.

The sediment flux (sediment discharge per unit area imparted by the velocity current) is determined from the product of flow velocity and sediment concentration as shown in Eq. (1). The unit sediment discharge is obtained from the integral of the sediment flux. With point measurements, this is accomplished by using the trapezoidal rule as sketched in Fig. 2b with p = 5. The measured total unit sediment discharge corresponds to the summation from 0 to p from the free surface to the bed, or:



A. Point sampling B. Point measurements

Fig. 2. Measured sediment load from point velocity and sediment concentration

$$q_{tm} = \sum_{0}^{p} 0.5\left(C_i \mathrm{v}_i + C_{i+1}\mathrm{v}_{i+1}\right)\left(h_{i+1} - h_i\right) \tag{1}$$

where, q_{tm} is the total measured unit sediment discharge based on p point samples, C_i and v_i are respectively the sediment concentration and flow velocity at a given point i, and h_{i+1} - h_i is the vertical layer thickness.

By increasing the number of point measurements, the accuracy of the calculated total measured unit sediment discharge is expected to increase. This occurs because the concentration and velocity profiles have a higher degree of certainty. By including additional discrete points the summation of the measured sediment concentration profile will be closer to the actual sediment discharge. However, when both the Rouse number Ro and the ratio of the lowest point sampling depth to the water column depth ($P = z_p/h$) are large, this approach can lead to highly inaccurate results. When bed load is dominant (large Ro), most of the sediment transport will occur below the lowest sampling point location (Dade and Friend, 1998; Julien, 2010). Thus, guidelines are needed to define the range of applicability of point measurement techniques in terms of u_*/ω (or Ro) and P. To quantify the accuracy of sediment transport from point measurements, the concept of partial sediment discharge is introduced. The partial sediment load is the sediment transported in the water column between the free surface and the lowest sampling point p. It may be interesting to point out that the partial sediment load is normally called the measured load (Julien, 2010). However, the term "partial load" is preferred in this article to avoid possible confusion between sediment quantities that have been measured and calculated. Therefore, the measured partial unit sediment discharge q_{pm} can be determined from 0 to p-1, using the rectangular rule (Fig. 2b):

$$q_{pm} = \sum_{0}^{p-1} 0.5\left(C_i \mathrm{v}_i + C_{i+1}\mathrm{v}_{i+1}\right)\left(h_{i+1} - h_i\right) \tag{2}$$

In Section 3, the analysis of q_{pm}/q_{tm} will determine whether point sampling is appropriate to measure most of the sediment load that is transported. Also, the investigation of z_p (or P) will be used to quantify the increase in measured sediment load as the sampling points get closer to the bed.

2.2 SEMEPP procedure

This analysis (SEMEPP) expands upon the Series Expansion of the Modified Einstein Procedure (SEMEP) developed by Shah-Fairbank et al. (2011). While SEMEP was based on depth-integrated samples, the SEMEPP analysis is based on point measurements. SEMEP has been shown to work well in sand-bed rivers (Shah-Fairbank et al., 2011). When the suspended sediment concentration profiles became uniform (low Ro) the momentum correction factor for sediment β and the von Kármán constant κ become irrelevant. One of the main features of SEMEPP has been to simply report the results as a function of u_* and ω. This parametric description does not require any assumption regarding the values of κ and β. The ratio u_*/ω is favored over the Rouse number as part of the overall improvements of the MEP procedure. For the comparison of u_*/ω with the Rouse number in this study, it has been assumed that β = 1 and κ = 0.4. The value of ω is calculated as: $\omega = \frac{8\nu}{d_s}\left\{\left[1+0.0139 d_*^3\right]^{0.5} - 1\right\}$.

The main features of SEMEPP include the following: (1) fit the concentration profile to the point concentration measurements to determine the reference concentration C_b and Ro; (2) fit a logarithmic velocity profile to the point velocity data to determine u_* and depth of zero velocity z_o; (3) use the series expansion algorithm of Guo and Julien (2004) to solve the Einstein integrals; and (4) calculate the sediment load by integrating the advective flux of sediment (Cv) using parts (1) and (2). SEMEPP is therefore an extension of the MEP to point measurements. The reason we continue to refer to the procedure as the MEP is because the procedure uses field measurements and the Einstein integrals. Improvements beyond the existing MEP are expected because point concentration measurements can define the sediment concentration profiles, which the depth-integrated samples of the original MEP could not do. The use of the series expansion should improve the accuracy of calculating the Einstein integrals over other methods. Finally, guidelines based on u_*/ω will also remove any uncertainly associated with β and κ.

2.3 Sediment load calculation using SEMEPP

The SEMEPP procedure requires point measurements to define the velocity distribution and the sediment concentration along a vertical profile (water column). The turbulent velocity profile (law of wall) within a river is based on the logarithmic velocity distribution (Keulegan, 1938).

$$\mathrm{v} = \frac{u_*}{\kappa} \ln \frac{z}{z_o} \tag{3}$$

where v is the velocity at a distance z above the bed, u_* is the shear velocity, κ is the von Kármán constant assumed equal to 0.4, and z_o is the vertical elevation where the velocity equals zero. In SEMEPP, the values of u_* and z_o are determined by fitting Eq. (3) to the entire velocity profile obtained from the point velocity measurements. Alternatively, when the friction slope S_f of the river is available, the shear velocity u_* can be determined based on the following relationship $u_* = \sqrt{ghS_f}$.

