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Similarity in catchment response
2. Moving rainstorms

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Abstract. The influence of storm motion on runoff is explored, with a focus on dimensionless hydrologic similarity parameters. One- and two-dimensional physically based runoff models are subjected to moving rainstorms. A dimensionless storm speed parameter Ut_e/L_p, where U is the storm speed, t_e is the runoff plane kinematic time to equilibrium, and L_p is the length of the runoff plane, is identified as a similarity condition. Storm motion effects on the peak discharge are greatest when the storm is traversing a one-dimensional runoff plane in the downslope direction at a dimensionless speed of $Ut_e/L_p = 0.5$. This conclusion holds for all values of the dimensionless storm sizes L_s/L_p where L_s is the length of the storm in the direction of motion. Simulations with a two-dimensional rainfall-runoff model confirm the applicability of this similarity parameter on natural watershed topography. Results indicate that the detailed simulation of storm motion is necessary when the storm is moving near the velocity of maximum effect, which is considerably slower than typical storm velocities.

1. Introduction

The influence of storm motion must be considered when applying remotely sensed precipitation data in distributed hydrologic modeling. The importance of storm motion on computed outflow hydrographs dictates the level of detail of storm motion which must be simulated. If storm motion has only a minor effect on outflow, stationary rainstorms may be used in simulations [e.g., *Saghafian et al.*, this issue]. Conversely, if the effects of storm motion are large, it is important to specifically model the changes in location of precipitation with time. Conceptually, the identification of hydrologic similarity conditions which relate storm and watershed characteristics to storm motion effects will assist in identifying conditions when storm motion may play a major role in catchment response.

1.1. Previous Work

Previous studies regarding the influence of storm motion on outflow hydrographs have not identified a specific dimensionless similarity parameter. The earliest studies were performed using physical laboratory apparatus [*Yen and Chow*, 1969; *Roberts and Klingeman*, 1970; *Black*, 1972]. All physical studies to date have documented an effect of storm motion on the magnitude and timing of outflow hydrographs.

More recent studies have relied upon numerical techniques. One of the earliest numerical studies of storm motion effects was performed by *Surkan* [1974]. The spatial and temporal variability of precipitation intensity was studied by *Niemczynowicz* [1984a] using a conceptual model of an urban area based on Manning's resistance equation and the continuity equation. Dimensional analysis was applied to describe the relationship between moving storms and runoff hydrographs. *Niemczynowicz* [1984b] also applied this model to urbanized catchments in the city of Lund, Sweden. This study showed that the maximum discharge with the steepest rising limb occurred when the storm was moving down the catchment at a speed approximately equal to the average flow velocity within the urban storm sewer. This analysis did not identify a descriptive response parameter.

A distributed numerical model based on the time-area concept was used by *Foroud et al.* [1984] to examine storm motion effects on a watershed in Quebec. The maximum storm motion effect on the peak discharge was observed when the storm was moving downstream at a velocity equal to the channel flow velocity. The effect of storm motion on conceptual catchment response was examined by *Watts and Calver* [1991]. Using a physically based runoff model, results indicated that storms moving downstream produce earlier and higher peak outflows, and that error will most likely be added to lumped parameter models at small storm speeds. Similarly to Foroud, Watts and Calver found that the maximum difference in the magnitude of the peak response between storms moving in upstream and downstream directions occurs when the storm speed is approximately equal to the average channel velocity.

1.2. Objectives

The objectives of this paper are to (1) identify a dimensionless hydrologic similarity parameter which relates storm speed

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to the influence of storm motion on the peak hydrograph discharge from a one-dimensional runoff plane; (2) determine the suitability of the one-dimensional results in describing the storm motion effect on a two-dimensional runoff geometry; and (3) ascertain the sensitivity of a two-dimensional surface to storm speed and direction, respectively. To achieve these objectives, both one- and two-dimensional physically based runoff models are applied to test the suitability of the similarity parameter using simple planar and complex watershed topography, together with rectangular, constant intensity, moving rainstorms.

2. Methodology

2.1. Time to Equilibrium

The kinematic time to equilibrium for turbulent flow on a rectangular runoff plane is given by *Henderson and Wooding* [1964]:

$$t_e = [nL_p/S_0^{1/2}i^{2/3}]^{3/5} \quad (1)$$

The variable t_e provides a characteristic runoff response time which incorporates the combined effects of land surface parameters such as runoff plane roughness (n), length (L_p), and slope (S_0), and the rainfall intensity i. The application of t_e as a similarity parameter which describes runoff sensitivity to spatially varied runoff plane characteristics and rainfall was presented by *Julien and Moglen* [1990] and *Ogden and Julien* [1993]. Runoff sensitivity to temporally varied rainfall was also analyzed in terms of t_e by *Ogden and Julien* [1993]. Based on previous applications of t_e in the development of dimensionless hydrologic similarity parameters, it is hypothesized that this variable may prove descriptive in the analysis of moving rainstorms.

