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Runoff Sensitivity to Temporal and Spatial Rainfall Variability at Runoff Plane and Small Basin Scales

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Surface runoff sensitivity to spatial and temporal variability of rainfall is examined using physically based numerical runoff models. Rainfall duration t_r and temporal sampling interval δt are varied systematically, and normalized by the time to equilibrium t_e. The relative sensitivity R_s is defined as the total volume of outflow variability over 50 Monte Carlo simulations normalized by the rainfall volume and the coefficient of variation of rainfall. Relative sensitivity to temporal rainfall variability increases with both t_r and δt. An asymptotic R_s value proportional to $(\delta t/t_e)^{1/2}$ is approached as $t_r \gg t_e$. Two-dimensional surface runoff simulations with spatially variable rainfall, without temporal variability, on two watersheds indicate that R_s decreases as t_r/t_e increases. Normalized R_s versus t_r/t_e curves are identical for two watersheds and a one-dimensional overland flow plane. These findings indicate that spatial variability is dominant when $t_r < t_e$, while temporal variability dominates when $t_r > t_e$, particularly for larger values of $\delta t/t_e$.

INTRODUCTION

When remotely sensed input data is applied to a runoff model, there are discrete temporal and spatial sampling resolutions of the input data which must be considered as possible sources of error. This error will adversely affect the runoff calculated by the model and produce variability in computed outflow due solely to the temporal and spatial resolution of the input data. It is important to explore the sources of this variability before precipitation data from remote sensors such as telemetered rain gauges or weather radar are applied to two-dimensional physically based runoff models. Additionally, it is important that the magnitude of the effect of each factor be determined. This will allow the proper application of remotely sensed input data in the runoff modeling process and provide additional understanding of the relation between model accuracy and input data temporal and spatial resolution.

Overland flow modeling requires information about numerous hydrologic parameters. The parameters most important when physically simulating overland flow are topography, surface roughness, soil infiltration characteristics, and the distribution, duration, and intensity of precipitation. Although the intensity of precipitation varies continuously in space and time, hydrologic models typically incorporate simplifying assumptions which apply single parametric values to rainfall intensity in space and time. The model output therefore fails to reflect the effects of temporal and spatial variability in the input.

Erichsen and Nordseth [1984] examined the validity of existing computational runoff simulation techniques. There are also many existing methods to model spatial and temporal variability of precipitation on watersheds. The literature gives several examples, particularly one-dimensional models which have been modified to simulate two-dimensional watersheds. More recently, physically based two-dimensional runoff models have been developed by *James and Kim* [1990] and *Julien and Saghafian* [1991].

The effect of the temporal variation of rainfall intensity on overland flow was analytically studied by *Hjelmfelt* [1981]. Results indicate that the peak discharge from an overland flow plane will be slightly larger for rainfall at constant intensity than for rainfall with a typical thunderstorm distribution. Hjelmfelt also notes that the correct value for time to equilibrium is essential for the determination of the peak discharge. The study concludes that difficulty in estimating the time to equilibrium for temporally varying precipitation limits the consideration of temporal variability for design purposes.

A linear and nonlinear reservoir routing method, proposed by *Diskin et al.* [1984], is capable of accepting spatially and temporally varying precipitation input. Another approach for modeling rainfall runoff with spatial and temporal variability in precipitation intensity was proposed by *Niemczynowicz* [1984]. This model of a conceptual urban area was based on Manning's equation and the continuity equation and used dimensional analysis to describe the relationship between moving storms and runoff hydrographs. Later, *Niemczynowicz* [1984] applied this model to urbanized catchments in the city of Lund, Sweden. This study showed that the maximum discharge with the steepest rising limb occurred when the storm was moving down the watershed at a speed approximately equal to the average flow velocity. This author's work focuses on the magnitude of the peak discharge and the relationship between storm velocity and peak discharge.

Storm and Jensen [1984] developed a theoretically and physically based runoff model capable of simulating nonurbanized watersheds with spatially and temporally varying precipitation inputs. This model is also capable of modeling subsurface flows; however, the paper presents a limited

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compilation of results. The effects of spatially variable precipitation on runoff have been studied by many researchers, including *Woo and Brater* [1962], *Jensen* [1984], and *Wood et al.* [1988]. *Richardson* [1989] performed a detailed study of overland flow dynamics using a one-dimensional finite element overland flow runoff model. The finite element runoff model was calibrated using the laboratory data of *Yen and Chow* [1968].

