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B. Saghafian, P.Y. Julien*

Engineering Research Center, Colorado State University, Fort Collins, CO 80523, USA

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Time to equilibrium for spatially variable watersheds

B. Saghafian, P.Y. Julien*

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Abstract

An algorithm for calculating the time to equilibrium of complex watersheds is developed for spatially varied watershed characteristics and excess rainfall intensity. The wave travel time through the flow path originating from the hydraulically most remote point in the basin essentially determines the equilibrium time of the system. The wave travel time is computed for two distinct phases: overland flow and channel flow. The hydraulically most remote point is stationary for all spatially uniform rainstorms but may vary for spatially distributed storms. The time to equilibrium varies inversely with rainfall intensity raised to the power 0.4. Using a raster-based approach, algorithms were developed to determine the distance–drainage area relationship and to perform the numerical integration of the time to equilibrium. Although approximate, the algorithm provides maps showing the isochromes of surface runoff and travel time to the outlet, along with time–area histograms for uniform or spatially distributed rainstorms. Isochrones determine the response time required for water, dilute suspensions and pollutants to reach the watershed outlet under complete equilibrium. The effects of infiltration in delaying the time to equilibrium are significant only when δ/K is large.

1. Introduction

The concept of time to equilibrium t_e of a watershed is widely used in simple forms for two major purposes. The first is to define the nature of surface runoff hydrographs from storms of duration t_r, namely partial ($t_r \leqslant t_e$) or complete equilibrium ($t_r > t_e$), which is obtained under uniform rainfall. The concept of complete vs. partial equilibrium hydrographs has been well documented by Woolhiser (1975, 1977). The second is to find the time associated with the maximum potential surface runoff from a watershed under constant rainfall intensity, which is reached at a time equal to or greater than t_e. Time to equilibrium is primarily a function of storm characteristics

* Corresponding author.

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and watershed physical and geometrical properties. Thus, t_e is representative of how a given storm interacts with the watershed and is indicative of the basin response time excited by that storm.

The watershed equilibrium time is often confused with the watershed time of concentration t_c. The time of concentration may be defined as the time needed for the most remote point of the basin to contribute to the surface runoff at the basin outlet, whether or not outlet runoff reaches steady state. If the duration of a constant intensity storm is equal to or greater than the time for the contribution of the most remote point to the outlet, steady state is reached at the equilibrium time; thus $t_c = t_e$. Here one needs to stress that the most remote point relative to the outlet is implied in a hydraulic sense and not in a geometrical sense. Moreover, this most remote point may not be a unique point for all spatially distributed rainstorms. Hence the most remote point is not necessarily the farthest point relative to the outlet but instead any point with the longest wave travel time to the outlet under a given storm condition.

2. Background

The relationship between the kinematic time to equilibrium, catchment characteristics, and rainfall properties has been analytically derived for well-defined simple geometries. For example, using the method of characteristics, the kinematic time to equilibrium was derived by Lighthill and Whitham (1955) and Henderson and Wooding (1964) for a rectangular plane sloping in a single direction. Using the same method, Agiralioglu (1984, 1988) derived integral forms for t_e of converging and diverging planes.

In the first of a series of classic papers, Wooding (1965a) developed analytical solutions for a hydraulic model of kinematic flow over a plane V-shaped catchment under uniform rainfall of finite duration and for a line stream outflow laterally supplied by the overland catchment discharge. The overland flow and channel flow were treated separately. A dimensionless time parameter λ was introduced as a measure of the ratio of stream response time (i.e. stream equilibrium time) to catchment response time (i.e. overland equilibrium time). Large values of λ cause delay in the arrival time of flood peak and reduce its amplitude. For small values of λ, the stream outflow tends toward the catchment outflow (Wooding, 1965b, 1966).

In reality, the absolute steady state is achieved only when the rainfall duration is infinite. For practical purposes, however, a virtual equilibrium condition often replaces the absolute equilibrium condition. Larson (1965), for example, selected the 'time to virtual equilibrium' to be the time when the discharge reaches 97% of steady-state discharge. However arbitrary, the 'time to virtual equilibrium' may be effectively used as the upper limit, though not in the exact sense, for a rainfall duration which produces a partial equilibrium hydrograph. Machmeier and Larson (1968) derived a relationship for the time to virtual equilibrium t_{ve}, the time at which flow rate is 97% of equilibrium flow, for the case of a synthesized model watershed of 54.66 km^2 in size, having a channel system with defined physical characteristics. The procedure involved disaggregating the watershed into several units of overland plane and then routing the lateral inflow in the channel network. The results indicated that t_{ve} varied inversely with the 0.23 power of rainfall rate, results similar to those of Ben-Zvi (1984).

