Total Sediment Load from SEMEP Using Depth-Integrated Concentration Measurements

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Abstract: This study improves total sediment load calculations on the basis of depth-integrated sediment concentration measurements for channels with significant sediment transport in suspension. The series expansion of the modified Einstein procedure (SEMEP) removes most of the empiricism found in the existing modified Einstein procedures (MEP). SEMEP calculations require field measurements of flow discharge, depth-integrated suspended sediment (SS) concentration, and suspended particle sizes. SEMEP calculates the Rouse number, Ro, from the median particle size measured in suspension d_{50ss}. On the basis of the sediment discharge measurements collected from 14 rivers, the accuracy of sediment discharge calculations depend on the ratio of the shear velocity u_* to the settling velocity ω. SEMEP performs accurately (error less than 25%) and without bias when $u_*/\omega > 5$. Calculations are also acceptable, but less accurate when u_*/ω is between two and five. Both SEMEP and MEP should not be used when the value of $u_*/\omega < 2$, and a simplified formulation on the basis of bed sediment discharge is recommended when $u_*/\omega < 2$. **DOI:** 10.1061/(ASCE)HY.1943-7900.0000466.

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Introduction

Sediment transport in rivers is complex, and accurate estimates of total sediment loads remain rather difficult to obtain. The total sediment load can normally be examined on the basis of either sediment source, modes of sediment transport, or measurement method. The sediment sources are identified as bed material load and wash load (fine particles not found in the bed). The modes of sediment transport are classified as either in suspension or near the bed. The measurement method refers to the amount of sediment measured by a suspended sediment (SS) sampler. Suspended sediment can be measured by using either a depth-integrated sampler or a point sampler, but the analysis in this paper is focused exclusively on depth-integrated sediment concentration measurements. Fig. 1(a) provides a graphical depiction of how total sediment load can be determined. One of the main challenges is that all size fractions transported in suspension can not be found in the bed material. Therefore sediment transport calculations on the basis of the bed material cannot accurately quantify the wash load. The procedure examined in this research is on the basis of the measured sediment discharge through field measurements of the sediment concentration. The primary objective is then to determine the total sediment discharge using extrapolations closer to the bed to determine the unmeasured sediment discharge.

This article focuses specifically on methods to estimate the total sediment discharge in rivers from field measurements of flow discharge, depth-integrated SS concentration, and suspended particle size. Depth-integrated samplers cannot measure the entire SS zone or water column. As a result, the total sediment discharge is estimated by extending the velocity and concentration profiles to the bed. Because the sediment concentration located near the bed can be very high, total sediment discharge calculations can be extremely variable. Thus, the accuracy of total sediment discharge calculations depends on whether the measured sediment concentrations are high enough and representative of concentrations in the unmeasured zone (i.e., near the bed). Therefore, there is a need to determine the range of conditions for which the total sediment discharge can be accurately estimated from depth-integrated SS concentration measurements.

The modified Einstein procedures (MEP) was first developed by Colby and Hembree (1955) to determine the total sediment discharge of sand bed channels on the basis of field concentration measurements obtained from a depth-integrated SS sampler, a bed material sample, and a flow discharge measurement. The method used the combination of the Einstein bed load function (Einstein 1942, 1950) and an extension of the velocity and concentration profiles to the unmeasured zone to estimate a total sediment discharge. The MEP was tested with data collected from the Niobrara River in Nebraska. The SS and bed material measurements were divided by sediment size classes (or bins), and the Rouse number (Ro $= \omega/\beta_s \kappa u^*$) was fitted by trial and error for a single overlapping bin. The procedure then empirically varied Ro with the settling velocity raised to the power of 0.7 for all remaining bins (Colby and Hembree 1955). In addition, the procedure arbitrarily divided the Einstein bed load transport rate by 2. Subsequent modifications of the MEP have been proposed (Burkham and Dawdy 1980; Colby and Hubbell 1961; Holmquist-Johnson et al. 2009; Lara 1966; Pemberton 1972; Shen and

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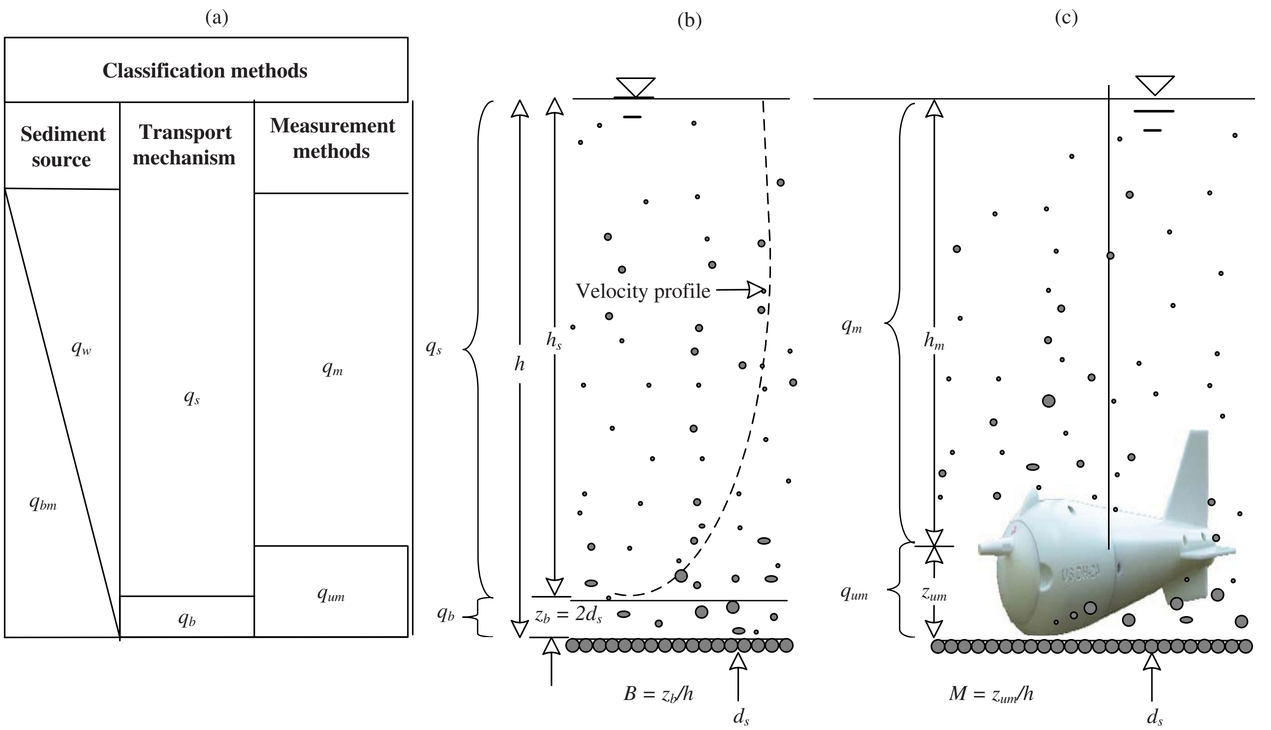

