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Suitability of simplified overland flow equations

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Abstract. A dimensionless formulation of the acceleration terms of the Saint-Venant equations is presented for one-dimensional overland flows under either laminar or turbulent conditions. For stationary storms over a plane surface of uniform roughness, dimensionless analytical expressions are derived during the rising limb for the local acceleration a_l^*, and during equilibrium for the convective acceleration a_c^* and the pressure gradient a_p^* ((13), (14), and (15), respectively). In terms of the order of magnitude, the three acceleration terms are inversely proportional to the kinematic flow number K. At equilibrium, the pressure gradient a_p^* is also inversely proportional to the square of the Froude number Fr. The relative magnitude of the acceleration terms for supercritical overland flow ($a_l^* > a_c^* > a_p^*$) differs from subcritical overland flow ($a_p^* > a_l^* > a_c^*$), which in all cases contrasts with open-channel flows ($a_p^* > a_c^* > a_l^*$). The kinematic wave approximation is therefore only suitable when both K and Fr are large. Improvements using the diffusive wave approximation are only possible for subcritical overland flow. Both the diffusive wave and the quasi-steady dynamic wave approximations are not suitable for supercritical overland flow. The analysis of moving storms corroborates these findings in that the local acceleration exceeds the convective acceleration. These effects are particularly pronounced during the rising limb of overland flow hydrographs for downstream moving rainstorms.

Introduction

The solution to overland flow problems is crucial because upland areas generally provide a significant source of water for surface runoff. In turn, knowledge of surface runoff is essential to determine the timing and magnitude of floods and to estimate soil erosion losses and nonpoint source pollutant transport from upland areas. Overland flow originates from rainfall, snowmelt, or saturation excess on the soil surface. Rainfall excess over a surface of slope S_0 and length L_0 generates surface runoff of thickness H_0 and Froude number F_0 at the downstream end of the upland surface. Complete equilibrium hydrographs occur when the rainfall duration exceeds the time to equilibrium T_e, while rainstorms shorter than T_e generate partial equilibrium hydrographs.

The governing equations for one-dimensional overland flow derived by Saint-Venant are unfortunately not prone to simple analytical solutions, except in very simplified cases. Prior analyses of the relative magnitude of the terms of the momentum equation for open channels lead to several levels of approximation depending on the desired level of accuracy. In their early treatment of flood movement in long rivers, *Lighthill and Whitham* [1955] concluded that all acceleration terms of the momentum equation could be neglected when the Froude number is less than two. Investigations focused primarily on the role played by three acceleration terms: (1) the local acceleration term denoted as a_l describing the temporal variability in velocity, (2) the convective acceleration term a_c describing the spatial variability in velocity, and (3) the pressure gradient term a_p describing the spatial variability in flow depth. Subsequent reviews of the relative magnitude of these acceleration terms by *Henderson* [1966], *Ponce and Li* [1979], and *Weinmann and Laurenson* [1979] suggested that $a_p > a_c > a_l$ for open-channel flows.

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Simplifications of the momentum equation, for computation of overland and channel flow, have been developed based upon the relative magnitude of the acceleration terms. These simplifications are known as (1) the quasi-steady dynamic wave, which neglects the local acceleration term a_l, (2) the diffusive wave, which neglects both the local and convective acceleration terms a_l and a_c, and (3) kinematic wave approximation, which neglects all three acceleration terms.

Woolhiser and Liggett [1967] normalized the momentum equation to simplify overland flow investigations and defined the kinematic flow number $K_0 = S_0L_0/H_0F_0^2$; they found that the kinematic wave approximation is suitable to overland flow when K_0 is greater than 10, which describes most actual flow situations. *Woolhiser* [1975] also showed that the rising outflow hydrograph compares favorably with the kinematic wave approximation even when $F_0 < 2$.

Al-Mashidani and Taylor [1974] solved the Saint-Venant equations for subcritical, critical, and supercritical flows while imposing critical flow conditions at the lower boundary; they confirmed the findings of *Woolhiser and Liggett* [1967] regarding the suitability of the parameter K_0, although higher values of K_0 were needed to use the kinematic wave approximation. *Al-Mashidani and Taylor* [1974] also showed that it is not necessary to impose the condition that $F_0 < 2$ to justify the kinematic wave approximation and that the downstream boundary condition has little or no influence on the outflow hydrograph.