The sediment concentration profile is described by the Rouse equation (Rouse, 1937). The suspended sediment distribution described in Eq.(4) provides the concentration at a specified distance z above the bed. (4)

$$C = C_b \left(\frac{h-z}{z} \frac{z_b}{h-z_b} \right)^{Ro} \tag{4}$$

where C is the concentration at an elevation z above the bed, Cb is the concentration at the reference elevation zb = 2ds above the bed and h is the total depth of flow. In SEMEPP, the values of Ro and Cb are determined by fitting the Rouse power function [Eq. (4)] to the concentration measurements. Figure 3 shows a schematic of the calculation procedure from SEMEPP.

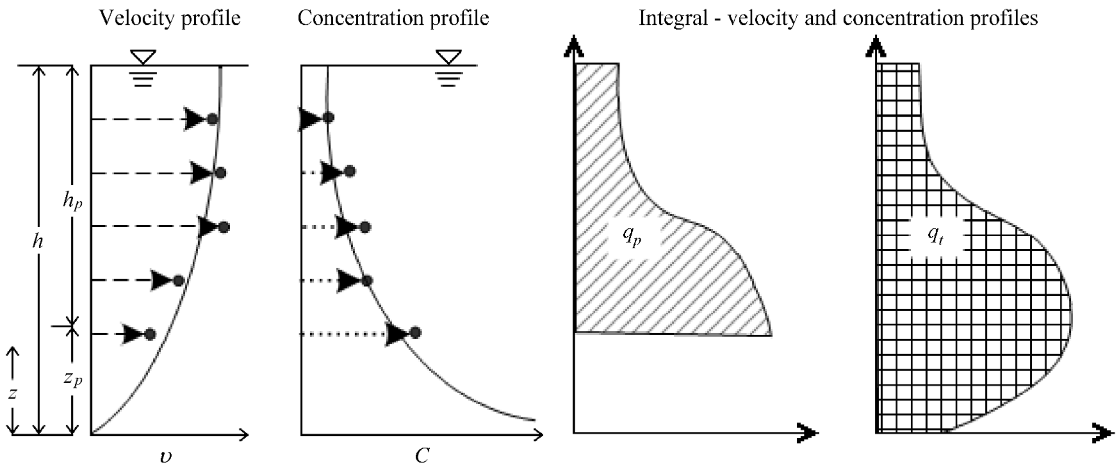


Fig. 3 Sketch of the sediment load calculation from SEMEPP

The unit sediment discharge by advection is given by the depth-integration of the product of sediment concentration C and flow velocity v. SEMEPP provides calculations of the unit total sediment discharge q_t, the unit suspended sediment discharge q_s and the partial unit suspended sediment discharge q_p respectively.

$$q_t = \int_{z_0}^{h} Cvdz \tag{5a}$$

$$q_s = \int_{z_b}^{h} Cvdz \tag{5b}$$

$$q_p = \int_{z_p}^{h} Cvdz \tag{5c}$$

where h is the total flow depth, z_b is the depth of the bed layer defined by Einstein as the reference depth of $2d_s$, and z_p is the elevation above the bed of the lowest point measurement. The values of q_p and q_t are sketched in Fig. 3. The ratio of q_p/q_t defines the fraction of the total sediment load transported in the sampling depth as calculated by SEMEPP.

From SEMEPP, the partial unit sediment discharge is then evaluated from Eq. (5c), after considering Eqs. (3) and (4), and can be expanded as follows:

$$q_p = \int_{z_p}^{h} C_b \left(\frac{h\ \ z}{z} \frac{z_b}{h\ \ z_b} \right)^{\mathrm{Ro}} \frac{u_*}{\ } \ln\left(\frac{z}{z_o} \right) dz \tag{6}$$

$$q_p = \frac{C_b h u_*}{\ } \left(\frac{B}{1\ \ B} \right)^{\mathrm{Ro}} \left\{ \ln\left(\frac{h}{z_o} \right) J_{1P} + J_{2P} \right\} \tag{7}$$

$$J_{1P} = \int_{P}^{1} \left(\frac{1\ \ z_*}{z_*} \right)^{\mathrm{Ro}} dz_* \tag{8}$$

$$J_{2P} = \int_{P}^{1} \ln z_* \left(\frac{1\ \ z_*}{z_*} \right)^{\mathrm{Ro}} dz_* \tag{9}$$

where, q_p is the partial unit sediment discharge calculated from SEMEPP, $z_b = 2d_s$ is the reference elevation, z_p is the nozzle height above the bed for the lowest measured point sample (Fig. 3), the bed layer ratio is $B = z_b/h$, the partial depth ratio is $P = z_p/h$, the integration variable is the relative depth $z_* = z/h$, and J_{1P} and J_{2P} are the modified Einstein integrals for the partial sediment load. In SEMEPP, u_* and z_o are determined from the velocity profile assuming $\kappa = 0.4$, while Ro and C_b are determined from the point concentration measurements. The Einstein integrals for partial load J_{1P} and J_{2P} in Eqs. (8) and (9) are solved with the series expansion algorithm of Guo and Julien (2004).