2.2. Equivalent Storms

Equivalent moving storms were defined by *Yen and Chow* [1969] as storms moving at different speeds with the same duration of rainfall at each point on the watershed and identical total rainfall volume on the catchment. To maintain constant rainfall volume between equivalent storms moving at different speeds, *Yen and Chow* [1969] held the precipitation intensity constant and varied the size of the storms. This definition requires the length of equivalent storms be directly proportional to the storm speed. However, this definition of equivalent moving storms is not applicable for analyses based on t_e because the constant rainfall rate requirement fixes the value of t_e for all storms. In reality, the equivalent moving storms defined by *Yen and Chow* [1969] are equivolume and equi-intensity moving storms.

An alternate definition of equivalent storms is proposed here to allow consideration of t_e in the analysis of storm motion effects on runoff characteristics. The new definition specifies, as did *Yen and Chow* [1969], that the volume of rainfall on the catchment be equal. However, this new definition requires that the physical size of equivalent storms must also be equal. By this new definition equivalent storms of equal size, moving at different speeds, must have rainfall intensities which vary in proportion to the ratio of the storm speeds:

$$i_2 = i_1|U_2/U_1| \quad (2)$$

The time to equilibrium for each equivalent storm will be different, varying with $i^{-0.4}$. When the storm size L_s is smaller than the length of the runoff plane L_p, t_e is calculated assuming that rain falls over the entire runoff plane, to maintain consistency in the calculation of t_e.

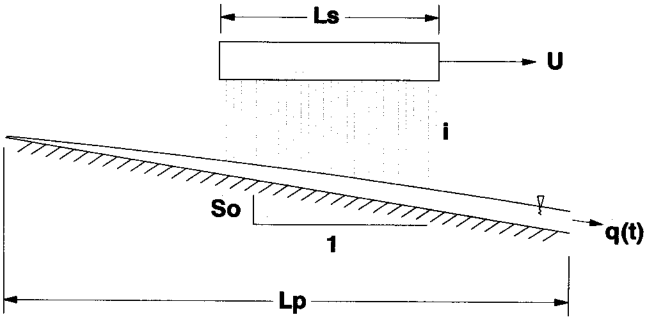

Figure 1. Schematic representation of one-dimensional experiment.

3. Results

3.1. One-Dimensional Runoff Simulations With Moving Storms

This experiment is designed to examine the effect of storm motion on a simple runoff plane and to identify a descriptive similarity parameter. Relevant runoff parameters (runoff plane slope S_0, length L_p, and Manning roughness coefficient n are arbitrarily assigned constant values of 10%, 100 m, and 0.02, respectively. The rainfall rate i, storm length L_s, and storm velocity U are independent variables. The experimental methodology is represented in Figure 1.

One-dimensional runoff routing was performed using the CASC finite element runoff model [*Julien and Moglen*, 1990], which provides a numerical solution to the kinematic wave form of the equations of motion by the Galerkin weighted residual method. Simulations are performed with excess rainfall on an impervious surface. The algorithm enables the simulation of time-varying storms [*Ogden and Julien*, 1993], stationary storms with variable overland flow parameters [*Julien and Moglen*, 1990], and moving storms [*Richardson*, 1989].

Given that a storm is large compared to the size of the runoff plane, $L_s \gg L_p$, and the storm speed is relatively slow such that $L_s/U > t_e$, the runoff plane will reach a maximum equilibrium discharge $q_p = iL_p$. In this case there will be no discernable effect of storm motion on peak discharge. In other words, the peak discharge will be the same, independent of storm speed. Of interest, therefore, are situations where $L_s/L_p \leq 1$ and $L_s/U < t_e$, when partial equilibrium hydrographs arise and storm motion effects will be observed. The storm motion effect on peak discharge q_p is assumed to be a functional relation of the form

$$\frac{q_p}{iL_p} = f\left(\frac{L_s}{L_p}, \frac{Ut_e}{L_p}\right) \quad (3)$$

In this experimental methodology, values of L_s/L_p tested ranged from 0.3 to 2.5, while Ut_e/L_p was varied between -1.2 and $+1.2$.