Morris [1979] compared the solution for the zero-depth

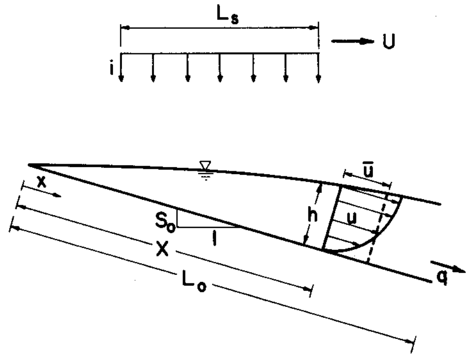

Figure 1. One-dimensional overland flow diagram for a block moving storm.

gradient to the critical flow lower boundary condition. The difference between the two solutions decreases as the value of K_0 increases. *Morris* [1979] concluded that for a range of F_0 and K_0 which covers most practical overland flow situations, the effect of the choice of lower boundary condition has no significant effect on flow depth and velocity profiles. *Morris and Woolhiser* [1980] introduced the additional criterion which combines F_0 and K_0 and reported that the kinematic wave approximation fails when $F_0 < 0.5$ and when $F_0^2 K_0 < 5$. *Morris and Woolhiser* [1980] recommended using the different wave approximation for some cases where $F_0^2 K_0 < 5$, and they showed that the diffusive wave approximation is satisfactory at low values of F_0 when K_0 is large. *Vieira* [1983] recommended using the kinematic wave approximation when $K_0 > 50$, and when $5 < K_0 < 20$ the kinematic wave or the diffusive wave approximations are suitable solutions to overland flow. *Govindaraju et al.* [1988] confirmed that the diffusive wave approximation is valid for small F_0 and large K_0.

More recently, *Hromadka and De Vries* [1988] recommended that use of the kinematic wave method for channel routing in watershed models be reconsidered. The controversial discussions to the paper led to the analysis of *Ponce* [1991], who concluded that the kinematic wave theory can be improved by extending it to the realm of diffusion waves.

These studies suggest that selection criteria based on the factors K_0 or F_0 are not as simple as originally perceived, and that the downstream boundary condition can influence the selection of appropriate simplifications of the momentum equation. The complications arising in the analysis of overland flow justifies a second look at the relative magnitude of the acceleration terms of the momentum equation.

The objective of this paper is to examine the relative magnitude of the terms of the momentum equation fo[r] one-dimensional overland flow over a plane surface wit[h] constant surface roughness. More specifically, the loca[l] acceleration term during the rising limb is compared with th[e] convective acceleration and pressure gradient terms unde[r] complete equilibrium. The approach differs from *Woolhise[r] and Liggett*'s [1967] normalizing procedure in that the anal-ysis is applicable at any location on the overland flow plan[e] and encompasses laminar and turbulent flow conditions fo[r] both stationary and moving storms.

Overland Flow Dynamics

A simplified diagram for one-dimensional overland flo[w] over a plane of length L_0 and slope S_0 with constan[t] roughness is presented in Figure 1. A block moving storm o[f] length L_s with velocity U generates surface runoff from th[e] excess rainfall rate i. At a distance X from the uppe[r] boundary, the unit discharge q varies in time and reaches [a] maximum value of $q_m = iX$ under equilibrium condition[s]. The flow depth h and the average flow velocity $\bar{u} = q/$[h] depend on the friction slope S_f. Additional variables includ[e] the gravitational acceleration g and the kinematic viscosity [ν] of the fluid.

The governing one-dimensional overland flow equation[s] derived by Saint-Venant describe conservation of mass an[d] momentum in space x and time t:

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = i(x, t) \quad (1)$$

$$\frac{1}{g}\frac{\partial \bar{u}}{\partial t} + \frac{\bar{u}}{g}\frac{\partial \bar{u}}{\partial x} + \frac{\partial h}{\partial x} = S_0 - S_f \quad (2)$$

respectively.