Fig. 1. Definition sketch

Hung 1983). Shah-Fairbank (2009) provided a detailed review of these methods and their relative contributions. Several field conditions were identified where the overlapping bin approach produced unrealistic results and significant errors in the calculations. This led to a fundamental review of the overlapping bin approach.

The primary purpose of this study is to develop a more reliable and simplified total sediment discharge calculation procedure, on the basis of the data collected from a depth-integrated sediment sampler. The objective is to develop a new MEP approach that would not be on the basis of overlapping bins, but would use both the particle size and concentration of sediment in suspension. The proposed procedure is called series expansion of the modified Einstein procedure (SEMEP) and does not require overlapping bins. It also became important to examine the accuracy of SEMEP through comparisons with field measurements of the total sediment load. The aim is to define the range of conditions for which the method is most suitable, as well as conditions for which the method should not be applied.

SEMEP Formulation

The sediment flux by advection can be described by the product of sediment concentration C and flow velocity v. As shown in Fig. 1(b), the turbulent velocity profile is assumed to follow a logarithmic distribution (Keulegan 1938):

$$v = \frac{u_*}{\kappa} \ln\left(\frac{30z}{k_s}\right) \tag{1}$$

where v = velocity measured at elevation z above the bed, u_* = shear velocity, κ = von Kármán constant normally close to 0.4, z = vertical distance above the bed, and k_s = boundary roughness height. Einstein (1950) assumed the value of k_s is equal to $2d_{65}$ of the bed material.

The concentration profile is estimated from the Rouse equation (Rouse 1937):

$$C = C_{z_b}\left(\frac{h-z}{z}\frac{z_b}{h-z_b}\right)^{\mathrm{Ro}} \tag{2}$$

$$\mathrm{Ro} = \frac{\omega}{\beta_s \kappa u_*} \tag{3}$$

where C = concentration at an elevation z above the bed, h = flow depth, Ro = Rouse number, ω = settling velocity, and C_{z_b} = reference concentration at an elevation z_b. Einstein (1950) used $z_b = 2d_s$ above the bed with d_s equals d_{65} of the bed material. It is often assumed that the momentum correction factor for the sediment β_s is equal to one and this is the value adopted in this study.

SS Discharge

The unit SS discharge q_s can then be determined by integrating the product of the velocity v and concentration profile C:

$$q_s = \int_{z_b}^{h} Cv\,\mathrm{d}z \tag{4}$$

The unit SS discharge q_s can be calculated by volume q_{sv} (L^2/T) when using a volumetric sediment concentration. Conversions to unit sediment discharge by mass q_{sm} (M/LT) and weight q_{sw} (M/T^3) are shown in Eq. (5):

$$q_{sw} = gq_{sm} = \rho_s g q_{sv} \tag{5}$$

where g = gravitational acceleration and ρ_s = mass density of the sediment. Eq. (6) refers to the unit sediment discharge by weight. The total unit sediment discharge is calculated by adding the bed and SS discharges per unit width, as described in Fig. 1(a):

$$q_t = q_b + q_s = q_b + \int_{z}^{h} Cv\,\mathrm{d}z \tag{6}$$

where q_t = unit total sediment discharge and q_b = unit bed sediment discharge. Einstein (1950) suggested the following relationship between the unit bed sediment discharge and reference concentration:

$$C_{z_b} = \frac{q_b}{v_{z_b} z_b} = \frac{q_b}{11.6 u_* z_b} \quad (7)$$

where v_{z_b} = reference velocity in the bed layer determined as $11.6u_*$. Therefore, Eqs. (8) and (9) are obtained by substituting Eqs. (1) and (2) into Eqs. (4) and (6). Eq. (7) is rearranged and substituted into Eq. (6) to describe the bed sediment discharge:

$$q_s = 0.216\, q_b \frac{B^{\mathrm{Ro}-1}}{(1-B)^{\mathrm{Ro}}} \left\{ \ln\left(\frac{30h}{d_s}\right) J_{1S} + J_{2S} \right\} \quad (8)$$

$$q_t = q_b + 0.216\, q_b \frac{B^{\mathrm{Ro}-1}}{(1-B)^{\mathrm{Ro}}} \left\{ \ln\left(\frac{30h}{d_s}\right) J_{1S} + J_{2S} \right\} \quad (9)$$

$$J_{1S} = \int_B^1 \left(\frac{1-z^*}{z^*}\right)^{\mathrm{Ro}} \mathrm{d}z^* \quad (10)$$

$$J_{2S} = \int_B^1 \ln z^* \left(\frac{1-z^*}{z^*}\right)^{\mathrm{Ro}} \mathrm{d}z^* \quad (11)$$

where $B = 2d_s/h$, $z^* = z/h$, and J_{1S} and J_{2S} are known as the Einstein integrals and are evaluated within the SS zone. The SEMEP formulation uses depth-integrated SS samples, which assume $\kappa = 0.4$, $z_b = 2d_s = 2d_{65}$ of the bed material, $\beta_s = 1$, and ω is based on the median grain diameter of the measured SS d_{50ss}. The values of J_{1S} and J_{2S} are also determined by using the series expansion given by Guo and Julien (2004).