The total unit sediment discharge is calculated based on the sum of the bed and suspended sediment discharge. The bed load transport function q_b in the bed layer of thickness $z_b = 2d_s$ and the near-bed velocity v_b are based on Einstein's bed load analysis (1942, 1950).

$$q_t = q_b + q_s; \quad \text{where} \quad q_b = C_b \mathrm{v}_b z_b; \quad \text{and} \quad \mathrm{v}_b = 11.6 u_* \tag{10}$$

$$q_t = q_b + \int_{z_b}^{h} C_b \left(\frac{h\ \ z}{z} \frac{z_b}{h\ \ z_b} \right)^{\mathrm{Ro}} \frac{u_*}{\ } \ln\left(\frac{z}{z_o} \right) dz \tag{11}$$

$$q_t = C_b \mathrm{v}_b z_b + \frac{C_b h u_*}{\ } \left(\frac{B}{1\ \ B} \right)^{\mathrm{Ro}} \left\{ \ln\left(\frac{h}{z_o} \right) J_{1S} + J_{2S} \right\} \tag{12}$$

$$J_{1S} = \int_{B}^{1} \left(\frac{1\ \ z_*}{z_*} \right)^{\mathrm{Ro}} dz_* \tag{13}$$

$$J_{2S} = \int_{B}^{1} \ln z_* \left(\frac{1\ \ z_*}{z_*} \right)^{\mathrm{Ro}} dz_* \tag{14}$$

where, q_t is the total calculated unit sediment discharge, q_b is the total unit bed load discharge, V_b is the flow velocity in the bed layer of thickness $z_b = 2d_s$, $B = z_b/h = 2\ d_s\ /h$, and J_{1S} and J_{2S} are the Einstein integrals for the suspended sediment zone. SEMEPP uses u_*, z_o, Ro and C_b from the velocity and concentration profiles, and q_b is determined from C_b, $\mathrm{v}_b = 11.6\ u_*$. The Einstein integrals for the suspended sediment load J_{1S} and J_{2S} in Eqs. (13) and (14) are solved with the series expansion algorithm developed by Guo and Julien (2004).

3 Experimental verification of -SEMEPP

3.1 Laboratory and field data

To validate SEMEPP, accurate point measurements were analyzed for both laboratory and river conditions. The data sets listed in Table 1 include point measurements (velocity and concentration) from Coleman's laboratory experiments, the Enoree, Middle Rio Grande and Mississippi Rivers. These data sets were obtained from previously published sources and compiled by Shah-Fairbank (2009). The total data sets include 124 verticals (water columns) and a total of 801 point velocity and point concentration measurements. These data sets were selected to provide a wide range in flow depth (0.017 m < h < 33.5 m), unmeasured depth (0.006 m < z_p < 2.2 m), measured total sediment transport [(0.01 kg $\mathrm{m}^{-1} \cdot \mathrm{s}^{-1}$) < q_t < (32 kg $\mathrm{m}^{-1} \cdot \mathrm{s}^{-1}$)] and relative submergence (1,600 < h/d_s < 530,000). This also allowed for a continuous range of values for u_*/ω and h_p/d_s to identify the domain of applicability of SEMEPP.

Coleman's laboratory data were measured in a laboratory flume (15 m long by 33 cm wide). The flow velocity and sand concentration profiles were collected for sand particles in suspension ranging in size from 0.088 mm to 0.5 mm (Coleman, 1981, 1986).

The Enoree River data set is based on field measurements made in relatively regular and straight reach (Anderson, 1942). At the sampling location, the sand-bed river was 15 m wide with near vertical banks. Numerous samplers regularly spaced on a sounding cable provided concurrent measurements of flow velocity and suspended sediment concentrations. The particle measured in suspension ranged from 0.07 to 0.99 mm.

Table 1 Main characteristics of the data set

Data Set	No. of verticals sampled	No. of points samples	h (m)	z_p (m)	d_s (mm)	h/d_s * 1000
Coleman Lab Data (Coleman, 1986)	6	12	0.170 to 0.172	0.006	0.1	1.6
Enoree River, SC (Anderson, 1942)	6	8	0.9 to 1.6	0.06 to 0.103	0.2 to 0.25	3.2 to 6.2
Middle Rio Grande at Bernalillo, NM (Nordin and Dempster, 1963)	4	4	0.7 to 0.8	0.27 to 0.37	0.07	11.5 to 12.5
Mississippi River. MS (Akalin, 2002)	108	6	6.4 to 33.5	0.4 to 2.2	0.4 to 0.6	15 to 530

The United States Bureau of Reclamation took point suspended sediment and velocity measurements on the Middle Rio Grande (Nordin and Dempster, 1963). The velocity measurements were taken using a Price current meter and the suspended sediment measurements were obtained using a US P-46. The particle measured in suspension ranged from less than 0.0625 to 0.5 mm, however the median particle size was always within the sand range.