The three partial derivatives on the left-hand side of (2) denote dimensionless forms of local acceleration $a_l = (1/g)(\partial \bar{u}/\partial t)$, convective acceleration $a_c = (\bar{u}/g)(\partial \bar{u}/\partial x)$, an[d] pressure gradient $a_p = \partial h/\partial x$, respectively. This on[e-]dimensional formulation of the momentum equation neglec[ts] both lateral inflow and the momentum contribution due t[o] rainfall intensity $\bar{u}i/h$. It can be demonstrated by multiplyi[ng] and dividing $\bar{u}i/h$ by $\bar{u}X$ that the rainfall momentum [is] proportional to the convective acceleration term throug[h] $\bar{u}i/h = \bar{u}\bar{u}iX/X\bar{u}h = \bar{u}^2/X \cong \bar{u}\partial\bar{u}/\partial x$. Since both terms yie[ld] similar results, the following analysis focuses on the conve[c]tive acceleration term.

An additional relationship such as the Darcy-Weisba[ch] equation relates resistance to flow with the friction slope S[$_f$] and can be rewritten in the following stage-discharge rel[a]tionship:

Table 1. Parameters α and β for Four Types of Overland Flow [*Julien and Simons*, 1985]

Flow Type	α	β
Laminar	$8gS_f/k\nu$	3.0
Turbulent flow, smooth boundary, (Blasius)	$1/\nu^{0.14}\ [8gS_f/0.22]^{0.57}$	1.72
Turbulent flow, rough boundary (Manning)	$S_f^{0.5}/n$	1.67
Turbulent flow, rough boundary (Chezy)	$CS_f^{0.5}$	1.5

$$q = \alpha h^{\beta} \tag{3}$$

The parameters α and β are summarized in Table 1 for four overland flow types: (1) laminar sheet flow with constant value of the resistance parameter k, (2) turbulent flow over a smooth boundary as described by the Blasius equation, (3) turbulent flow over a rough boundary with a constant value of Manning n, and (4) turbulent flow over a rough boundary with very small relative roughness with constant value of the Chezy coefficient C. It is important to notice that by definition of the resistance relationship, the coefficient α is a function of the friction slope S_f.

Normalization of the Governing Equations

Woolhiser and Liggett [1967] normalized the momentum equation after dividing each parameter in (1) and (2) by a combination of the normal depth H_0 at the downstream end of the plane L_0, the corresponding normal flow velocity V_0, and the maximum rainfall intensity i_0. The resulting dimensionless parameters (denoted with an asterisk) are defined: $h_0^* = h/H_0$, $u_0^* = \bar{u}/V_0$, $X_0^* = x/L_0$, $t_0^* = V_0t/L_0$, and $i_0^* = i/i_0 = 1$. With this normalization, both the continuity equation (1) and the momentum equation (2) can be rewritten in the following normalized dimensionless form:

$$\frac{\partial h_0^*}{\partial t_0^*} + \frac{\partial u_0^* h_0^*}{\partial x_0^*} = 1 \tag{4}$$

$$\frac{\partial u_0^*}{\partial t_0^*} + u_0^* \frac{\partial u_0^*}{\partial x_0^*} + \frac{1}{F_0^2}\frac{\partial h_0^*}{\partial x_0^*} = \frac{S_0 L_0}{H_0 F_0^2}\left(1 - \frac{u_0^{*2}}{h_0^*}\right) \tag{5}$$

Note that the kinematic flow parameter $K_0 = S_0L_0/H_0F_0^2$ appears on the right-hand side of (5). This normalization provides tremendous insight into overland flow characteristics, particularly regarding the role played by the parameters F_0 and K_0. The analysis of the relative magnitude of the acceleration terms on the left-hand side of (5) is somewhat difficult because the values of the derivatives, such as $\partial u_0^*/\partial t_0^*$, are unknown even though u_0^*, h_0^*, x_0^*, and t_0^* can be determined at the downstream end under equilibrium conditions. The following treatment introduces modifications of the parameters F_0 and K_0. The proposed analysis evaluates the relative magnitude of the acceleration terms of the momentum equation for the rising and equilibrium portions of an overland flow hydrograph including laminar and turbulent overland flows under stationary and moving rainstorms.