Depth-Integrated Sediment Concentration Measurements

As shown in Fig. 1(c), depth-integrated sediment concentration measurements define the measured unit sediment discharge. It is evaluated by integrating the product of flow velocity and volumetric sediment concentration, from the nozzle height z_{um} (unmeasured depth) to the free surface (i.e., $z = h$):

$$q_m = \int_{z_{um}}^{h} Cv\mathrm{d}z \quad (12)$$

where q_m = measured unit sediment discharge and z_{um} = unmeasured depth or nozzle elevation above the bed, as shown in Fig. 1(c). Therefore, Eq. (13) is obtained by inserting Eqs. (1) and (2) into Eq. (12). The only difference from Eqs. (4) and (12) is the change in the limits of integration:

$$q_m = 0.216 q_b \frac{B^{\mathrm{Ro}-1}}{(1-B)^{\mathrm{Ro}}} \left\{ \ln\left(\frac{30h}{d_s}\right) J_{1M} + J_{2M} \right\} \quad (13)$$

$$J_{1M} = \int_M^1 \left(\frac{1-z^*}{z^*}\right)^{\mathrm{Ro}} \mathrm{d}z^* \quad (14)$$

$$J_{2M} = \int_M^1 \ln z^* \left(\frac{1-z^*}{z^*}\right)^{\mathrm{Ro}} \mathrm{d}z^* \quad (15)$$

where J_{1M} and J_{2M} = modified Einstein integrals evaluated in the measured zone and $M = z_{um}/h$. In SEMEP, these integrals are solved directly by using the series expansion formulation of Guo and Julien (2004). The novelty in SEMEP is that q_b is calculated directly on the basis of q_m because all other parameters in Eq. (13) are known. This is one of the unique features of SEMEP because there is no need to selectively use the Einstein bed load function or to arbitrarily divide the bed sediment discharge intensity by two, as was done in earlier MEP formulations.

The following steps show how to use SEMEP to determine q_b, q_s and q_t on the basis of q_m:

1. Calculate Ro on the basis of hydraulic parameters and the median grain size in suspension (d_{50ss}) by using Eq. (3).
2. Determine the limits of integration (h, z_{um}, and z_b).
3. Use the series expansion (SEMEP) to calculate the Einstein and the modified Einstein integrals (J_{1S}, J_{2S}, J_{1M}, and J_{2M}).
4. Given the value of q_m, B, Ro, h/d_s, J_{1M}, and J_{2M}, calculate q_b directly by using Eq. (13).
5. Calculate q_s and q_t by using Eqs. (8) and (9), respectively.
6. Estimate the total sediment discharge from the unit sediment discharge and the channel width.

SEMEP Testing

A large sediment transport data set collected from 14 rivers was used for testing SEMEP. Data were compiled by Shah-Fairbank (2009) and included detailed data sets collected for the Platte River (Kircher 1981) and many other U.S. streams (Williams and Rosgen 1989). This data set is unique in that each site has a complete record including total sediment discharge measurements from both depth-integrated and Helley-Smith samplers. Table 1 summarizes the type of data available for testing each river. Complete data sets contain: water discharge Q, flow velocity V, channel width W, average flow depth h, slope S, water temperature T, measured unit SS discharge with depth-integrated samplers q_m, measured unit sediment discharge from Helley-Smith samplers, particle size distributions of the material found both in the bed and measured in the SS sampler, and measured unit total discharge at a constricted section q_t.

The detailed database compiled by Shah-Fairbank (2009) contains over 300 complete measurements. The values of u_*/ω from the data set varied from 0.5–15,000, and the ratio of h/d_s ranged from 50–12,000. The data sets are grouped as Platte River, U.S. streams with SS, and U.S. streams from Colorado. The measured total sediment discharges are compared with the SEMEP calculations in Fig. 2. Open symbols are used when $u_*/\omega < 5$, and full symbols are used when $u_*/\omega > 5$. The sediment transport rates vary by more than seven orders of magnitude and the SEMEP calculations are very close to the line of perfect agreement. The 100% error margin of the sediment transport calculations is also shown between the dotted lines in Fig. 2. Overall, the calculations are in closer agreement with the measurements when $u_*/\omega > 5$ and when the measured total sediment discharge is large, i.e., greater than approximately 20,000 tons/day.

Statistical Analysis

The degree of accuracy of SEMEP is evaluated through a statistical analysis. Three main parameters are examined: (1) the mean percent error (MPE); (2) the coefficient of determination; and (3) the concordance correlation coefficient.

The MPE is a measure of the relative error (Ott and Longnecker 2001) and reflects a bias in the calculations:

$$\mathrm{MPE} = \frac{\sum_{i=1}^{n} \frac{(X_i - Y_i)}{X_i}}{n} \quad (16)$$

where X_i = measured total sediment discharge, Y_i = calculated total sediment discharge, and n = number of samples. If the value of MPE is greater than zero, then SEMEP underpredicts the total sediment discharge.

Table 1. Total Sediment Discharge Measurements on Several Rivers

Category	River	Number of samples	Q	V	W	h	S	T°	Sediment discharge q_s	Sediment discharge q_b	Grain size Bed	Grain size SS[a]
Platte River	North Platte River, NE	17	√	√	√	√		√	√	√	√	√
	South Platte River, CO and NE		√	√	√	√		√	√	√	√	√
	Platte River, NE		√	√	√	√		√	√	√	√	√
U.S. streams with SS[a]	Susitna River near Talkeetna, AK	37	√	√	√	√	√	√	√	√	√	√
	Chulitna River below Canyon near Talkeetna, AK	43	√	√	√	√	√	√	√	√	√	√
	Susitna River at Sunshine, AK	37	√	√	√	√	√	√	√	√	√	√
	Snake River near Anatone, WA	31	√	√	√	√	√	√	√	√	√	√
	Toutle River at Tower Road near Silver Lake, WA	19	√	√	√	√	√	√	√	√	√	√
	North Fork Toutle River near Kid Valley, WA	5	√	√	√	√	√	√	√	√	√	√
	Clearwater River at Spalding, ID	35	√	√	√	√	√	√	√	√	√	√
U.S. streams from Colorado	Mad Creek Site 1 near Empire, CO	5	√	√	√	√	√	√	√	√	√	
	Craig Creek near Bailey, CO	21	√	√	√	√	√	√	√	√	√	
	North Fork of South Platte River at Buffalo Creek, CO	20	√	√	√	√	√	√	√	√	√	

[a]SS = suspended sediment.