By taking t_{ve} to be the particle travel time from the watershed farthest point to the outlet, Golany and Larson (1971) tried to develop a general relationship for t_{ve}. The physical and geomorphological characteristics used previously by Machmeier and Larson (1968) were incorporated in a synthesized watershed model that routes the runoff on overland planes and in channel network. This model was used to estimate the constants in the t_{ve} equation.

Hjelmfelt (1978) provided a method to calculate the time of concentration t_c for constant rainfall intensity and transient infiltration. He pointed out that the outflow increases after reaching that time. He also inferred that as a general trend the varying infiltration during a storm has a significant effect on the time of concentration and on the shape of the runoff hydrograph.

Akan (1986) developed a mathematical model based on kinematic flow and the Green–Ampt infiltration equation to calculate the time of concentration t_c of overland flow on a rectangular plane surface. He expressed t_c, in a dimensionless form, in terms of two dimensionless parameters. More recently, Akan (1988) attempted to determine the time of concentration and the peak runoff discharge from an infiltrating converging basin. He used a similar approach to that of a rectangular plane and provided several charts for t_c and peak discharge for storms with duration equal to t_c.

Based on this background information, the literature clearly lacks in the development of a time to equilibrium formulation applicable to large watersheds. Numerical solutions are required when dealing with two-dimensional watersheds subjected to spatially distributed rainstorms. Accordingly, the primary objectives of this paper are to (1) develop a formulation for time to equilibrium of watersheds under distributed rainfall and (2) propose and implement a raster-based (cellular) numerical approach to compute time to equilibrium, which also results in a 'travel time' map and time–area histogram of a watershed under equilibrium conditions.

Time to equilibrium for small and large watersheds is a very important parameter governing the linearity of the rainfall–runoff process. When the rainfall duration t_r exceeds the time to equilibrium t_e, the surface runoff discharge increases linearly with rainfall intensity in the sense of the rational formula. Accordingly, linear methods such as the unit hydrographs (UH) and instantaneous unit hydrographs (IUH) are applicable. On the other hand, when the rainfall duration is short compared with the time to equilibrium, partial equilibrium hydrographs are obtained for which the relationship between rainfall and surface runoff is non-linear. Surface runoff calculations based on linear techniques such as UH and IUH are not applicable and methods that account for the non-linearity of surface runoff must be used.

In recent investigations (Julien and Moglen, 1990; Ogden and Julien, 1993), the concept of time to equilibrium was applied to impervious planes and to small semi-arid watersheds. When considering the magnitude and variability of peak discharge owing to spatial and temporal variability in watershed and rainfall characteristics, there is similarity at different scales when plotting the runoff response versus the ratio of rainfall duration to time to equilibrium. To examine whether the similarity characteristics also extend to large watersheds, one must be capable of calculating the time to equilibrium for large watersheds.

3. Analytical formulation

3.1. General approach

In this paper we assume that surface runoff over a watershed occurs in two distinct but consecutive phases, namely overland flow and channel flow. Although it is difficult to separate these two phases clearly, overland flow is generally slower owing to shallow flow depths, whereas channel flow is usually confined to a limited width boundary with faster flood wave celerity. A simple technique is proposed that accounts for channelized flow on overland planes. For a basin to reach steady state at the time to equilibrium, the wave originating from the hydraulically most remote point in the basin must reach the basin outlet by traveling through both the overland and channel flow phases. The wave travel time is first formulated in a general form and then separately computed for these two phases.

Let us assume, for the time being, rainfall of constant intensity falling over an impervious watershed. As a wave originates from a point at a distance $x = X_1$ and travels to $x = X_2$, in either overland or channel flow phase, the wave travel time t_w may be written as

$$t_w = \int_{x_1}^{x_2} \frac{\mathrm{d}x}{c} \tag{1}$$

where x is the distance measured along the flow path and c is the wave celerity. The celerity of the wave depends upon an expression for resistance as well as the wave type. In general, the wave celerity is expressed by

$$c = \frac{\partial Q}{\partial A_x} = \frac{\partial Q}{\partial h}\frac{\mathrm{d}h}{\mathrm{d}A_x} \tag{2}$$

where Q is the discharge, h is the flow depth, and A_x is the flow cross-section area. With regard to the wave type, the kinematic wave approximation is appropriate for most overland flow conditions (Woolhiser, 1975). This approximation will be eventually applied to simplify the general form.