Stationary Storms

With reference to Figure 1, analytical expressions for flow depth and velocity are derived for long ($L_s > L_0$) stationary storms ($U = 0$) over a plane surface (constant slope S_0) under a constant rainfall intensity i. It is assumed that the overland flow plane is initially dry ($h = 0$ and $q = 0$) prior to the beginning of precipitation at time $t = 0$. *Woolhiser and Liggett* [1967] clearly demonstrated that according to the kinematic wave approximation at high values of K_0, the flow depth increases linearly with time $h = it$ during the rising limb until the flow depth conveys the equilibrium discharge ($q_m = iX$). At the inception of runoff, the statement $h = it$ is always exact regardless of the momentum equation because it stems solely from continuity (1). Generally speaking, $h = it$ is considered a reasonable approximation during the rising limb, except near the time to equilibrium.

The time to equilibrium T_e is obtained from (3) with $h = iT_e$ under equilibrium discharge $q_m = iX = \alpha(iT_e)^{\beta}$, or

$$T_e = i^{((1/\beta)-1)}(X/\alpha)^{1/\beta} \tag{6}$$

Note that this general form of the time to equilibrium is valid at any location on the plane for any of the four flow conditions summarized in Table 1. The time to equilibrium from (6) is consistent with the full dynamic momentum equation because α is defined in terms of the friction slope.

Under equilibrium conditions, the flow depth is space dependent but time invariant. The relationships for flow depth and velocity ($\bar{u} = q/h$) obtained from (3) under equilibrium discharge ($q = iX = \alpha h^{\beta}$) are

$$h = (iX/\alpha)^{1/\beta} \tag{7}$$

$$\bar{u} = \alpha(iX/\alpha)^{(\beta-1)/\beta} \tag{8}$$

respectively.

The Froude number $Fr = \bar{u}/(gh)^{1/2}$, obtained from (7) and (8), and the kinematic flow number $K = S_0X/hFr^2$ obtained from (7) and (9), are written as a function of the dimensionless distance from the upstream end of the overland flow plane ($X^* = X/L_0$):

$$Fr = g^{-1/2}\alpha^{3/2\beta}(iX)^{(1-(3/2\beta))} = F_0X^{*(1-(3/2\beta))} \tag{9}$$

$$K = gS_0\alpha^{-2/\beta}i^{((2/\beta)-2)}X^{((2/\beta)-1)} = K_0X^{*((2/\beta)-1)} \tag{10}$$

respectively. Both Fr and K from (9) and (10) vary along the plane. They are slightly different from F_0 and K_0 previously defined at the downstream end of the plane by *Woolhiser and Liggett* [1967]. Both formulations are equivalent when $X^* = 1$. Example calculations of the Froude number using (9) indicate that overland flow generated by uniform excess rainfall intensities of 1 in/hour (7.05×10^{-6} m/s) are supercritical ($Fr > 1$) at any slope steeper than 0.01 over a smooth surface longer than 50 m. The corresponding kinematic flow number is large ($K > 150$). In supercritical flow, backwater effects cannot propagate upstream and downstream boundary conditions are not required.

Analytical Description of Acceleration Terms

A normalized dimensionless form of the momentum equation is obtained after dividing (2) by the bed slope S_0:

$$[a_l^* + a_c^* + a_p^*] = 1 - S_f/S_0$$

where (11)

$$a_l^* = \frac{1}{gS_0}\frac{\partial \bar{u}}{\partial t} \qquad a_c^* = \frac{\bar{u}}{gS_0}\frac{\partial \bar{u}}{\partial x} \qquad a_p^* = \frac{1}{S_0}\frac{\partial h}{\partial x}$$

The local acceleration term a_l^* is zero during equilibrium but varies during the rising and falling limbs. On the other hand, the convective acceleration term a_c^* and the pressure gradient term a_p^* vanish when $h = it$ during the rising limb because h does not vary with x at the downstream end of the plane. The following analysis first focuses on defining the maximum values of each term and then compares the

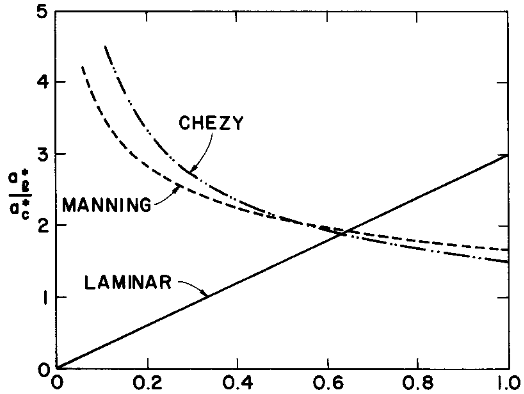

Figure 2. Ratio of a_l^*/a_c^* for stationary storms.

relative magnitude of these maximum values. Therefore the local acceleration term a_l^* which is largest during the rising limb is compared with the convective acceleration term a_c^* and the pressure gradient term a_p^* which are largest at equilibrium.