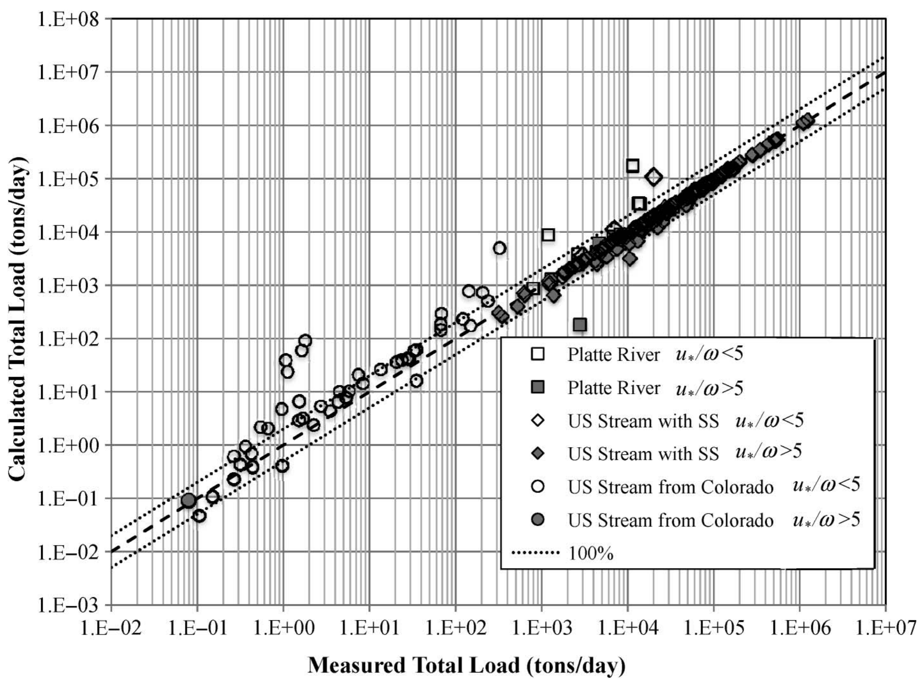

Fig. 2. SEMEP calculated versus measured total sediment discharge in tons/day

The coefficient of determination R^2 is calculated (Ott and Longnecker 2001) as

$$R^2 = \left(\frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{n}(X_i - \bar{X})^2 \sum_{i=1}^{n}(Y_i - \bar{Y})^2}} \right)^2 \tag{17}$$

where $\bar{X}$ = mean measured total sediment discharge and $\bar{Y}$ = mean calculated total sediment discharge. As the value of R^2 approaches 1, there is less variation between the measured and calculated sediment discharge.

The concordance correlation coefficient ρ_c evaluates the degree to which pairs of observations fall on the line of perfect agreement at 45° line from the origin (Lin 1989):

$$\rho_c = \frac{2s_{xy}}{s_x^2 + s_x^2 + (\bar{X} - \bar{Y})^2} \tag{18}$$

$$s_{xy} = \frac{\sum_1^n (X_i - \bar{X})(Y_i - \bar{Y})}{n - 1} \tag{19}$$

$$s_x^2 = \frac{\sum_1^n (X_i - \bar{X})^2}{n - 1} \tag{20}$$

$$s_y^2 = \frac{\sum_1^n (Y_i - \bar{Y})^2}{n - 1} \tag{21}$$

where s_{xy} = covariance, s_x = variance of the measured total sediment discharge, and s_y = variance of the calculated total sediment discharge.

The results of the statistical data analysis are summarized in Table 2. Overall, the results indicate that when $u_*/\omega > 5$, the MPE is close to zero and the values of R^2 and ρ_c are closer to 1.

Table 2. Statistical Results for SEMEP

Rivers	u_*/ω	n	MPE	R^2	ρ_C
Platte River	>5	6	0.12	0.71	0.76
	<5	11	−0.17	0.62	0.71
U.S. streams with SS[a]	>5	203	0.02	0.99	0.99
	<5	4	−0.11	0.99	0.81
U.S. streams from Colorado	>5	1	0.02	–	–
	<5	45	0.26	0.82	0.84
All data	>5	210	0.01	0.98	0.99
	<5	60	−2.09	0.93	0.90

[a]SS = suspended sediment.

Overall, the MPE equaled 0.01 for all data when $u_*/\omega > 5$. Thus, it is concluded that SEMEP is highly accurate and without bias when $u_*/\omega > 5$.

The accuracy of the SEMEP method is also graphically investigated as a function of u_*/ω. Fig. 3 indicates that the error is lowest when $u_*/\omega > 5$. Indeed, more than 90% of the samples have less than 25% error when $u_*/\omega > 5$.

Ratio of Suspended to Total Sediment Discharge

The ratio q_s/q_t of the unit SS discharge to the total sediment discharge can be calculated using Eqs. (8) and (9):

$$\frac{q_s}{q_t} = \frac{0.216\ \frac{B^{\mathrm{Ro}-1}}{(1-B)^{\mathrm{Ro}}}\{\ln(\frac{30h}{d_s})J_{1S} + J_{2S}\}}{1 + 0.216\ \frac{B^{\mathrm{Ro}-1}}{(1-B)^{\mathrm{Ro}}}\{\ln(\frac{30h}{d_s})J_{1S} + J_{2S}\}} \tag{22}$$