The United States Army Corps of Engineers collected data near the Old River Control Complex of the Mississippi River in 1998 by the Waterways Experiment Station. The equipment used for the measurements includes: Acoustic Doppler Current Profiler, US P-63, drag bucket, Price current meter and depth sounder. Data at four sites (Union Point, Line 13, Line 6 and Tarbert Landing) were used and compiled by Akalin (2002). The particle measured in suspension ranged from 0.0625 to 2 mm.

3.2 Total sediment load comparison

The measured total sediment load was determined from Eq. (1). The calculated total sediment load was determined from SEMEPP using Eq. (12). A comparison of the calculated and measured total loads is shown in Fig. 4 with each river represented by a distinct symbol. A twenty five percent error band has been placed on the figure. As the measured unit total sediment discharge increases, a higher degree of accuracy is achieved between measured and calculated (SEMEPP) total sediment discharge.

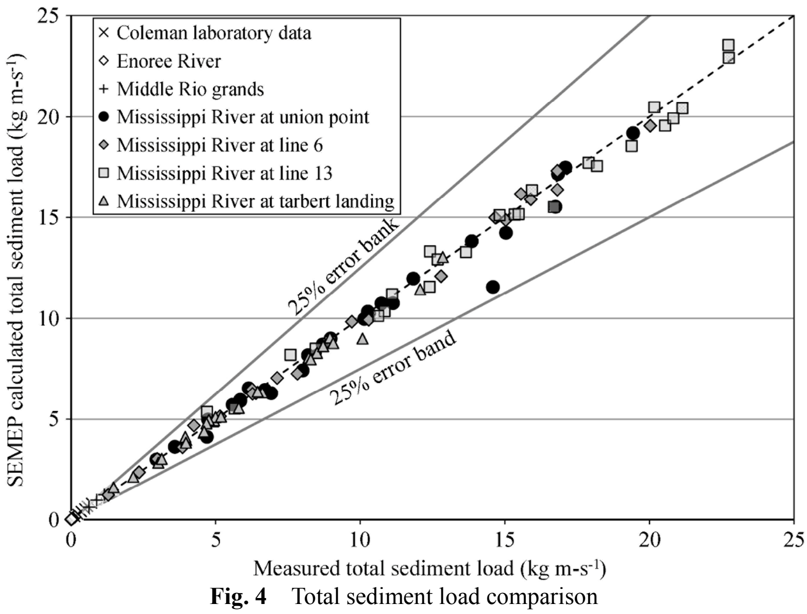


Fig. 4 Total sediment load comparison

3.3 Applicability of SEMEPP

This section addresses the first issue raised in the introduction: under what conditions can the total sediment load be accurately determined using point measurements. In other words, under what conditions can SEMEPP calculate most of the sediment load transported in a sand-bed channel? The answer to this question is found in the ratio of the partial to the total sediment load q_p/q_t. Point measurements can effectively measure most of the transport rate when the ratio is high. On the other hand, low values of q_p/q_t describe conditions where most of the sediment is transported as bed load, in which case, point measurement techniques would not be appropriate.

From the measurements, the ratio of the partial to total unit sediment discharge q_{pm}/q_{tm} is determined for each sample based on Eqs. (1) and (2). SEMEPP is also used to determine the calculated values of the ratio of partial to total unit sediment discharge q_p/q_t based on Eqs. (7) and (12) for each sample. For the comparison, representative samples covering a wide range of h/d_s were selected and the results are shown in Fig. 5. The measured values of the ratio q_{pm}/q_{tm} are compared to the ratios q_p/q_t calculated using SEMEPP. Measurements are shown as points, while the series of lines represent SEMEPP calculations obtained from varying the value of u_* to generate a continuous line. There is more variability in the percentage of the total load that is measured when u_*/ω is between 2 to 5. The results of the calculations seem increasingly accurate when $u_*/\omega > 10$. Figure 5 shows that the SEMEPP calculated values of q_p/q_t match the measurements particularly well at higher values of u_*/ω. Overall, point measurement techniques like SEMEPP are considered well-suited when $u_*/\omega > 5$, at least 60% of the total sediment load is measured when 90% of the water column is sampled. As the sampling depth is reduced the variability between the measured and calculated ratio of q_p/q_t increases, however there is less variability between the measured and calculated ratios as $u_*/\omega > 5$.