During the rising limb of a hydrograph, the flow depth, velocity, and discharge at a given point change primarily with time, while space derivatives are comparatively small. The depth-averaged velocity $\bar{u} = q/h$ varies with time from $h \cong it$. The following expression for the local acceleration a_l^* is obtained from (8) after neglecting $\partial\alpha/\partial t$:

$$a_l^* \cong \frac{(\beta - 1)}{gS_0} \alpha i^{\beta-1} t^{\beta-2} \tag{12}$$

This approximate relationship deviates from an identity only because of two approximations, $h \cong it$ and $\partial\alpha/\partial t \cong 0$. The first approximation is valid at the beginning of the storm because both $q = 0$ and $\partial q/\partial x = 0$ at the early rise. The second approximation relates to changes in friction slope and α with time. The time derivative of the friction slope, which is nearly constant, remains small compared to the time derivative of flow depth, which is the dominant factor during the rising limb.

The local acceleration term a_l^* from (12) can be rewritten as a function of the following dimensionless parameters: (1) the kinematic number K from (10), (2) the dimensionless time $t^* = t/T_e$, with T_e from (6), and (3) the distance $X^* = X/L_0$:

$$a_l^* \cong \frac{(\beta - 1)}{K} t^{*\,\beta-2} = \frac{(\beta - 1)}{K_0} t^{*\,\beta-2} X^{*\,(1-(2/\beta))} \tag{13}$$

Notice in (13) that the formulation using K does not involve the parameter X^*, which justifies our preference for using K instead of K_0.

After equilibrium, the flow depth, velocity, and discharge at a given point change primarily with space, while time derivatives vanish. Analytical relationships for a_c^* and a_p^* follow from $q = iX$ and (7) after considering that α is nearly constant, or neglecting $\partial\alpha/\partial x$:

$$a_c^* \cong \frac{\beta - 1}{\beta K} = \left(\frac{\beta - 1}{\beta K_0}\right) X^{*\,(1-(2/\beta))} \tag{14}$$

$$a_p^* \cong \frac{1}{\beta K Fr^2} = \left(\frac{1}{\beta K_0 F_0^2}\right) X^{*\,((1/\beta)-1)} \tag{15}$$

These two approximate relationships deviate from strict identities only because α, or the friction slope, is nearly constant. It is acknowledged that the friction slope and α vary with space. The space derivative of the friction slope, which is nearly constant, remains small compared to the space derivatives of flow depth and velocity, which are the dominant factors during equilibrium because $\partial\bar{u}h/\partial x = i$.

All three dimensionless acceleration terms a_l^*, a_c^*, and a_p^* from (13), (14), and (15) are inversely proportional to the kinematic flow number K, or K_0, and only a_p^* depends on the Froude number Fr, or F_0. These three expressions can be used to assess the friction slope for overland flow with stationary storms. Furthermore, a_l^* can be related to the friction slope by $a_l^* = 1 - (S_f/S_0)$ during the rising limb of the hydrograph (when $0 < t^* < 1$), while a_c^* in conjunction with a_p^* can be related to the friction slope by $a_c^* + a_p^* = 1 - (S_f/S_0)$ when the flow is steady and nonuniform under equilibrium conditions ($t^* \geq 1$).