A Monte Carlo technique was used to study the effect of rain gauge sampling resolution on a distributed watershed model by *Krajewski et al.* [1991]. This study used a one-dimensional distributed parameter model based on the kinematic wave approximation. The findings of this study indicated that model response is more sensitive to temporal resolution than spatial resolution of the input precipitation data. Additionally, these researchers found that a lumped parameter model consistently underestimated peak flows. They also concluded that a two-dimensional distributed model would be better suited for modeling the problem of optimal spatial and temporal resolution.

Julien and Moglen [1990] investigated the effects of spatial variability on overland flow hydrographs using a one-dimensional, kinematic wave, finite element model. The specific areas of interest explored were analysis of the differences in model response for the cases of partial and complete equilibrium; comparison of the relative influence of the spatially varying quantities slope, width, rainfall intensity, and roughness coefficient; and a sensitivity analysis of required grid spacing. The effect of parameter variability was examined by randomly perturbing the values of the four spatially varied parameters in both a correlated and uncorrelated fashion. Julien and Moglen found the effect of all perturbations is a function of t_r/t_e, or the ratio of the rainfall duration t_r to the time to equilibrium t_e of the plane under a constant precipitation intensity. One finding of this study was that the relative magnitudes of the effects of perturbations are different, but when each is normalized with respect to the value at $t_r/t_e = 1.0$, the effects were identical. Therefore a scalar multiple of the spatial variability in any one of the four spatially variable parameters can simulate spatial variability in the other three parameters. More importantly, they found that as t_r/t_e exceeds 2.0, the relative sensitivity of the model to perturbations in all spatial runoff plane and rainfall characteristics becomes small.

OBJECTIVES

There are four specific objectives of this study. The first objective is to verify the findings of *Julien and Moglen* [1990] regarding runoff sensitivity to rainfall spatial variability, using two-dimensional simulations. The second objective is to quantitatively determine the sensitivity of one-dimensional overland flow to temporal precipitation variability. The third objective is to verify the one-dimensional temporal variability study in two-dimensional simulations. The fourth and final objective is to draw a conclusion regarding runoff relative sensitivity R_s due to the separate influences of rainfall spatial and temporal variability as a function of rainfall duration, rainfall data temporal resolution, and runoff area time to equilibrium.

GOVERNING EQUATIONS AND MODELING TECHNIQUE

The state variables governing one-dimensional overland flow are the length of the horizontal projection of the overland flow plane L, the depth of flow h, the average velocity of flow $\bar{u}$, the surface slope S_o, the excess rainfall rate in space and time $i(x, t)$, and the discharge per unit width q. The hydraulic roughness of the plane n and the gravitational acceleration g are also important variables.

There are a host of different approaches in the literature for overland flow routing. The partial differential equations which express conservation of mass (1) and momentum (equations (2) and (3)) in two dimensions are expressed as

$$\frac{\partial h}{\partial t} + \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} = e \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\left(S_{ox} - S_{fx} - \frac{\partial h}{\partial x}\right) \tag{2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = g\left(S_{oy} - S_{fy} - \frac{\partial h}{\partial y}\right) \tag{3}$$

where

- u, v flow velocities in x and y direction, respectively;
- S_{ox}, S_{oy} bed slopes in x and y directions, respectively;
- S_{fx}, S_{fy} friction slopes in x and y directions, respectively;
- g acceleration due to gravity;
- h flow depth;
- q_x unit discharge in x direction;
- q_y unit discharge in y direction;
- e excess rainfall rate, equal to i–f;
- i incident rainfall intensity;
- f infiltration rate.

The left-hand sides of (2) and (3) contain the local and convective acceleration terms, while the right-hand sides of the equations denote the forces per unit mass acting in the x and y directions, respectively.

The applicability of the simplified equations of motion have been discussed in the literature [*Richardson* 1989; *Ponce and Li*, 1979; *Ponce et al.*, 1978]. *Richardson* [1989] has shown that the full dynamic and kinematic forms of the momentum equation are most applicable for the simulation of overland flow. The most widely used overland flow routing technique is the kinematic wave form of the equation of motion, which neglects the acceleration and pressure terms. The diffusive wave formulation neglects all three acceleration terms on the left-hand side of (2) and (3). The advantages of the diffusive wave formulation over the kinematic waveform are its applicability in regions of small slope and/or high roughness, and the ability to store water on the watershed surface. These advantages justify its application in overland flow simulations on natural basin topography.