Eq. (22) shows that the ratio of suspended to total sediment discharge is only a function of u_*/ω and the ratio of flow depth h to grain diameter d_s. In this analysis, the simple ratio of u_*/ω has been preferred to the Rouse number ($\mathrm{Ro} = \omega/0.4u_*$) because u_*/ω does not require the empirical determination of the von Kármán constant. The results of the ratio q_s/q_t as a function of u_*/ω and relative submergence h/d_s are plotted in Fig. 4. Fig. 4 shows the SEMEP calculations of the ratio q_s/q_t using Eq. (22) at constant values of h/d_s while varying the value of u_*/ω. This graph is highly instructive. When the value of u_*/ω is less than one, the various lines associated with h/d_s converge. This occurs because the majority of the sediment transport takes place close to the bed and sediment transport does not depend on flow depth in this zone. When the value of u_*/ω is greater than one, more sediment is transported in suspension and the lines associated with relative submergence have a tendency to diverge. This occurs because flow depth becomes more important when sediment is in suspension. Overall, it is found that q_s/q_t depends largely on u_*/ω and much less on h/d_s. As the value of h/d_s approaches infinity and the value of u_*/ω is greater than 2.5 the ratio q_s/q_t approaches one, which suggests that the sediment will move primarily in suspension, and bed sediment discharge becomes a negligible fraction of the total sediment discharge.

Dominant Modes of Sediment Transport

As the MEP approach uses measured SS to determine total sediment discharge, it will be most accurate when most of the sediment is transported in suspension, i.e., large values of u_*/ω. On the other hand, low values of u_*/ω should correspond to low sediment transport rates. The ratio of suspended to total sediment discharge q_s/q_t is used to determine the dominant mode of sediment transport (Dade and Friend 1998; Julien 1995). The modes of sediment transport are a function of the shear velocity u_* and the settling velocity ω of the particle size in suspension. The curves generated from SEMEP [Eq. (22) and Fig. 4] can be used to determine the value of u_*/ω corresponding to user-specified ratios of bed sediment discharge and SS discharge. By using Fig. 4, it can be stated that SEMEP is very useful in streams and rivers with sediment predominantly transported in suspension (i.e., u_*/ω is greater than 2).

Low Sediment Transport Rates

Fig. 4 shows that more than 25% of the sediment is transported as bed sediment discharge when $u_*/\omega < 2$. As the lines in Fig. 4 converge ($q_s/q_t = 0.25$) and u_*/ω is less than 2, the ratio of q_s/q_t becomes proportional to the ratio of u_*/ω, or inversely proportional to Ro, because the sediment discharge is not a function of h/d_s:

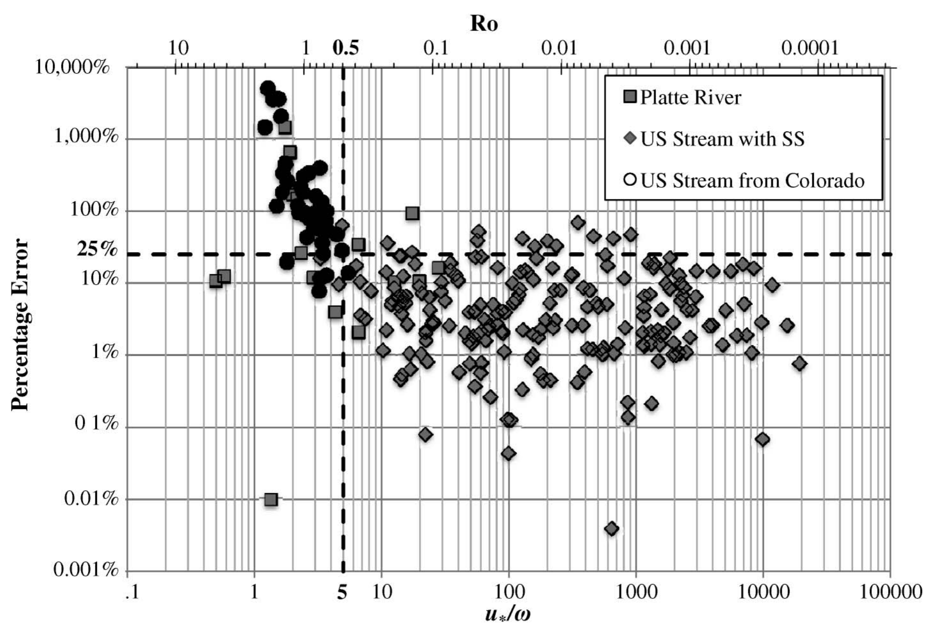

Fig. 3. Accuracy of SEMEP as a function of u_*/ω

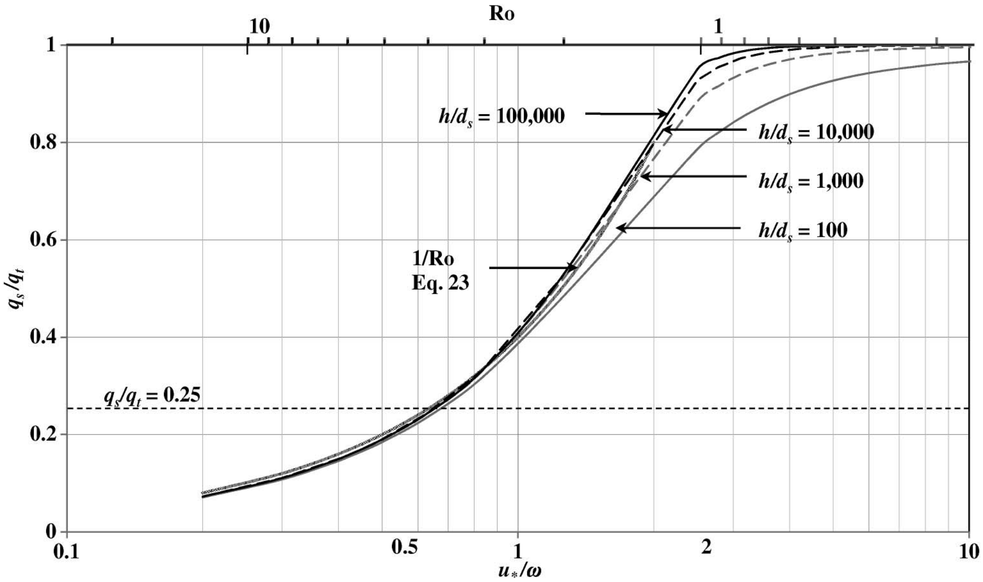

Fig. 4. SEMEP curves of q_s/q_t as a function of u_*/ω and h/d_s

$$\frac{q_s}{q_t} \cong 0.4\frac{u_*}{\omega} = \frac{1}{\text{Ro}} \quad \text{when } \frac{u_*}{\omega} \leq 2 \tag{23}$$

When $u_*/\omega < 2$, the value of q_t can thus be calculated directly from the unit bed sediment discharge and the ratio of u_*/ω:

$$q_t \cong q_b\left(\frac{1}{1 - \frac{0.4u_*}{\omega}}\right) \quad \text{when } \frac{u_*}{\omega} \leq 2 \tag{24}$$

This simplification allows the user to estimate q_s/q_t, q_b/q_t, and q_t without solving the Einstein integrals, as long as q_b is known. Thus, when $u_*/\omega < 2$, bed load equations are recommended and the MEP should not be used.