Relative Magnitude of Acceleration Terms

The ratio of a_l^* during the rising limb to a_c^* at equilibrium is obtained from (13) and (14):

$$a_l^*/a_c^* = \beta t^{*\,\beta-2} \tag{16}$$

Note that this ratio is independent of both the kinematic flow number and the Froude number but varies solely with dimensionless time and the exponent β of the resistance equation. For turbulent flow, as represented by either the Chezy or Manning equations, β is less than two, which indicates that the minimum value of the ratio a_l^*/a_c^* for turbulent flows must be larger than β. This minimum value (1.5 for Chezy and 1.67 for Manning) will occur when the dimensionless time is unity. For laminar flow, the ratio of a_l^*/a_c^* has a minimum value of zero, when $t^* = 0$. The implication of this observation is that a_l^* will be greater than a_c^*, as is illustrated in Figure 2, except for laminar flow when t^* is less than about 0.3. For a complete equilibrium hydrograph on a plane surface with uniform roughness, the ratio of the two largest acceleration terms is obtained when $t^* = 1$. The largest value of a_l^* always exceeds the maximum value of a_c^* in all cases because $\beta > 1$ for both laminar and turbulent flows. It is concluded that for complete equilibrium hydrographs, the local acceleration term always exceeds the convective acceleration term regardless of the kinematic flow number. This demonstrates that the quasi-steady wave approximation does not necessarily improve upon the diffusive wave approximation because the neglected term a_l^* is larger than the term a_c^* retained in this simplification.

The ratio of the convective acceleration a_c^* to the pressure gradient term a_p^* under complete equilibrium conditions is obtained from (14) and (15):

$$a_c^*/a_p^* = (\beta - 1)Fr^2 = (\beta - 1)F_0^2 X^{*\,(2-(3/\beta))} \tag{17}$$

This ratio does not depend on the kinematic flow number but varies with the exponent β and the Froude number. The values of the Froude number above which a_c^*/a_p^* exceed unity are $Fr > 0.7$ for laminar flow ($\beta = 3$); $Fr > 1.22$ Manning equation ($\beta = 5/3$); and $Fr > 1.4$ for Che equation ($\beta = 1.5$). Considering all cases, a_c^* exceeds when the flow is supercritical with $Fr > 1.4$, while $a_c^* < a_p^*$ for subcritical flows with $Fr < 0.7$.

The ratio of the local acceleration a_l^* during the rising limb to the pressure gradient a_p^* at equilibrium is obtained from (13) and (15):

$$a_l^*/a_p^* = \beta(\beta - 1)Fr^2t^{*\beta-2} = \beta(\beta - 1)F_0^2t^{*\beta-2}X^{*(2-(3/\beta))} \quad (18)$$

This ratio is independent of the kinematic flow number but varies with flow type, Froude number, and dimensionless time t^*. For complete equilibrium hydrographs ($t^* \geq 1$), the values of the Froude number above which $a_l^* > a_p^*$ are $Fr > 0.4$ for laminar flow ($\beta = 3$); $Fr > 0.95$ for Manning equation ($\beta = 5/3$); and $Fr > 1.15$ for Chezy equation ($\beta = 1.5$). These criteria are similar to those for a_c^*/a_p^*. Therefore a_l^* exceeds a_p^* when the flow is supercritical with $Fr > 1.4$, while $a_l^* < a_p^*$ for subcritical flows with $Fr < 0.4$.

The results can be summarized as follows: (1) $a_l^* > a_c^* > a_p^*$ for supercritical flows with $Fr > 1.4$ and (2) $a_p^* > a_l^* > a_c^*$ for subcritical flows with $Fr < 0.4$.

Moving Storms

The relative magnitude of the acceleration terms of the momentum equation for block moving rainstorms is analyzed numerically, as analytical solutions become overly complex. The one-dimensional full-dynamic wave model FDCASC tested by *Richardson* [1989] simulates runoff under spatially and temporally varied rainfall precipitation. This model simulates one-dimensional overland flow for upslope and downslope block moving storms, as is sketched in Figure 1. The dimensionless discharge $q^* = q/iL_0$ varies with dimensionless time $t^* = t/T_e$. Two dimensionless storm parameters define the length of the storm $L_* = L_s/L_0$ and the storm celerity $U^* = UT_e/L_0$. Positive and negative values of U^* correspond to storms moving down and up the plane, respectively.