A resistance equation is required to provide an energy balance. Practical resistance equations are empirically derived from the analysis of flow variables and field data. A widely applicable roughness equation is the Manning equation in SI units:

$$U = \frac{1}{n} R_h^{2/3} S_f^{1/2} \quad (4)$$

where n is the Manning roughness coefficient, $U = (u^2 + v^2)^{1/2}$ is the flow velocity, R_h is the hydraulic radius, and $S_f = (S_{fx}^2 + S_{fy}^2)^{1/2}$.

Roughness equations are typically of the form $q = (q_x^2 + q_y^2)^{1/2} = \alpha h^\beta$, which for the Manning equation $\alpha = S_f^{1/2}/n$ and $\beta = 5/3$.

Numerical modeling of overland flow has evolved considerably in recent years with the proliferation of digital computers. The CASC one-dimensional kinematic wave rainfall runoff model [*Julien et al.*, 1988] provides a finite element numerical solution by the Galerkin weighted residual method. This model employs the Manning equation and neglects infiltration, therefore assuming that all rainfall is excess rainfall. The algorithm enables the simulation of time varying storms, stationary storms with variable overland flow parameters [*Julien and Moglen*, 1990], and one-dimensional moving storms [*Richardson*, 1989]. A more rigorous description of the model formulation, simulation examples for stationary and moving storms, and comparison with observed data is published in the CASC user's manual [*Julien et al.*, 1988].

The CASC2D watershed runoff model employs a raster-based, two-dimensional solution of the diffusive wave formulation of the St. Venant equations [*De St. Venant* 1871] for overland flow. The model was developed at Colorado State University by *Julien and Saghafian* [1991] and includes *Green and Ampt* [1911] infiltration calculation and diffusive wave channel routing routines. Watershed data are stored in raster format at a user defined spatial resolution. Typically, applied spatial resolutions range from 125 to 200 m. Watershed characteristics within each grid element (e.g., roughness coefficient, soil characteristics, etc.) are treated as constant quantities. The CASC2D runoff model can simulate moving storms and incorporate weather radar estimated rainfall rates [*Ogden*, 1992]. Complete details of the model formulation, options, data requirements, and evaluation are published in the CASC2D user's manual [*Julien and Saghafian*, 1991].

The precipitation model by *Rodriguez-Iturbe and Eagleson* [1987] was selected to generate two-dimensional rainfall fields used in this study. This model employs statistical clustering techniques and simulates the birth, growth, and decay of convective precipitation within a large mesoscale area (LMSA) which is several hundreds of kilometers on a side. The model assumes that within the LMSA, smaller regions of intense rainfall occur, which are referred to as small mesoscale areas (SMSA). Rainfall is more intense within the SMSA's because of the clustering of rain cells, in accordance with a Neyman-Scott process. The spatial distribution of rainfall clusters within the LMSA is taken as a Poisson process. The number of spatial points of rainfall within each SMSA is also determined by a Poisson distribution, with rainfall intensity being an exponentially distributed variable.

Topographical data from two basins, namely, Macks Creek and Taylor Arroyo, were used in the tests. The Macks Creek watershed is located in mountainous southwestern Idaho, covers an area of 31.64 km^2, and has a mean slope of 9.4%. The Taylor Arroyo watershed in southeastern Colorado covers an area of 121 km^2 and has a mean watershed slope of 1.6%. Both watersheds are located in semi-arid regions and are dominated by Hortonian runoff production mechanisms.