Incipient Motion

The initiation of motion is commonly described by the Shields' diagram. The modified Shields diagram (Cheng and Chiew 1998, 1999; Julien 1995) plots the Shields parameter τ_* as a function of the dimensionless particle diameter d_*:

$$\tau_* = \frac{hS}{(G-1)d_{50ss}} \tag{25}$$

$$d_* = d_{50ss}\left(\frac{(G-1)g}{v^2}\right)^{\frac{1}{3}} \tag{26}$$

Simons and Richardson performed studies in large laboratory flumes (up to 8 ft wide and 150 ft long) to determine flow resistance and sediment transport rates. They conducted 339 equilibrium runs within the database (Guy et al. 1966) includes water discharge, flow depth, average velocity, water surface slope, SS concentration and gradation, total sediment concentration and gradation, and bed configuration. Fig. 5 shows the data of Guy et al. (1966) on a modified Shields diagram. This figure also shows lines of constant u_*/ω as a function of τ_* and d_*. Clearly, there is no sediment transport when $u_*/\omega < 0.2$.

In summary, Table 3 delineates the primary modes of sediment transport. There are four predominant modes of sediment transport: suspended load, mixed load, bed load, and no transport. The

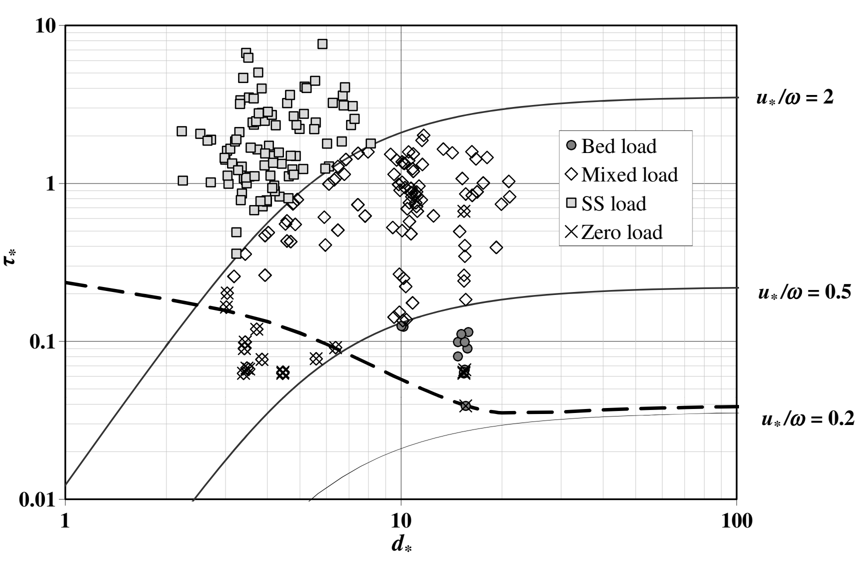

Fig. 5. Shields parameter versus d_* and u_*/ω for flume data

Table 3. Dominant Mode of Sediment Transport and Recommended Calculation Procedure

u^*/ω	Ro	Dominant mode of transport	Sediment calculation procedure
< 0.2	> 12.5	No motion	
0.2–0.5	5–12.5	Bed load	Bed load equation
0.5–2	1.25–5	Mixed load	Einstein or bed material load equation
2–5	0.5–1.25	Suspended load	MEP or SEMEP
> 5	< 0.5		SEMEP with high accuracy

Note: Ro is calculated assuming that $\kappa = 0.4$.

delineation criteria are as follows: bed load is dominant when q_s/q_t is less than 0.2. Indeed, the suspended load is less than 25% of the bed load and hence the suspended load can be considered relatively small compared with the bed load. Likewise, suspended load is dominant when q_s/q_t is greater than 0.8. In this case, the bed load transport is less than 25% of the suspended load and hence the bed load can be considered relatively small. Mixed load is considered between these two cases and corresponds to cases where neither the bed load nor the suspended load can be considered very small compared to the other.

The results indicate that when $u_*/\omega > 5$, SEMEP performs accurately (percent error $< 25\%$ in Fig. 3) and without bias (MPE < 0.01 in Table 2). SEMEP calculations are acceptable, but less accurate, when u_*/ω is between two and five. Both SEMEP and MEP should not be used when $u_*/\omega < 2$ because most of the sediment transport is not in suspension. Sediment transport calculation methods on the basis of the particle sizes of the bed material are therefore recommended instead of SEMEP when $u_*/\omega < 2$. Graphically, the dominant mode of sediment transport and recommended sediment transport procedure is illustrated in Fig. 6.

Ratio of Measured to Total Sediment Discharge

The ratio of the measured to total unit sediment discharges q_m/q_t is useful in practice. When this ratio is large, the extrapolation to determine the unmeasured sediment discharge will only add a small fraction to the measured sediment discharge, thus providing accurate estimates of total sediment discharge:

$$\frac{q_m}{q_t} = \frac{0.216\ \frac{B^{\mathrm{Ro}-1}}{(1-B)^{\mathrm{Ro}}}\{\ln(\frac{30h}{d_s})J_{1M} + J_{2M}\}}{1 + 0.216\ \frac{B^{\mathrm{Ro}-1}}{(1-B)^{\mathrm{Ro}}}\{\ln(\frac{30h}{d_s})J_{1S} + J_{2S}\}} \qquad (27)$$

Eq. (27) is solved here using SEMEP from depth-integrated sediment concentration measurements, particle size distribution, and hydraulic parameters. The values of J_{1M} and J_{2M} are solved on the basis of the series expansion of the Einstein integrals evaluated within the measured zone.