The relationship between flow depth h, flow velocity V, and unit discharge q for a general resistance law can be written as

$$V = \alpha h^{\beta-1}; \qquad q = \alpha h^{\beta} \tag{3}$$

where α and β are resistance parameters. Based on flow cross-section, the area A_x and the hydraulic radius R can be determined as functions of flow depth h, either precisely through geometrical relationships or approximately through rating curves. In equation form, let us assume

$$A_x = a_1 h^{b_1}; \qquad R = a_2 h^{b_2} \tag{4}$$

where a_1, a_2, b_1, and b_2 are constants for a given cross-section. Substituting the discharge $Q = A_x V$ from (3) and (4) in the wave celerity Eq. (2) yields

$$c = \frac{\beta + b_1 - 1}{b_1} \alpha h^{\beta - 1} \tag{5}$$

At this stage, we must evaluate flow depth at the equilibrium state. As a wave originates from the hydraulically most remote point P_r in the basin and it travels downstream along a certain kinematic path x, each flow cross-section reaches equilibrium just as the wave from P_r passes through. Each cross-section along flow path x is assumed to have a certain drainage area (upslope area). The equilibrium discharge passing through a given cross-section p would be the spatially integrated rainfall rate over the drainage area associated with that cross-section. This can be mathematically expressed by

$$Q_e(x_p, y_p) = \int_0^{A(x_p, y_p)} i(x, y) \mathrm{d}A \tag{6}$$

where $Q_e(x_p, y_p)$ is the equilibrium discharge passing through cross-section p, (x_p, y_p) are the rectangular coordinates of cross-section p, $A(x_p, y_p)$ is the total drainage area at p, $i(x, y)$ is rainfall intensity at coordinates (x, y), and (x, y) are rectangular coordinates of any point draining to p. The steady-state discharge for a uniform rainfall, for instance, would simplify to equal the rainfall rate times the drainage area. The equilibrium flow depth is computed based on $Q_e = A_x(h_e)V(h_e)$ using (3) and (4) as follows:

$$h_e = \left(\frac{Q_e}{a_1 \alpha}\right)^{\frac{1}{(\beta + b_1 - 1)}} \tag{7}$$

By substituting h_e in the wave celerity Eq. (5) and then back into the wave travel time Eq. (1) one obtains

$$t_w = \int_{x_1}^{x_2} \frac{\mathrm{d}x}{\alpha \left(\frac{\beta + b_1 - 1}{b_1}\right) \left(\frac{Q_e}{a_1 \alpha}\right)^{\frac{\beta - 1}{\beta + b_1 - 1}}} \tag{8}$$

Let

$$\gamma = \frac{\beta - 1}{\beta + b_1 - 1} \tag{9}$$

which yields

$$t_w = \int_{x_1}^{x_2} \frac{\gamma b_1}{(\beta - 1)\alpha^{1-\gamma}} \left(\frac{a_1}{Q_e}\right)^{\gamma} \mathrm{d}x \tag{10}$$

Equation (10) is the general wave travel time relationship which accounts for basin characteristics, including slope, roughness, drainage area density, flow width, and flow cross-section, as well as rainfall properties reflected in spatial distribution of rainfall rate. When the flow is turbulent over the major portion of a watershed, the Manning resistance equation can be applied. Alternate formulations for laminar flow are possible (e.g. Woolhiser, 1975; Julien and Simons, 1985). The flow velocity in SI units is obtained from

$$V = \frac{1}{n} R^{2/3} S_f^{1/2} = \frac{1}{n} (a_2 h^{b_2})^{2/3} S_f^{1/2} \tag{11}$$

where S_f is friction slope; and α, β, and γ are expressed by

$$\alpha = \frac{1}{n} a_2^{2/3} S_f^{1/2}; \qquad \beta = \frac{2}{3} b_2 + 1; \qquad \gamma = \frac{2b_2}{2b_2 + 3b_1} \tag{12}$$

Consequently, t_w for the Manning resistance equation transforms into

$$t_w = \int_{x_1}^{x_2} (1 - \gamma) \left(\frac{a_1}{Q_e}\right)^{\gamma} \left(\frac{n}{a_2^{2/3} S_f^{1/2}}\right)^{1-\gamma} \mathrm{d}x \tag{13}$$

It should be noted that the equation retains its generality with respect to the spatial variability in rainfall. When the friction slope is replaced by the bed slope S_o for the kinematic wave approximation, the derived time is called kinematic time to equilibrium.