The peak discharge from each simulation is extracted from the outflow hydrographs for equivalent storms moving at dif-

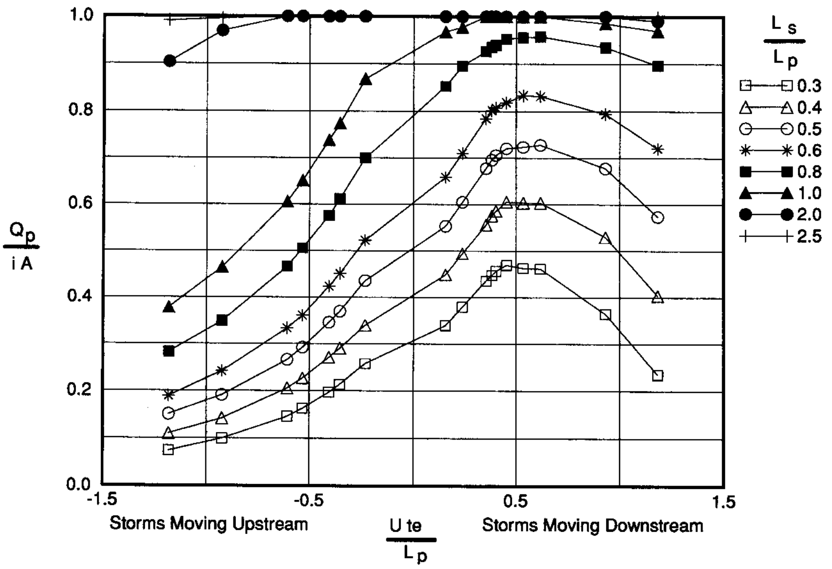

Figure 2. Effect of storm motion on peak discharge from one-dimensional plane.

ferent speeds and nondimensionalized by the rainfall rate i and runoff plane area A. Figure 2 presents the results of simulations as a family of curves where each curve represents equivalent storms of a specified storm length L_s/L_p. Note that each curve also corresponds to a constant rainfall volume.

The influence of storm motion on the peak discharge from a one-dimensional runoff plane is unambiguous for partial equilibrium hydrographs. For turbulent flow using Manning's equation, the maximum peak discharge occurs when the storm moves in the downslope direction at a critical speed equal to $U = L_p/2t_e$ for all values of $L_s/L_p < 1.0$. Additionally, the vertical alignment of the curves in Figure 2 justifies the use of L_p/t_e as a characteristic response velocity. The peak discharge is less than maximum if the storm moves faster or slower than this critical speed. For cases where $L_s/L_p \geq 1.0$ the maximum peak discharge is equal to the equilibrium discharge; in no case could the maximum discharge exceed the equilibrium discharge.

3.2. Two-Dimensional Runoff Simulations With Moving Storms

Previous studies of the influence of storm motion on the discharge from watersheds have noted a correlation between storm speed and quantities such as the average channel or storm sewer velocity. However, these velocities are difficult to quantify without a priori knowledge of discharges. In this portion of this paper the dimensionless similarity parameter developed in the one-dimensional runoff plane analysis is tested with a physically based runoff model in two-dimensional watershed simulations. This technique requires knowledge of the average excess rainfall rate.

The CASC2D watershed runoff model was used on elevation data from the Macks Creek experimental watershed [*Robins et al.*, 1965]. Both the model and watershed are discussed by *Saghafian et al.* [this issue]. A model grid size of 125 m was selected to represent the basin topography in sufficient detail. To maintain consistency with the one-dimensional analysis, the basin is assumed to be impervious in that all rainfall is taken as excess rainfall. A constant value of the Manning roughness coefficient equal to 0.04 is applied for all overland flow.

All channels are assumed to be wide, and all flow is routed as overland flow. This assumption may limit the applicability of these results to catchments which are dominated by overland flow. Nonetheless, the presented data analysis technique uses dimensionless terms which are valid for identical dimensionless parameter values, independent of calibration. As in the one-dimensional case, the results are analyzed using the time to equilibrium. The absence of specific channel routing in these simulations does not invalidate the results because the inclusion of channel routing would change the time to equilibrium for a given rainfall intensity. However, the bimodal distribution of wave speeds (overland and channel) may alter watershed response to storm speeds.

The basin kinematic time to equilibrium was determined using the method outlined by *Saghafian et al.* [this issue]. A different time to equilibrium was determined for each equivalent storm, as the rainfall rate is a function of L_s/L_p. The basin characteristic length L_p was taken as the length of the longest flow path in the watershed.