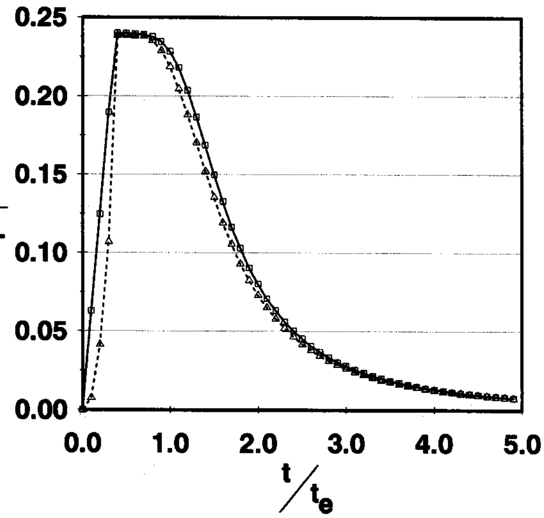

Fig. 1*a*. Hydrograph envelope for $t_r/t_e = 0.4$ and $\delta t/t_e = 0.1$.

Infiltration was neglected in this study. It is impossible to separate the sole effect of rainfall spatial or temporal variability on runoff in the presence of spatially and temporally varying infiltration. By neglecting infiltration, all rainfall is taken as excess rainfall, effectively eliminating separate consideration of the effects of spatial and temporal variability of rainfall and infiltration rate. Furthermore, the selected data analysis method requires conservation of outflow mass, precluding the consideration of infiltration. The applicability of our results is limited to runoff situations where either soils are of very low permeability or rainfall rates are much greater than infiltration rates.

Channel routing was neglected and all flow was modeled as overland flow. The inclusion of channel routing would have added considerable computational effort to this study. The presence of channels would modify the value of t_e for a given rainfall rate. Similarly, the value of the overland flow roughness coefficient affects t_e. The value of Manning's n used in this study was held constant throughout the basin and was equal to 0.04. All results are expressed in terms of t_e, thus enabling the conversion of the results to different runoff geometries which include channel routing or spatially variable roughness coefficient.

HYDROGRAPH ENVELOPES

The hydrograph envelope procedure used by *Julien and Moglen* [1990] was selected to reduce the output data into an interpretable form. Hydrograph envelopes arising from spatially variable rainfall on a one-dimensional runoff plane can be seen in that paper. A hydrograph envelope is generated by finding, for each set of 50 equivalent systems, the highest and lowest simulated discharges at each computational time step. These envelopes show the maximum variation in discharge as a function of time along the hydrograph. Four typical hydrograph envelopes arising from temporally varying rainfall are shown in Figures 1*a* and 1*b* for partial equilibrium hydrographs ($t_r < t_e$) and in Figures 1*c* and 1*d* for complete equilibrium hydrographs ($t_r \geq t_e$). In these

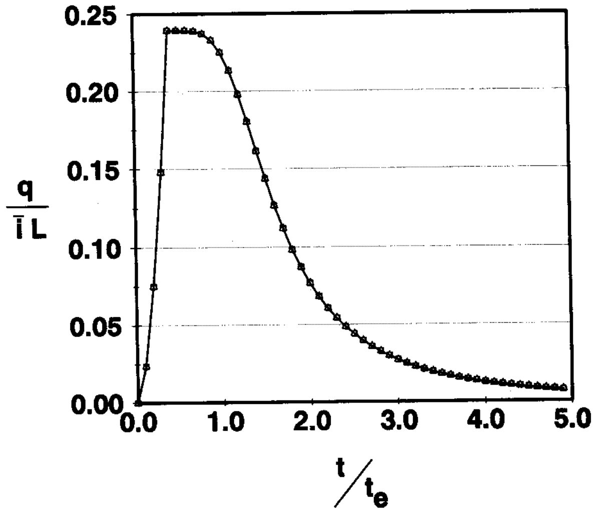

Fig. 1*b*. Hydrograph envelope for $t_r/t_e = 0.4$ and $\delta t/t_e = 1.1$.

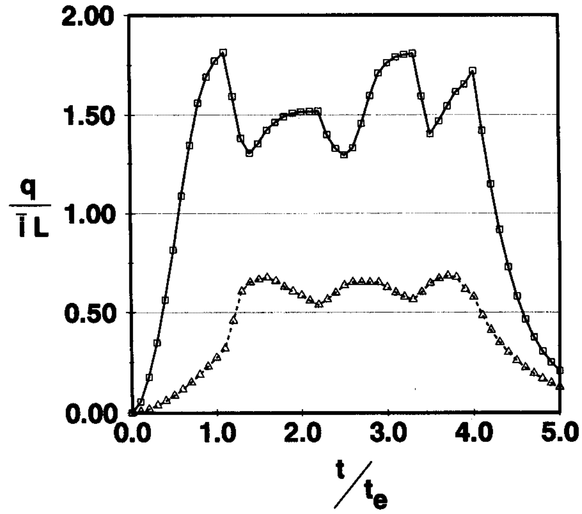

Fig. 1*d*. Hydrograph envelope for $t_r/t_e = 4.0$ and $\delta t/t_e = 1.1$.

four hydrograph envelopes the discharge is made dimensionless by dividing by the steady state system inflow $\bar{i}L$, where $\bar{i}$ represents the mean rainfall rate and L is the overland flow plane horizontal projection.