If the wave travel time is computed for $X_1 = 0$ and $X_2 = L$, where L is the total length along the hydraulically longest flow, then t_w represents the kinematic equilibrium time for the portion of the basin draining along length L:

$$t_e = \int_{0}^{L} (1 - \gamma) \left(\frac{a_1}{Q_e}\right)^{\gamma} \left(\frac{n}{a_2^{2/3} S_0^{1/2}}\right)^{1-\gamma} \mathrm{d}x \tag{14a}$$

or if the rainfall intensity i is uniformly distributed in space:

$$t_e = \frac{1}{i^{\gamma}} \int_{0}^{L} (1 - \gamma) \left(\frac{a_1}{A}\right)^{\gamma} \left(\frac{n}{a_2^{2/3} S_0^{1/2}}\right)^{1-\gamma} \mathrm{d}x \tag{14b}$$

The total time to equilibrium t_e can be separated into two phases: the travel time for overland flow t_{eo} and for channel flow t_{ec}. Explicitly:

$$t_e = t_{eo} + t_{ec} \tag{15}$$

We now analyze the overland flow and channel flow phases separately. It should be noted that the channel flow phase considers the cumulative runoff from the upstream overland flow phase.

3.2. Overland flow

The parameter values corresponding to the flow over an overland plane are as follows: $a_1 = W$; $b_1 = 1$; $a_2 = 1$; $b_2 = 1$; $\gamma = 2/5$; $L = L_{ov}$; $S_o = S_{ov}$; where W is the local overland plane width and L_{ov} is the total length of the overland plane starting from the hydraulically most remote point. The bed slope S_{ov} would then be the local overland plane bed slope. Therefore, the kinematic time to equilibrium for turbulent overland flow from Eq. (14a) becomes

$$t_{eo} = \frac{3}{5}\int_0^{L_{ov}} n^{3/5}\left(\frac{W}{Q_e}\right)^{2/5}\frac{1}{S_{ov}^{3/10}}\,\mathrm{d}x \tag{16}$$

If variations of width W, drainage area A, surface roughness n, bed slope S_o, and rainfall intensity i along the hydraulically longest flow path are known, the overland flow travel time can be estimated by simplifying the above equation. For example, in the case of a uniform rectangular plane under uniform rainfall intensity i, the equilibrium discharge of rectangular plane $(Q_e)_{\mathrm{rec}}$ is given by

$$(Q_e)_{\mathrm{rec}} = iA(x) = ixW \tag{17}$$

Solving (16) yields the following well-known formula:

$$(t_{eo})_{\mathrm{rec}} = \frac{n^{3/5}L_{ov}^{3/5}}{i^{2/5}S_{ov}^{3/10}} \tag{18}$$

Saghafian (1992) tested (14b) for converging and diverging planes. The calculated values of time to equilibrium matched those reported by Agiralioglu (1984, 1988). The raster-based algorithm for surface runoff calculations has also been tested against runoff measurements from small overland flow plans. Saghafian (1992) showed good agreement with the experimental data of Schaake (1965), Dickinson et al. (1967) and Woolhiser (1969).

3.3. Channel flow

Once a kinematic wave enters the channel network, it travels faster depending upon flow depth and channel cross-section geometry. Certain relationships for $A_x(h,x)$, $R(h,x)$, $S_o(x)$, and $Q_e(x)$, or alternatively $i(x,y)$ and $A(x)$, must be obtained to allow the calculation of the travel time integral. If the channel system consists of several segments with uniform cross-section, then the integral may be performed piecewise. For the jth channel segment of uniform cross-section and constant roughness from $X_1 = L_{j-1}$ to $X_2 = L_j$, we can estimate the travel time from (14a) by

$$t_{ec,j} = (1 - \gamma_j)(a_{1j})^{\gamma_j} \left(\frac{n_j}{a_{2j}^{2/3}} \right)^{1-\gamma_j} \int_{L_{j-1}}^{L_j} \frac{\mathrm{d}x}{Q_e^{\gamma_j} S_0^{(1-\gamma_j)/2}} \tag{19}$$

where $t_{ec,j}$ is the travel time through the jth channel. If the bed slope is essentially uniform for the jth channel, the bed slope may be removed from the integral and approximated by

$$S_{0j} \approx \frac{E_{j-1} - E_j}{L_j - L_{j-1}} \tag{20}$$

where E denotes the bed elevation. The substitution for Q_e requires knowledge of spatial variability of rainfall and also that of drainage area along the channel path. Numerical methods to evaluate Q_e are discussed for impervious and pervious watersheds.

4. Time to equilibrium of nearly impervious or saturated watersheds

On nearly impervious watersheds (high ratio of rainfall intensity over permeability), the calculation of time to equilibrium from Eq. (14) requires good estimates of drainage area A and consequently equilibrium discharge Q_e. A comprehensive raster-based approach is proposed and the tools were developed whereby numerical approximations can be made for the variables inside the integral t_e in Eq. (14). The output of this approach can also be used for other applications such as the time–area technique of runoff estimation.