Downslope Moving Storms

This analysis summarizes typical results from *Richardson*'s [1989] investigation on one-dimensional moving rainstorms. For instance, the acceleration terms for the rising and equilibrium portions of a downslope moving storm with

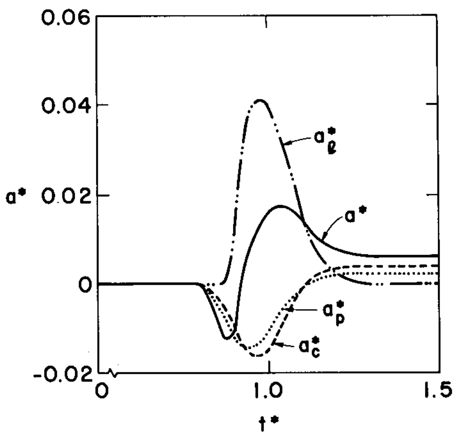

Figure 3. Acceleration terms for a downslope moving storm ($U^* = 1$).

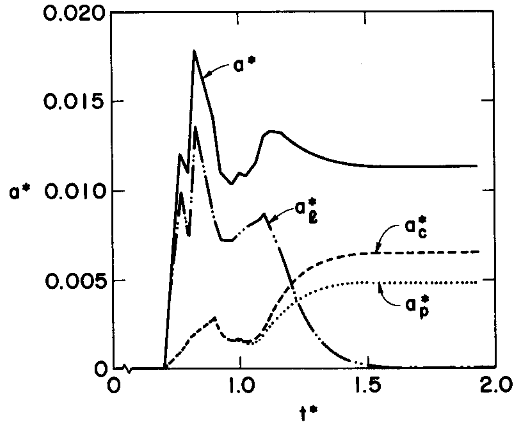

Figure 4. Acceleration terms for an upslope moving storm ($U^* = -1$).

celerity $U^* = 1$ and storm length $L^* = 1.5$ are shown in Figure 3. The other parameters for this case are $F_0 = 1.63$ and $K_0 = 105$. Under these conditions, the values of the acceleration terms for stationary storms from (13), (14), and (15) are $a_l^* \sim 0.006$, $a_c^* \sim 0.004$, and $a_p^* \sim 0.002$, respectively; thus $a_l^* > a_c^* > a_p^*$.

For the downslope moving storm in Figure 3, all three acceleration terms are zero prior to runoff, after which the term a_l^* increases rapidly as soon as the flood wave reaches the downstream end of the plane. It is found that the maximum value of $a_l^* = 0.04$ is roughly an order of magnitude larger than the corresponding value for an equivalent stationary storm $a_l^* = 0.006$. Both values of a_c^* and a_p^* are negative, and their absolute magnitude is smaller than a_l^* as predicted for supercritical stationary storms ($a_l^* > a_c^* > a_p^*$). It is clear from Figure 3 that for downslope moving storms, all the acceleration terms reach a maximum during the rising limb of the hydrograph. Equilibrium conditions are reached for sufficiently large storms. Figure 3 shows that the analytical expressions of a_c^* and a_p^* for stationary storms are reached asymptotically as $t^* > 1.3$. The numerical sum of all three acceleration terms ($a^* = a_l^* + a_c^* + a_p^*$) is also presented on the plots for a storm moving down a plane. During the rising limb, the sum of all three acceleration terms a^* is smaller than the single contribution because the values of a_l^* oppose those of a_c^* and a_p^*.

This analysis contrasts with the previous discussion pertaining to stationary storms. It is important to note that each acceleration term contributes to the value of the friction slope during the rising limb. The term a_l^* peaks at a positive value while the other two acceleration terms are negative, which indicates that using either the quasi-steady dynamic wave or the diffusive wave approximation results in "false improvement" of the computation of the friction slope. Indeed, a friction slope greater than the bed slope is calculated when it actually should be less than the bed slope.

Upslope Moving Storms

Upslope moving storms produce gradually rising limbs when compared with downslope moving storms. The time to equilibrium is consequently delayed, and a value of $L^* = 2$ ensures that the discharge reaches equilibrium. The values of the acceleration terms are plotted on Figure 4 for a storm moving up the plane at a dimensionless storm speed ($U^* = -1$). The other parameters for these simulations are the same as for the simulation of upslope moving storms.