As in the one-dimensional study the concept of the equivalent rectangular block moving storm is applied. The direction of storm motion must be considered as an additional factor which affects the magnitude of the effect of storm motion on the peak outflow. Equivalent storms with identical values of L_s/L_p are moved in eight compass directions, spaced 45° apart. Eight values of L_s/L_p ranging from 0.15 to 1.25 were tested.

Figure 3 shows the influence of storm motion on peak discharge for storms moving in northwest and southeast directions. As in the one-dimensional case the maximum effect of storm motion on runoff occurs at a dimensionless storm speed $Ut_e/L_p = 0.5$, with considerable dependence on the direction of motion. With reference to the figure depicting Macks Creek [*Saghafian et al.*, this issue, Figure 1], Macks Creek has two main channels. One main channel is oriented in a roughly east-west direction, while the other flows from the southwest to northeast. The storms moving in the northeasterly direction in Figure 3 move in the approximate mean downstream direction. Storms moving toward the northwest or southeast are nearly perpendicular to the channel which drains the larger southern

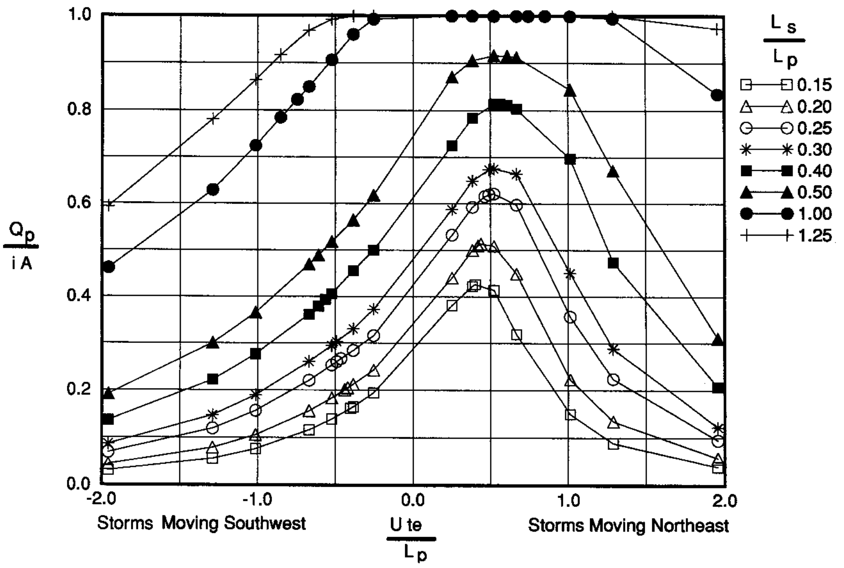

Figure 3. Effect of storm motion on peak discharge for storms moving along a northeast-southwest line.

portion of the watershed. Figure 4 shows peak outflow data produced by storms moving in northwest and southeast directions, representing storms more closely aligned with the channel which drains the larger southern portion of the watershed. The data in Figure 4 indicate that the dimensionless storm speed of maximum effect on peak discharge is approximately 0.20 for all values of L_s/L_p. Dimensionless speeds of maximum effect for all storm directions and values of L_s/L_p tested are listed in Table 1. From the data in Table 1 it is apparent that the direction of storm motion which has the largest effect on the peak outflow for the Macks Creek watershed lies between east and northeast, which is approximately in the mean downstream flow direction.

3.3. Discussion

Conclusive results from the one-dimensional study show that the storm motion effect on peak discharge on an impervious runoff plane is solely a function of Ut_e/L_p for $L_s/L_p \leq 1$ (Figure 2). In general, when the flow is turbulent (Manning equation), the effect of storm motion on peak discharge is most pronounced when the storm moves at a velocity of $L_p/2t_e$ in a downstream direction. Physically, this result indicates that for rainstorms moving in the downstream direction, the average kinematic speed of the flood wave is equal to $0.5(L_p/t_e)$. This inference is made from the fact that the peak outflow magnitude is reached when the flood wave and storm front reach the outlet simultaneously [*Yen and Chow*, 1969]. Note that this result is valid only for partial equilibrium storms.