In a first step, a watershed is discretized using a square grid mesh with a predetermined size given the required level of accuracy on one hand and practicality on the other. Ground elevation and surface roughness coefficients are assigned to each grid cell. The channel cross-section geometry of drainage channel cells is then determined. The following algorithms lead to the numerical solution of the integral t in Eq. (14).

The kinematic flow path is determined from the topography of the watershed. Trajectories for all the grid cells in the watershed are determined from digital elevation model (DEM) data. It is expected that all trajectories terminate at the outlet unless they lead to ponding areas such as lakes. The algorithm selects one grid cell and determines which of the surrounding grid cells has the lowest elevation. If the lowest neighboring cell is also lower than the original grid cell then the trajectory marches through; otherwise a depression has been intercepted. This action is repeated until the trajectory meets the outlet or a depression area, at which point this trajectory has reached its downstream bound. A similar process is performed for all of the grid cells and the results are stored in computer files.

The trajectories determined using this method are then employed to calculate the drainage area for each grid by counting how many trajectories pass through a given cell. The output consists of a list of grid cells specified by their column and row addresses, and their corresponding drainage area A. When available, a raster geographic information system (GIS) with capabilities similar to these algorithms can be substituted.

The trajectories and drainage area data are used in Eq. (14) to compute the wave travel time along all trajectories originating from primary cells, those grid cells which have no other cells draining into them. This process is done for all such trajectories because the hydraulically longest trajectory may not necessarily originate from the geometrically farthest cell. The true equilibrium time is the longest travel time of all the travel time values that are computed.

The time to equilibrium equation (14) is numerically integrated using the trapezoidal method. At each grid cell G along a trajectory x the drainage area is known and the bed slope is determined based on grid cell elevations along the trajectory x. If grid cell G is a channel cell, then the channel cross-sectional geometry is used in estimating t_{ec}. Overland flow width is taken as cell size if the grid cell is an overland cell. Finally, the t_e integral is computed along the length of the grid cell. This piecewise integration is continued until the outlet grid cell is encountered.

An alternate algorithm is capable of accounting for spatial variability of rainfall intensity. The equilibrium discharge is estimated from Eq. (6) as follows:

$$Q_e(j_p, k_p) = W^2 \sum_{(j_0,k_0)}^{(j_p,k_p)} i(j,k) \tag{21}$$

where j_0 and k_0 are, respectively, row and column numbers of primary grid cells draining into p; j_p and k_p are, respectively, grid cell row and column numbers at point p; j and k are, respectively, row and column numbers of any grid cell draining into p; W is grid cell length (or width); $i(j,k)$ is rainfall intensity over cell (j,k). The rainfall intensity is assumed uniform within each square grid cell. A simple alternative to Eq. (21) would be to compute the spatially averaged rainfall rate. However, the flow velocity distribution, and thus wave travel time, along all flow trajectories produced by a given rainfall spatial distribution is generally different from that produced by the spatially averaged rainfall. Moreover, spatial distribution of rainfall may relocate the position of the hydraulically most remote cell.

The proposed method of calculating time to equilibrium was applied to Macks Creek (Cline, 1988), a 32.2 km^2 steep semi-arid watershed located in southwest Idaho. Elevation over the Macks Creek watershed drops from about 1830 m in the mountains to about 1130 m at the outlet, with overland plane slopes up to 30%. Two principal channels with slopes averaging 5% collect surface runoff. Fig. 1 depicts the graphical topomap of the watershed overlaid by the geometrically longest trajectory. Both uniform and spatially varied rainstorms were examined. The computed equilibrium times matched the hydrograph times associated with discharge values, simulated by the two-dimensional raster-based rainfall–runoff model CASC2D (Julien and Saghafian, 1991), exceeding 98% of the equilibrium discharge. For Macks Creek watershed, the geometrically longest flow trajectory was identical to the hydraulically longest trajectory for uniform rainstorms. Once this trajectory is determined, it will remain the same for all other uniformly