The magnitude of acceleration terms for upslope moving storms differs largely from downslope moving storms. Runoff immediately increases at the downstream end of the plane and consequently, the flow depth, velocity, and discharge at the terminus of the plane are larger than for locations further upstream. All three acceleration terms are positive during the rising limb, the dominant term being a_l^*. As equilibrium conditions are approached, the value of a_l^* decreases to zero and the values of a_c^* and a_p^* approach the analytical values obtained for stationary storms as shown in Figure 4 when $t^* > 1.5$.

As in the case of downslope moving storms, all three acceleration terms contribute to the determination of the friction slope during the rising limb with downslope moving storms. However, for upslope moving storms, all acceleration terms are positive during the rise, and the use of either the quasi-steady or the diffusive wave approximation will adjust the friction slope in the right direction. The use of either approximation, however, is insufficient because the largest acceleration term a_l^* is neglected.

Conclusion

This investigation focuses primarily on the relative magnitude of the acceleration terms of the Saint-Venant equations applied to one-dimensional overland flow on a plane surface with uniform roughness. Both stationary and block moving rainstorms are considered and the analysis is applicable to laminar and turbulent flows. In absolute value, all acceleration terms (a_l^*, a_c^*, a_p^*) become small compared to S_0 when K is large, as given by (13), (14), and (15). When considering the relative magnitude of the acceleration terms for complete equilibrium hydrographs under stationary rainstorms, it is found that (1) the local acceleration term a_l^* during the rising limb is always larger than the convective acceleration term a_c^* at equilibrium, (2) $a_l^* > a_c^* > a_p^*$ for supercritical overland flow with $Fr > 1.4$, and (3) $a_p^* > a_l^* > a_c^*$ for subcritical overland flow with $Fr < 0.4$. These results on the relative magnitude of the acceleration terms are independent of the kinematic flow number K.

In supercritical overland flow, these results ($a_l^* > a_c^* > a_p^*$) are opposite to those for open channels ($a_l^* < a_c^* < a_p^*$) in that the diffusive wave approximation and the quasi-steady dynamic wave approximation do not improve upon the kinematic wave approximation because the additional term considered is smaller than the terms deleted from the approximation. On the other hand, the results for subcritical overland flow ($a_p^* > a_l^* > a_c^*$) support the use of the diffusive wave approximation but prohibit the use of the quasi-steady dynamic wave formulation.

The findings for moving rainstorms corroborate those of stationary storms in that $a_l^* > a_c^* > a_p^*$ for supercritical overland flow. Individual values of the acceleration terms for moving rainstorms can be up to one order of magnitude larger than for an equivalent stationary storm. Figures 3 and 4 clearly show that either the quasi-steady dynamic wave approximation (neglecting the term a_l^*) or the diffusive wave approximation (neglecting both a_l^* and a_c^*) would neglect terms significantly larger than a_p^*. This effect is found to be more pronounced during the rising limb of overland flow hydrographs for downslope moving storms.

Notation

a^* sum of three dimensionless acceleration terms.
a_c convective acceleration.
a_c^* dimensionless convective acceleration.
a_l local acceleration.
a_l^* dimensionless local acceleration.
a_p pressure gradient.
a_p^* dimensionless pressure gradient.
C Chezy resistance coefficient.
Fr Froude number.
F_0 downstream Froude number.
g gravitational acceleration.
h flow depth.
h_0^* dimensionless flow depth.
H_0 downstream normal flow depth.
i excess rainfall intensity.
i_0 maximum rainfall intensity.
i_0^* dimensionless rainfall intensity.
k laminar resistance parameter.
K kinematic flow number.
K_0 downstream kinematic flow number.
L_s storm length.
L^* dimensionless storm length.
L_0 runoff length.
n Manning resistance coefficient.
q unit discharge.
q^* dimensionless unit discharge.
q_m equilibrium unit discharge.
S_0 bed slope.
S_f friction slope.
t time.
t^* dimensionless time.
t_0^* dimensionless time from *Woolhiser and Liggett* [1967].
T_e time to equilibrium.
U storm velocity.
U^* dimensionless storm velocity.
$\bar{u}$ average flow velocity.
u_0^* dimensionless flow velocity.
V_0 downstream normal flow velocity.
X distance from upstream boundary.
x downstream coordinate.
x_0^* dimensionless downstream distance.
α coefficient of the stage-discharge relationship.
β exponent of the stage-discharge relationship.
ν kinematic viscosity of water.

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