The effect of block moving storm speed and direction on the peak outflow was calculated using the topography of Macks Creek watershed. The storm speed data were analyzed by a direct adaptation of the one-dimensional analysis with the additional consideration of storm direction. Results indicate that storm motion can have a significant effect on the outflow hydrograph peak. As in the one-dimensional case, this is partic-

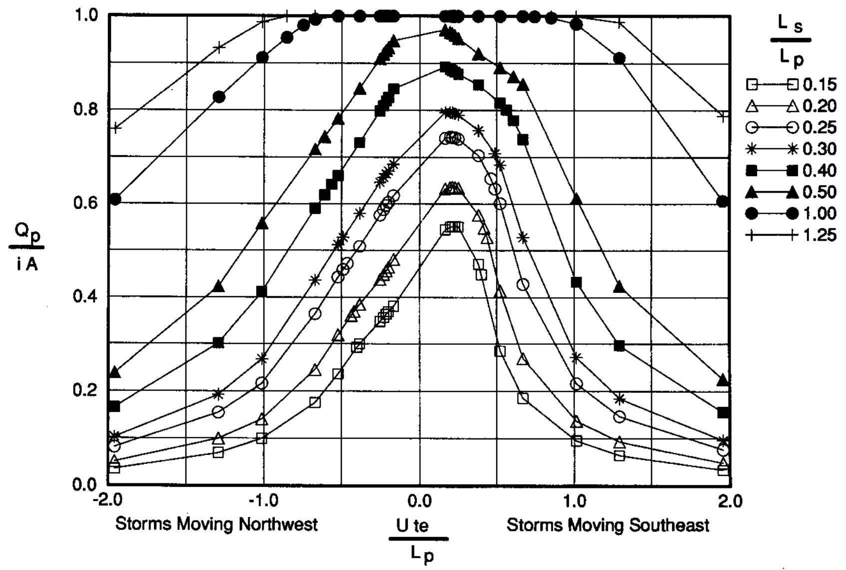

Figure 4. Effect of storm motion on peak discharge for storms moving along a northwest-southeast line.

Table 1. Dimensionless Storm Speeds Ut_e/L_p With Maximum Effect on Peak Outflow on Macks Creek for Different Storm Lengths and Directions

Storm Direction	L_s/L_p 0.15	0.20	0.25	0.30	0.40	0.50
North	0.18	0.18	0.25	0.25	0.25	0.30
Northeast	0.40	0.44	0.50	0.52	0.55	0.56
East	0.25	0.25	0.47	0.50	0.55	0.40
Southeast	0.20	0.20	0.20	0.19	0.16	0.14

ularly true when the dimensionless storm velocity Ut_e/L_p is near 0.5 and the storm is moving in the mean downstream direction (Figure 3). Therefore the storm speed of maximum effect on two-dimensional watershed topography is approximately $L_p/2t_e$.

Considering the time to equilibrium t_e from (1), with L_p assumed proportional to the square root of the watershed area, the storm speed of maximum effect U_m at $U_m t_e/L_p = 0.5$ may be defined as

$$U_m = \frac{1}{2}\frac{L_p}{t_e} = \frac{A^{0.2}S_0^{0.3}i^{0.4}}{2n^{0.6}} \qquad (4)$$

Accordingly, the storm velocity of maximum effect increases with drainage area, watershed slope, and rainfall intensity and decreases with surface roughness. For instance, the storm velocity of maximum effect on runoff for a 100-km^2 watershed (assuming $L_p = 10.0$ km) with $S_0 = 0.001$ and $n = 0.03$ under a rainfall intensity $i = 7 \times 10^{-6}$ m/s (1 inch/h) is $U_m = 0.18$ m/s or 0.65 km/h in the downstream direction. This speed is small compared to typical storm speeds. Conversely, a 50-km/h storm speed corresponds to dimensionless storm speeds ranging from -40.0 to $+40.0$.

4. Summary

In summary, it is preferable to model the motion of a storm across a basin in detail when considering partial equilibrium storms ($L_s/L_p < 1$) and when the storm is moving approximately at a dimensionless velocity of $Ut_e/L_p = 0.5$ in the downstream direction. The dimensionless storm velocities which influence the peak discharge range from -0.5 to 2.0, while dimensionless storm velocities are typically of the order of -40 to $+40$, depending on the watershed size, slope, roughness, and rainfall intensity. The specific conclusions of this study are as follows.

1. Using a one-dimensional runoff geometry, a hydrologic similarity parameter Ut_e/L_p defines the storm velocity which produces the maximum effect on the peak discharge for partial equilibrium hydrographs. This storm velocity U_m is equal to $L_p/2t_e$ in the downstream direction (Figure 2), independent of storm size.

2. On two-dimensional basin topography, storm motion has a maximum effect on the peak discharge when the storm speed is near $L_p/2t_e$ in a downstream direction (Figures 3 and 4), defining L_p as the length of the longest flow path in the basin.

3. Two-dimensional runoff geometries are considerably more sensitive to storm speed than direction. In general, increases in the peak discharge occur only when the storm is moving within 90° of the mean drainage direction.

4. Storm motion in an approximate upstream direction reduces the magnitude of the hydrograph peak.

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