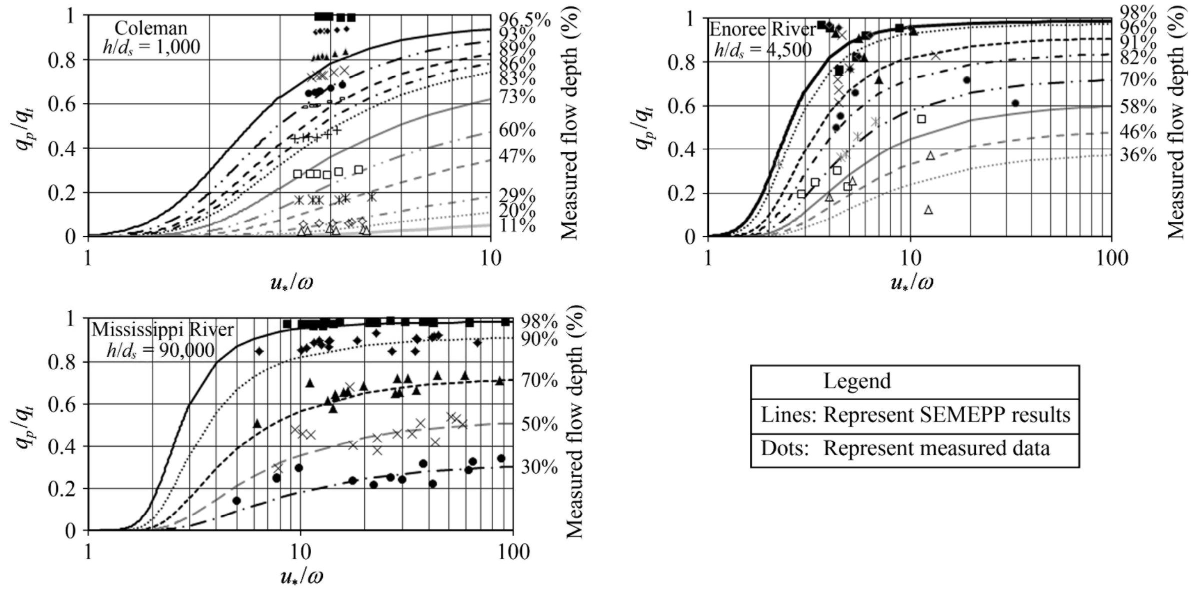


Fig. 5 Experimental data validation based on q_p/q_t vs u_*/ω and h_p/h

It is also observed in Fig. 5 that most of the sediment is transported near the bed when the values of $u_*/\omega < 2$. Indeed, only a small fraction (less than 20%) of the total sediment load could be measured by point samplers when $u_*/\omega < 2$, and this even when 90% of the water column is sampled. This means that the unmeasured sediment load is at least five times larger than the measured sediment load. This suggested that the based on SEMEPP method is not acceptable because bed load is the dominant mode of sediment transport.

3.4 Statistical analysis of the accuracy of SEMEPP

The degree of accuracy of SEMEPP is evaluated using the root mean square error (referred to as RMSE). RMSE is a measure of the absolute error (Ott and Longnecker, 2001) and identifying the precision.

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}\left(X_i - Y_i\right)^2}{n}} \tag{15}$$

where, X_i is the measured total load, Y_i is the calculated total load (SEMEPP) and n is the number of samples. The RMSE is always positive and low numbers are indicative of better calculations.

The mean percent error (referred to as MPE) is a measure of the relative error (Ott and Longnecker, 2001) and is useful in identifying the bias in the calculations.

$$\mathrm{MPE} = \frac{\sum_{i=1}^{n}\frac{\left(X_i - Y_i\right)}{X_i}}{n} * 100 \tag{16}$$

If the value of MPE is less than zero, then SEMEPP has a bias towards over predicting the total sediment load.

Table 2 contains a summary of the statistical parameters. The data show that the magnitude of both the sediment load and error (RMSE) increases with h/d_s, which suggest less precision. However, this is associated with high sediment loads and concentrations and the relative value of the error or bias (MPE) clearly decreases as the relative submergence h/d_s increases. In the cases of the Middle Rio Grande and Mississippi Rivers, the measured and calculated total sediment loads are within 25% of each other. It is thus concluded that point measurements have a tendency to provide an accurate total load calculation in deep sand-bed rivers even though the results may be less precise.

Table 2 Statistical results for the measured vs calculated sediment load q_t

Data set	h/d_s	% flow measured	h_p/d_s	Measured total load (kg m-s^{-1})		Measured vs. SEMEPP total load	
				Min	Max	MEP (%)	RMSE (kg m-s^{-1})
Coleman Lab Data	1,000	96%	965	0.13	0.71	-13.1	0.064
Enoree River, SC	4,500	98%	4,410	0.01	0.12	-7.9	0.005
Middle Rio Grande, NM	12,000	87%	10,436	0.62	1.43	-5.9	0.073
Mississippi - Tarbert	55,000	98%	53,900	1.47	12.86	1.4	0.296
Mississippi - Union Point	75,000	98%	73,500	2.95	19.43	1.7	0.7
Mississippi - Line 6	90,000	98%	88,200	1.28	20.03	0.4	0.332
Mississippi - Line 13	110,000	98%	107,800	4.70	32.35	0.7	0.798

3.5 Analysis of the effect of sampling depth h_p

This section addresses the second issue raised in the introduction: what percentage of the water column should be sampled for an accurate determination of the total sediment load. Both the channel bed material and flow depth will affect the results. Figure 5 readily shows an increase fraction of q_p/q_t as the sampling depth increases. The accuracy in the calculations of sediment load increases with the value of u_*/ω. Thus when $u_*/\omega = 10$, only 50% of the water column of the Mississippi River needs to be sampled for measured and calculated total load to be within 35%. On the other hand, 90% of the water column of the Enoree River will need to be sampled.