The measured unit sediment discharge q_m is a function of the sampling depth h_m. If the unmeasured flow depth z_{um} is constant, then the ratio q_m/q_t of the measured to total sediment discharges is evaluated from Eq. (27). The analysis of this equation shows that the ratio of q_m/q_t is a function of (1) the ratio of u_*/ω, (2) the ratio of h/d_s, and (3) the ratio of h_m/h. This calculation example uses sand sizes of 0.0625 mm $< d_s <$ 2 mm. The unmeasured flow depth is that of a standard depth-integrated sampler with z_{um} of 0.1 m, which corresponds to the height of a standard Helley-Smith sampler (Emmett 1980). Hence, this example corresponds to the case where the total sediment discharge can determined directly from the sum of the sediment discharges from the depth-integrated sampler and the Helley-Smith sampler. Fig. 7 plots values of the ratio q_m/q_t generated from Eq. (27) by varying the value of u_*/ω as a function of grain size, and the percentage of the flow measured h_m/h. The flow depth h is varied from 0.2 to 10 m. As a result, the series of lines in Fig. 7 show that q_m/q_t varies primarily with u_*/ω and h_m/h. These results confirm that the measured sediment discharge would only be a small fraction of the total sediment discharge when $u_*/\omega < 2$.

SEMEP Attributes and Limitations

Summary of the Main Attributes of SEMEP

The main differences between SEMEP and previous MEP algorithms are the following:

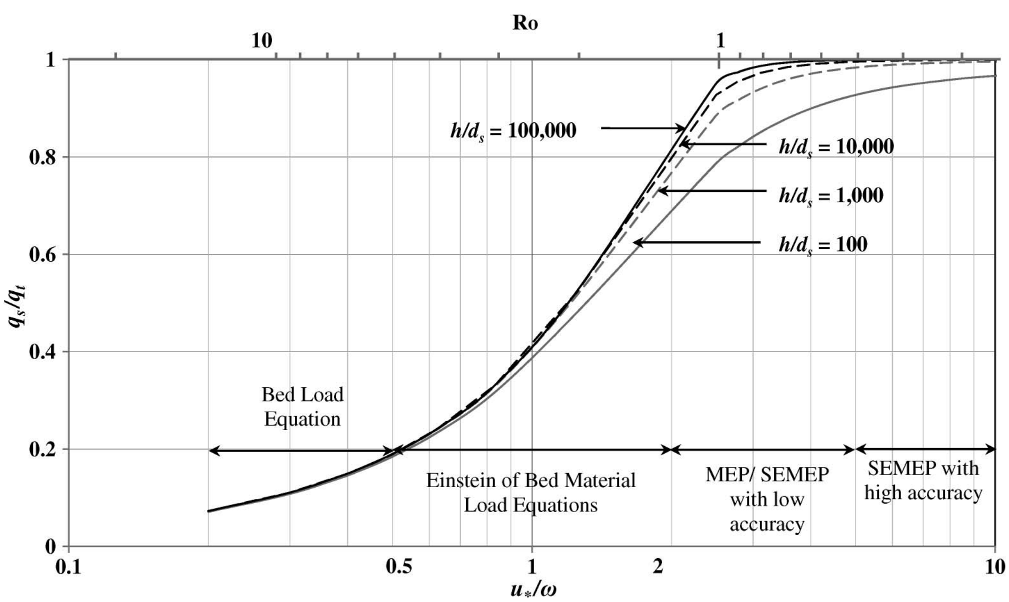

Fig. 6. Mode of sediment transport and recommended calculation procedure

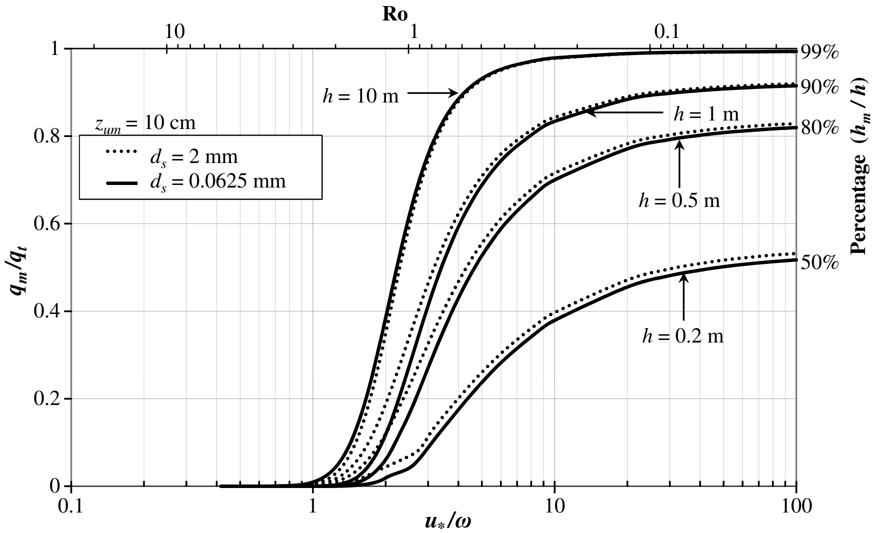

Fig. 7. Ratio q_m/q_t of measured to total sediment discharge versus u_*/ω and h_m/h

1. Calculation of the total sediment discharge on the basis of the median grain size of SS d_{50ss}, thus not requiring the SS and bed material samples to be divided into size classes or bins;
2. No regression fitting of Ro on the basis of the data from overlapping bins and thus the removal of the empirical power 0.7 from the original MEP;
3. Computation of bed sediment discharge on the basis of the measured sediment discharge and there is no need to preferentially select Einstein's bed load equation or arbitrarily divide the bed load intensity by two;
4. Evaluation of *Ro* on the basis of the ratio of settling velocity ω to shear velocity u_*, assuming $\beta_s = 1$ and $\kappa = 0.4$;
5. Use of the series expansion of Guo and Julien (2004) to solve the Einstein integrals; and
6. Estimation of the bed sediment discharge from the measured sediment discharge; hence the total sediment discharge will always be equal to or larger than the measured sediment discharge, which was not always the case with other MEP formulations.