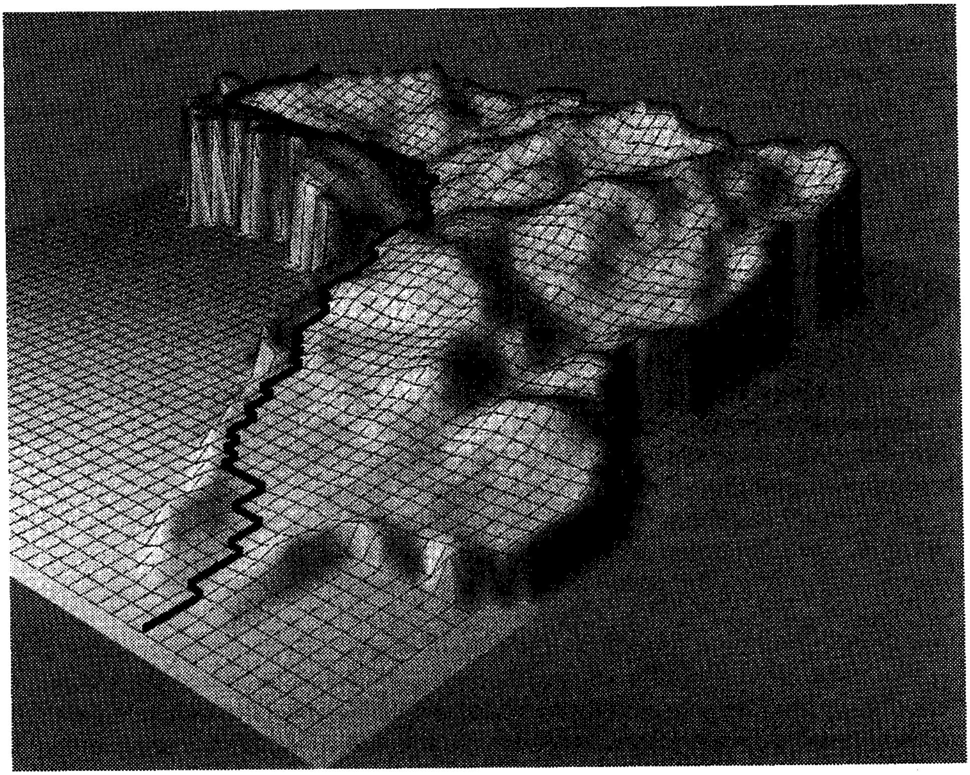

Fig. 1. Geometrically longest flow trajectory for Macks Creek.

distributed rainfall, and t_e will vary inversely with intensity raised to 0.4 power. However, for nonuniform storms concentrated on the upstream portion of the watershed, the hydraulically most remote cell relocated itself to a different cell as far as about half the distance of the geometrically most remote cell to the outlet, which had a longer overland plane length but shorter channel reach. For this particular distributed rainstorm the t_e was about 15% shorter than t_e determined assuming a spatially averaged rainfall intensity. Values for the parameter λ (as defined by Wooding, 1965a) turned out to be 0.25 and 0.2, respectively, for the uniform and nonuniform storms examined above. These low values of λ highlight the relative dominance of overland flow compared with channel flow over Macks Creek. Similarly, the hydraulically most remote point has a higher likelihood to be relocated when overland flow is the dominant process.

Fig. 2 shows the 'travel time' map for Macks Creek watershed under a uniform storm. In this figure the entire watershed area has been divided into a number of subareas (displayed with different intensity of gray shading), each subarea covering a certain range of wave travel time associated with cells contained in that subarea. The 30 min contour isochrones of travel time to the outlet are also depicted in Fig. 2. The consequent time–area histogram in increments of 1 min travel time for a low rainfall