To probe deeper into the effect of sampling depth, a sensitivity analysis on the accuracy of partial sampling is performed by varying the number of sediment concentration measurements used to fit the Rouse equation. The purpose of varying the number of point sediment samples is to determine what partial sampling depth could yield reasonable results in deep rivers. The total suspended sediment load using SEMEPP was re-calculated for the different sampling depths and the RMSE and MPE were determined for the entire data set containing 668 data points. The total sediment load calculated from SEMEPP was compared to the measured total load. Table 3 provides a representative summary of the RMSE and MPE results for various sampling depths. In general, the RMSE and MPE tend to increase as the sampling depth decreases. Under certain circumstances the statistical parameters decrease with reduced sampling depth. This occurs due to lower surface concentrations. Shallow rivers with h_p/h < 10,000 have a tendency to over predict total sediment load, thus resulting in a negative MPE. The Enoree River has a high negative MPE value due to the ratio of u_*/ω and the high amounts of washload.

The absolute percent error was determined for each of the 668 samples from:

$$\%Error = \frac{100*|X_i \quad Y_i|}{X_i} \tag{17}$$

Figure 6 provides a graphical display of all the results. Table 4 also summarizes the analysis of the average values of the absolute error percentage as a function of h_p/d_s at each site. The analysis clearly shows that a higher percentage (70%) of points have an error less than 25% when h_p/d_s increases.

This analysis shows that sediment load predictions in deep rivers are more accurate (lower error percentage) than shallow rivers and laboratory flumes, as shown in Table 4. It is concluded that a value of h_p/d_s > 10,000 (Table 3 and Fig. 6) would result in sufficiently accurate sediment load calculations from SEMEPP because approximately 60% of the samples have an absolute percent error less than 25%. This leads to results of practical significance. This implies that for a grain diameter of 0.1 mm, accurate sediment load calculations from point measurements require a minimum sampling depth in excess of 1 m.

4 Applicability of SEMEPP in sand-bed rivers

The domain of applicability of SEMEPP in shallow and deep sand-bed rivers is determined from a representative application example. The particle size is kept constant at 0.2 mm (fine sand), and the values of q_p/q_t are calculated for typical values of sampling depth. For instance, the value for z_p = 10 cm corresponds to the nozzle height of standard point samplers (e.g. P-61 and P-63). Based on the findings in Table 3, it is considered that point measurements would be useful when that h_p/d_s > 10,000 which corresponds to a minimum flow depth of 2 m.

The results in Fig. 7 show that regardless of the river size, once the value of u_*/ω > 5 it becomes possible to measure most (q_p/q_t > 50%) of the total sediment load from a standard sampler, z_p = 10 cm. For instance in Fig. 7c at a flow depth of 20 m (measuring approximately 99.5% of the water column), over 90% of the total sediment load (q_p/q_t) would be measured using point samples from 10 cm above the bed to the free surface when $u_*/\omega = 5$. In comparison, Fig. 7a shows a shallow stream where a maximum of 80% of the water column is measured and only 53% of the total sediment load being measured. This means that accurate sampling over a partial flow depth is best suited for deep rivers carrying fine sediment while measurements become less accurate in shallow and small streams. The main contribution of this article is that the percentage of the sediment load that is measured in shallow and deep sand-bed rivers can be determined as a function of the sampling depth z_p and u_*/ω.

Table 3 Statistical results for q_t as a function of sampling depth h_p

Data Set	h/d_s	% Flow Measured	h_p/d_s	Measured vs SEMEP - Total Load	
				MPE	RMSE
COLEMAN LAB DATA	1,000	93%	930	-0.157	0.075
	1,000	89%	895	-0.179	0.087
	1,000	86%	860	-0.200	0.099
	1,000	82%	824	-0.241	0.119
	1,000	73%	731	-0.316	0.159
	1,000	60%	596	-0.398	0.221
	1,000	47%	467	-0.238	0.170
	1,000	29%	286	-0.134	0.142
	1,000	20%	198	-0.036	0.105
	1,000	11%	110	-0.141	0.152
ENOREE RIVER - SC	4,500	96%	4,337	-0.057	0.003
	4,500	89%	4,014	-0.043	0.004
	4,500	82%	3,682	-0.070	0.005
	4,500	68%	3,040	-0.085	0.009
	4,500	56%	2,507	-1.018	0.245
	4,500	44%	1,981	-1.484	0.252
	4,500	31%	1,396	-2.092	0.282
MIDDLE RIO GRANDE - NM	12,000	68%	8,149	-0.058	0.076
	12,000	50%	5,974	-0.067	0.104
	12,000	33%	3,909	-0.052	0.082
MISSISSIPPI RIVER UNION POINT	75,000	90%	67,500	0.025	0.544
	75,000	70%	52,500	0.062	1.562
	75,000	50%	37,500	0.065	2.055
	75,000	30%	22,500	0.034	4.065
MISSISSIPPI RIVER LINE 13	110,000	90%	99,000	0.006	1.013
	110,000	70%	77,000	-0.004	1.322
	110,000	50%	55,000	0.063	1.884
	110,000	30%	33,000	0.090	3.029
MISSISSIPPI RIVER LINE 6	90,000	90%	81,000	0.024	1.124
	90,000	70%	63,000	0.045	1.333
	90,000	50%	45,000	0.052	2.093
	90,000	30%	27,000	-0.036	3.471
MISSISSIPPI RIVER TARBERT	55,000	90%	49,500	0.013	0.250
	55,000	70%	38,500	0.045	0.557
	55,000	50%	27,500	0.078	0.859
	55,000	30%	16,500	0.033	1.336