Summary of the Limitations of SEMEP

There are limitations associated with the development of SEMEP. The proposed procedure is on the basis of the series expansions of the Einstein integrals (Guo and Julien 2004). To apply the series expansion algorithm, the relative submergence h/d_s must be greater than 20 and u_*/ω must be greater than 0.42. This should cover just about all cases encountered in engineering practice. Other methods can be used to evaluate the Einstein integrals. These should yield comparable results as long as they are sufficiently accurate.

When calculating the u_*/ω, the fall velocity is evaluated on the basis of the median grain size in suspension. By removing the need for particle sizes found in both the bed and the suspension, the SEMEP procedure can be used in rivers with high wash load and with coarse armored beds. However, as the procedure calculates a total sediment discharge from the suspended load measurements, it is limited to sediment sizes finer than 2 mm. SEMEP should not be used to determine gravel transport rates as those fractions cannot be measured in suspension with standard depth-integrated samplers, e.g., P-61 and P-63.

Conclusions

This study improves total sediment discharge calculations on the basis of the measured concentration data from a depth-integrated sampler. SEMEP incorporates the following characteristics to remove the arbitrary empiricism of other MEP. SEMEP is on the basis of the median grain diameter d_{50ss} of sediment in suspension, and no bins are required. There is no empirical fitting between the bed material and sediment in suspension. SEMEP calculates bed sediment discharge on the basis of the measured SS load, and there is no need to use the Einstein bed load equation or arbitrarily divide the bed load intensity by two.

SEMEP was tested on 14 rivers within the United States, at sediment transport rates varying by more than seven orders of magnitude. It is concluded that SEMEP performs best when $u_*/\omega > 5$, and when the total sediment discharge is greater than 20,000 tons/day. The method is without bias (MPE $= 0.01$ when $u_*/\omega > 5$), and there is excellent agreement between calculations and measurements of the total sediment discharge ($R^2 = 0.98$ and $\rho_c = 0.99$). SEMEP is also recommended, but is less accurate, when u_*/ω is between 2 and 5. Both SEMEP and other MEP are not recommended when $u_*/\omega < 2$, and methods on the basis of the bed sediment discharge or bed material discharge [e.g., Fig. 6 and Eq. (24)] are recommended when $u_*/\omega < 2$.

Fig. 7 defines the ratio of the measured to total sediment discharge for sand sizes and standard samplers. It is concluded that q_m/q_t varies primarily with u_*/ω and h_m/h. These results confirm that depth-integrated sampling is useful in streams where $u_*/\omega > 2$.

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Notation

The following symbols are used in this paper:

B = ratio of bed layer thickness to flow depth $z_b/h = 2d_s/h$;
C = sediment concentration by volume;
C_{z_b} C_{zb} = reference sediment concentration by volume at the reference depth of $2d_s$;
d_{50ss} = median grain size of material in suspension from the depth-integrated sample (L);
d_s, d_{65} = particle size associated with material finer than 65% of the bed material sample (L);
g = gravitational acceleration (L/T^2);
h = average flow depth (L);
h/d_s = relative submergence;
h_m = sampling depth between the nozzle height and the free surface (L);
$J_{1M} = \int_M^1 (\frac{1-z^*}{z^*})^{\mathrm{Ro}} \mathrm{d}z^*$;
$J_{1S} = \int_B^1 (\frac{1-z^*}{z^*})^{\mathrm{Ro}} \mathrm{d}z^*$;
$J_{2M} = \int_M^1 \ln z^* (\frac{1-z^*}{z^*})^{\mathrm{Ro}} \mathrm{d}z^*$;
$J_{2S} = \int_B^1 \ln z^* (\frac{1-z^*}{z^*})^{\mathrm{Ro}} \mathrm{d}z^*$;
k_s = surface roughness height (L);
M = ratio of nozzle height to flow depth z_{um}/h;
MPE = mean percent error;
n = number of samples;
Q = water discharge (L^3/T);
q_b = unit bed load discharge by weight in the bed layer $z < 2d_s$ (M/T^3);
q_b/q_t = ratio of bed to total unit sediment discharge;
q_m = unit measured sediment discharge by weight from z_{um} to h (M/T^3);
q_m/q_t = ratio of measured to total unit sediment discharge;
q_s = unit SS discharge by weight (M/T^3);
q_{sm} = unit SS discharge by mass (M/LT);
q_{sv} = unit SS discharge by volume (L^2/T);
q_{sw} = unit SS discharge by weight (M/T^3);
q_s/q_t = ratio of suspended to total unit sediment discharge;
q_t = unit total sediment discharge by weight (M/T^3);
R^2 = coefficient of determination;
Ro = Rouse number;
S = slope;
s_x = variance in the measured data;
s_{xy} = covariance between the measured and calculated data;
s_y = variance in the calculated data;
T° = water temperature;
u_* = shear velocity (L/T);
V = measured depth-average flow velocity (L/T);
v = velocity at elevation z above the bed (L/T);
v_{z_b} = velocity at reference point z_b (L/T);
W = channel width (L);
X_i = measured total sediment discharge by weight (ML/T^3);
$\bar{X}$ = mean value of measured total sediment discharge by weight (ML/T^3);
Y_i = calculated total sediment discharge by weight (ML/T^3);
$\bar{Y}$ = mean value of calculated total sediment discharge by weight (ML/T^3);
Z = vertical elevation above the bed (L);
z_b = reference depth $2d_s$ (L);
z^* = ratio of z/h;
β_s = momentum correction factor, assumed to equal 1;
κ = von Kármán constant, assumed equal to 0.4 for this study;
ρ_c = concordance coefficient; and
ω = settling velocity of sediment particles in suspension (L/T).

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