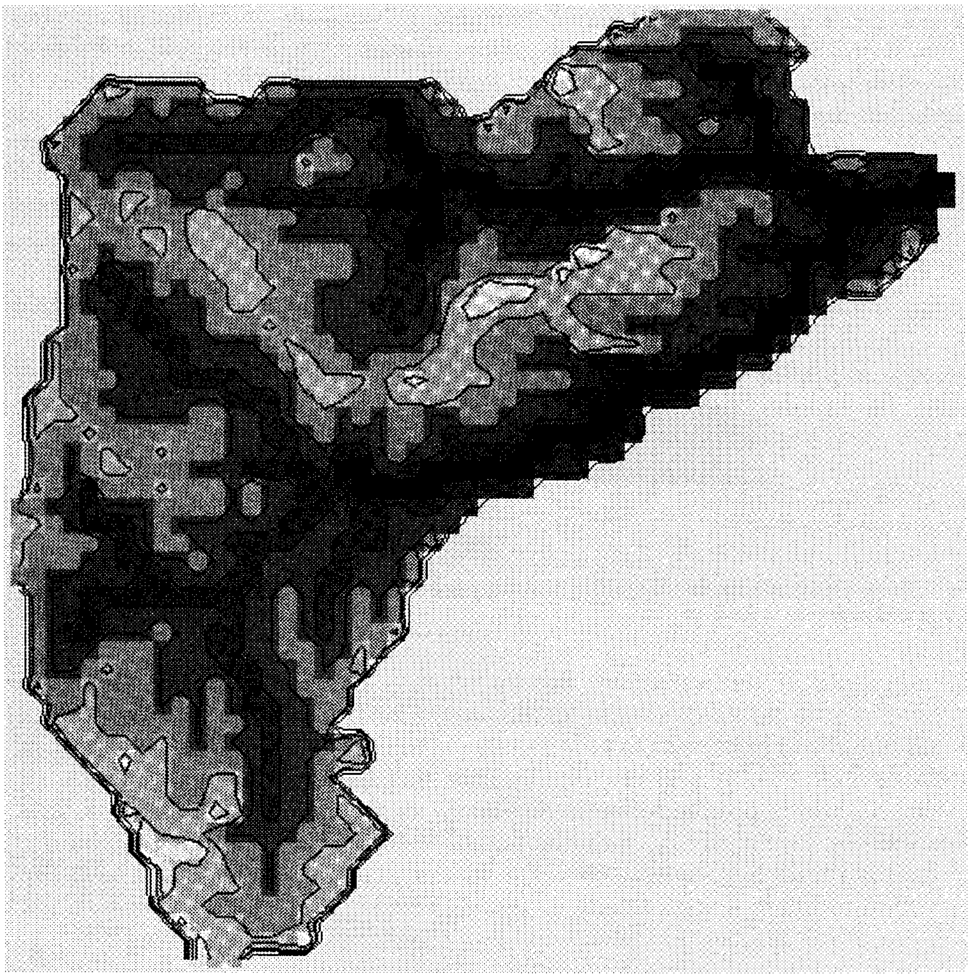

Fig. 2. Travel time map and 30 min contour isochrones for Macks Creek watershed under uniform rainfall (lighter shades imply longer travel time to the outlet).

intensity of 3.6 mm h^{-1} and grid length of 150 m is illustrated in Fig. 3. Similar figures can be developed for spatially distributed rainstorms. The travel time map and the time–area histogram may be effectively used in runoff estimation under complete equilibrium conditions based on a time–area technique.

The outcome of above the raster-based algorithm depends on the choice of computational grid scale, which determines the level of geometric simplification of the watershed and rainfall characteristics. For the Macks Creek watershed subject to uniform excess rainfall, an increase of 44% in equilibrium time was computed as the computational grid length increased by 400% from 150 m to 600 m. This effect is mainly attributed to topographic rescaling under coarser square grids, which generally results in smoothing of surface elevation, bed slope, and drainage

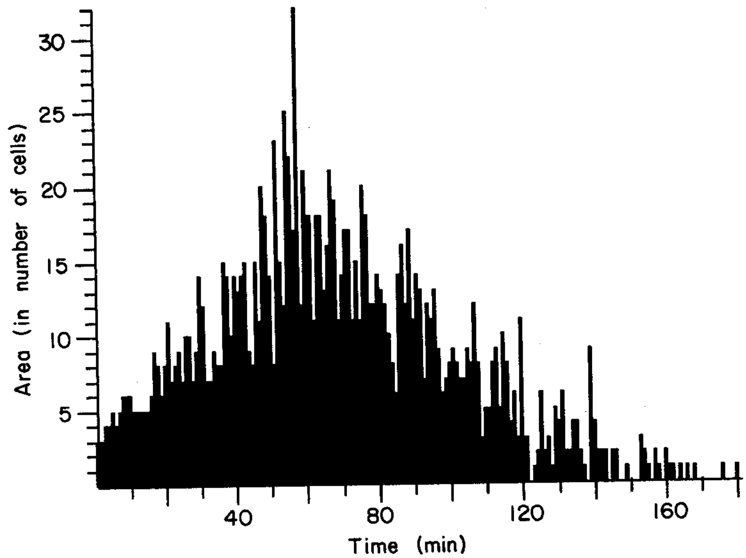

Fig. 3. Time–area histogram for the Macks Creek watershed subject to rainfall intensity of 3.6 mm h^{-1} at a 150 m grid size.

characteristics of the watershed. Besides affecting the terrain representation, an increase in grid cell size results in an increased proportion of overland flow, which drains slowly compared with the channel flow. Grid coarsening causes an increase in surface runoff time at grid cell sizes coarser that the overland runoff length. In watersheds where overland flow is dominant, as given by the parameter λ, it is important to have a grid cell size that is shorter than or equal to the physical size of surface runoff by overland flow. On Macks Creek, $\lambda = 0.25$, and a grid size resolution of 150 m is appropriate.