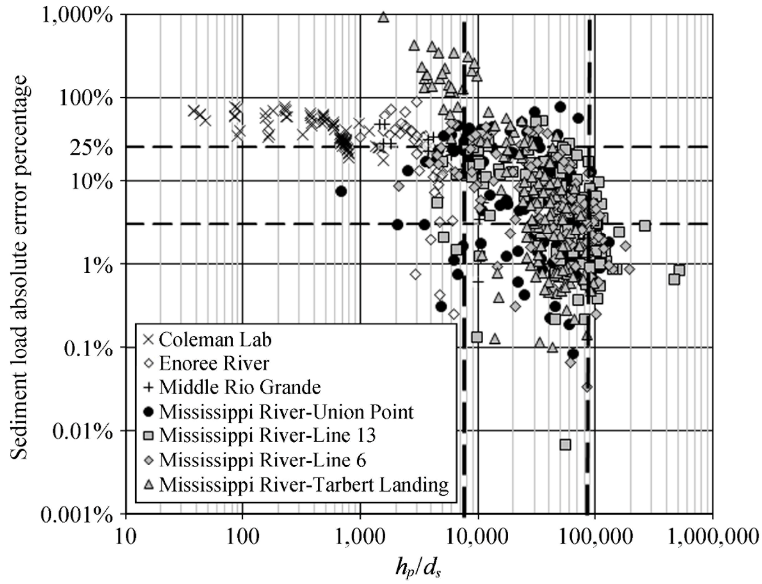


Fig. 6 Error analysis vs sampling depth h_p/d_s

Table 4 Summary of error analysis as a function of relative sampling depth h_p/d_s

Data set	Number of samples	h_m/d_s>1000 and % difference < 25%		h_m/d_s >10,000 and % difference < 25%	
		Number of samples	% of samples	Number of samples	% of samples
Coleman lab data	72	3	4.2%	0	0%
Enoree river, SC	43	23	53.5%	0	0%
Middle RIO GRande, NM	20	12	60.0%	3	15%
Mississippi - Tarbert	145	122	84.1%	104	71.7%
Mississippi - Union point	140	128	91.4%	116	82.9%
Mississippi - Line 6	115	99	86.1%	88	76.5%
Mississippi - Line 13	133	91	68.4%	91	68.4%
Total	668	478	71.6%	402	60.2%