5. Effect of infiltration

For a pervious watershed subject to a given rainstorm, the infiltration capacity of the soil decreases with time. The actual infiltration rate, however, may increase or decrease depending upon storm temporal variations. For uniform rainfall at a constant intensity, one should expect that the decreasing infiltration rate allows the flow rate to increase after the time of concentration is reached. If the rainfall duration is very long, surface runoff gradually approaches an asymptotic steady state. The equilibrium discharge is estimated based on the ultimate excess rainfall rate, which is equal to the point rainfall intensity minus the soil saturated hydraulic conductivity (K) integrated over the watershed area. Theoretically speaking, the equilibrium time on pervious watersheds is never reached because the infiltration rate reaches the saturated hydraulic conductivity after an infinite time. However, a time to virtual equilibrium (Larson, 1965) may serve as a measure of response time of pervious watersheds subject to uniform or distributed rainstorms. The time to virtual equilibrium provides an approximation rather than an exact time. The approximation is acceptable as long as the characteristic time of ponding is short compared with the total time to equilibrium, which is a reasonable approximation for large watersheds.

The virtual equilibrium time, t_{ve}, for pervious watersheds subject to a constant rainfall intensity is defined as follows:

$$t_{ve} = (t_s)_{\max} + t_e \tag{22}$$

where $(t_s)_{\max}$ is the maximum time along the longest trajectory for the point infiltration rate to reach the infiltration rate $f_s = \xi K$ at a rate slightly in excess of the saturated hydraulic conductivity at a level ξ, where $\xi > 1$; t_e is the equilibrium time from Eq. (14) for the steady excess rainfall intensity equal to $(i - K)$. The Green–Ampt infiltration equation is used:

$$K(t_s - t_p) = (F_s - F_p) - \delta \ln\left(\frac{F_s + \delta}{F_p + \delta}\right) \tag{23}$$

and

$$f_s = \xi K = K\left(1 + \frac{\delta}{F_s}\right) \tag{24}$$

where $\delta = (1 - S)\theta_e H_f$ is a function of the degree of initial saturation S, effective porosity θ_e, and capillary pressure head H_f; $t_p = K\delta/[i(i - K)]$ represents the ponding time $(i > K)$; $F_p = it_p$; F_s denotes the cumulative infiltration depth at the time t_s defined below.

The time t_s to reach near steady-state flow at the level $f_s = \xi K$ is obtained after combining Eqs. (23) and (24):

$$t_s = \frac{\delta}{K}\left[\frac{K^2}{i(i-K)} + \frac{1}{(\xi - 1)} - \ln\frac{\xi(i-K)}{(\xi - 1)i}\right] \tag{25}$$

This approach in computing t_s from (25) and t_{ve} from (22) was tested on Macks Creek. For $\xi = 1.05$, the CASC2D-generated hydrograph discharges at $t = t_{ve}$ were within 90% and 95% of the equilibrium discharge. For practical purposes, t_s is large only when δ/K is large. For near-impervious watersheds, $t_s \approx 17\delta/K$ as $K/i \to 0$, and for highly pervious watersheds, $t_s \to t_p$ as $K \to i$.

6. Summary and conclusions

A technique has been formulated to allow calculation of the kinematic time to equilibrium of complex watersheds associated with the hydraulically, not necessarily geometrically, most remote point in watersheds. The algorithm accounts for spatially variable watershed characteristics and spatially distributed rainfall intensity. Using a raster-based approach, algorithms were developed to determine flow trajectories, drainage area density, and the wave travel time for kinematic waves originating from primary cells. A time to virtual equilibrium t_{ve}, which reduces to t_e for impervious or saturated surfaces, was defined as an approximate measure of response time of pervious watersheds.

In conclusion, the time to equilibrium of natural watersheds is not easily calculated because of the complexities surrounding the spatial distribution of hydrologic parameters, such as topography, surface resistance, soil types, infiltration parameters and rainfall distribution, as well as the time-varying nature of the infiltration process. Although an exact solution is not likely to be found, reasonable approximations can be obtained from this algorithm applied to raster-based data sets. The main conclusions of this study are:

(1) the position of the hydraulically remotest point of a watershed depends on the spatial distribution of rainfall.

(2) Time to equilibrium is inversely proportional to the rainfall intensity raised to the power of 0.4.

(3) The isochrone map displays the surface runoff travel time required to route water and pollutants to a watershed outlet under complete equilibrium. The distribution of surface area within the isochrones in Fig. 3 can also be used to define complete equilibrium hydrographs at the outlet from the time–area method.

(4) The effects of infiltration on delaying the time to equilibrium are calculated from Eq. (25)—for practical purposes, t_s is large only when δ/K is large.

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