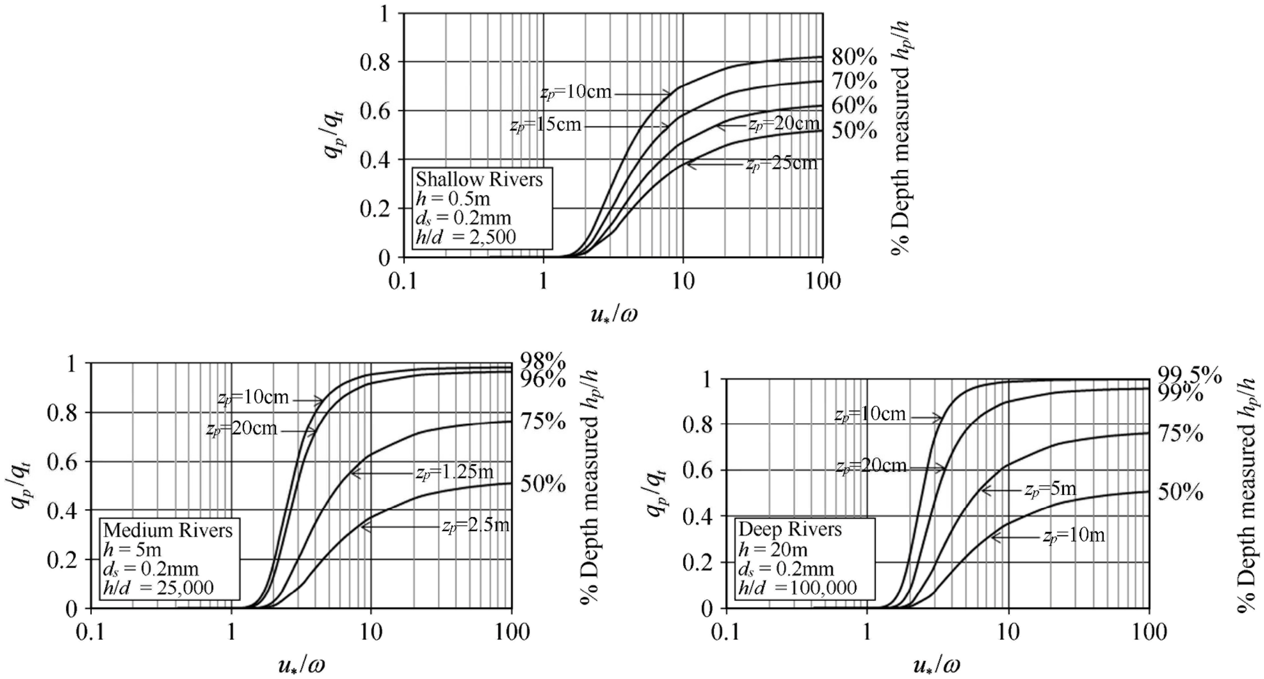


Fig. 7 Point sampling efficiency for sand-bed rivers

5 Conclusions

This study determines under what conditions point velocity and sediment concentration measurements can be used to calculate total sediment discharge in sand-bed rivers. A new total sediment discharge calculation algorithm based on the Series Expansion of the Modified Einstein Point Procedure (SEMEPP) is presented. This procedure provides uses to use point sediment data with the Modified Einstein approach for a high degree of reliability with evaluation of total sediment load. The procedure is tested with a wide range of laboratory and field data totaling 801 point measurements on 124 verticals at sediment concentrations less than 0.1 kg l^{-1}. The ratio of shear to fall velocities u_*/ω is very important to determine when point measurements can be effective in sand-bed rivers. Point measurement techniques like SEMEPP are well-suited when $u_*/\omega > 5$, in such case, at least 60% of the total sediment load is measured when 90% of the flow depth is sampled. The determination of sediment discharge from point measurements is most accurate in deep rivers when $u_*/\omega > 10$ and $h_p/d_s > 10{,}000$. The root mean square error (RMSE) increases (less precision) while the relative error (MPE) decreases (less bias) as h_p/d_s increases. The effect of sampling depth has also been investigated. For instance on the Mississippi at $u_*/\omega = 10$ from Fig. 5, only 36% of the total sediment load is measured when sampling 50% of the flow depth. In comparison, more than 80% of the total sediment discharge can be measured when sampling 90% of the water column. As illustrated in Fig. 7, the main contribution of this article is to quantify the percentage of the sediment load that is measured in shallow and deep sand-bed rivers as a function of the sampling depth z_p and u_*/ω. Finally, it is found that point measurements are not recommended for shallow rivers (h < 0.5 meters) or when $u_*/\omega < 2$ because they only capture a small proportion of the total sediment load.

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Notation

B		Ratio of bed layer thickness to flow depth $B = z_b /h = 2d_s/h$
C	M L^{-3}	Sediment concentration
C_b	M L^{-3}	Reference concentration at the reference depth of $2d_s$
C_i	M L^{-3}	Sediment concentration a location i
d_s	L	Particle diameter
h	L	Total flow depth
h/d_s		Relative submergence
h_p	L	Sampling depth from the free surface to the lowest sampling point
h_p/d_s		Ratio of sampling depth to particle size
h_p/h		Relative sampling depth
i		Sampling location with the vertical measured from the water surface

J_{1S}		$\int_B^1 \left(\frac{1 \quad z_*}{z_*}\right)^{\mathrm{Ro}} dz_*$ first Einstein Integral for the suspended load
J_{1P}		$\int_P^1 \left(\frac{1 \quad z_*}{z_*}\right)^{\mathrm{Ro}} dz_*$ first Einstein Integral for the partial load
J_{2S}		$\int_B^1 \ln z_* \left(\frac{1 \quad z_*}{z_*}\right)^{\mathrm{Ro}} dz_*$ second Einstein Integral for the suspended load
J_{2P}		$\int_P^1 \ln z_* \left(\frac{1 \quad z_*}{z_*}\right)^{\mathrm{Ro}} dz_*$ second Einstein Integral for the partial load
MPE		Mean Percent Error
n		Total number of verticals used particular statistical analysis
p		Number of sampling points along a vertical
P		Ratio of sampler nozzle height to flow depth $P = z_p/h$
q_b	$M L^{-1}T^{-1}$	Unit bed load discharge by mass in the bed layer $0 < z < z_b$
q_{pm}	$M L^{-1}T^{-1}$	Partial unit sediment discharge by mass from the point measurements
q_p	$M L^{-1}T^{-1}$	Partial unit sediment discharge by mass calculated from SEMEPP
q_{pm}/q_{tm}		Measured ratio of partial to total sediment discharge
q_p/q_t		Ratio of partial to total sediment discharge calculated from SEMEPP
q_s	$M L^{-1}T^{-1}$	Unit suspended sediment discharge by mass
q_t	$M L^{-1}T^{-1}$	Total unit sediment discharge by mass calculated from SEMEPP
q_{tm}	$M L^{-1}T^{-1}$	Total unit sediment discharge by mass from the point measurements
RMSE		Root Mean Square Error
Ro		Rouse number
u_*	$L T^{-1}$	Shear velocity $u_* = \sqrt{ghS_f}$
u_*/ω		Ratio of shear velocity to fall velocity
v	$L T^{-1}$	Velocity
v_i	$L T^{-1}$	Velocity at i
v_b	$L T^{-1}$	Velocity in the bed layer, $v_b = 11.6 u_*$
X_i	$M L^{-1}T^{-1}$	Measured sediment discharge
Y_i	$M L^{-1}T^{-1}$	Calculated sediment discharge
z	L	Vertical elevation above the bed
z_b	L	Reference elevation of $2d_s$ or bed layer thickness
z_o	L	Elevation above the bed where the velocity is zero
z_p	L	Nozzle elevation of the sampler above the bed
z_*		Ratio of z/h
β		Momentum correction factor for sediment, assumed $\beta = 1$
κ		Von Kármán constant, assumed $\kappa = 0.4$ in this study
ω	$L T^{-1}$	Fall velocity of sediment