Sensitivity, a measure of the model response to a change in an input parameter, is used to determine the importance of parameters and to optimize parameter values within a model. With an understanding of the general behavior of the model response to temporal input variability, the goal is to quantify the temporal sensitivity of the model output to each value of temporal sampling resolution $\delta t/t_e$.

With reference to Figures 1*a* through 1*d*, the volume of runoff ΔV, contained within the upper discharge $q_{\max}$ and the lower discharge $q_{\min}$ hydrograph envelopes, is obtained by

$$\Delta V = \sum_{\tau=0}^{\infty} (q_{\max} - q_{\min})_\tau w \Delta t \tag{5}$$

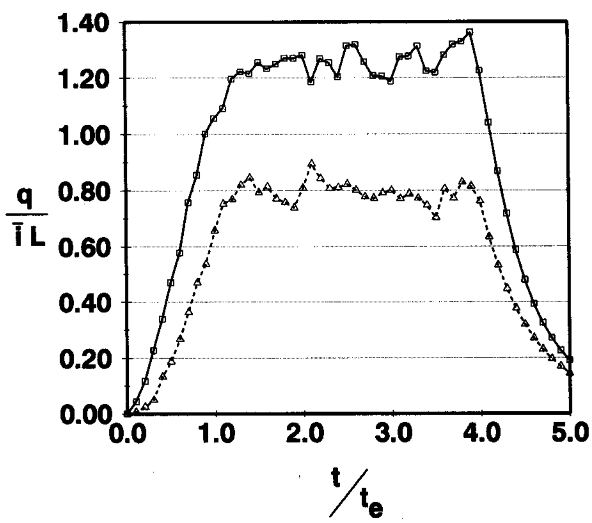

Fig. 1*c*. Hydrograph envelope for $t_r/t_e = 4.0$ and $\delta t/t_e = 0.1$.

where w is the runoff plane width and Δt is the computational time step.

The dimensionless hydrograph envelope volume V^* for the hydrograph envelopes is calculated using

$$V^* = \Delta V/V_r \tag{6}$$

where V_r is the input rainfall volume and is given by:

$$V_r = wL\delta t \sum_{\tau=0}^{t_r} i(\tau) \tag{7}$$

The magnitude of the dimensionless hydrograph envelope volume is indicative of the variability of the 50 Monte-Carlo outflow hydrographs used to generate the envelope.

The coefficient of variation C_v is defined as

$$C_v = \sigma/\mu \tag{8}$$

where σ is the standard deviation of the parameter of interest (in this case the standard deviation of the precipitation time series) and μ is the algebraic mean of the parameter. The value of C_v, 0.319, used throughout these analyses, is the theoretical value for a discrete uniform distribution. Relative sensitivity R_s, which represents the outflow variability normalized by input variability, is represented mathematically by

$$R_s = V^*/C_v \tag{9}$$

This quantity is called the relative sensitivity because the factors μ and V_r in the denominators of (6) and (8) make R_s a dimensionless quantity which is independent of the units used to measure μ and V_r.

The normalized relative sensitivity R_s^*, which represents the R_s function divided by its value at $t_r/t_e = 1.0$, is given by

$$R_s^* = \frac{R_s(t_r/t_e)}{R_s(t_r/t_e = 1.0)} \tag{10}$$

Normalized relative sensitivity is useful for comparing the shape of the R_s versus t_r/t_e function for different runoff geometries.

EFFECTS OF TWO-DIMENSIONAL SPATIAL VARIABILITY OF RAINFALL ON RUNOFF

The objective of this portion of the study is to examine the sensitivity of two-dimensional runoff geometries to the spatial variability of precipitation intensity, an extension of the study by *Julien and Moglen* [1990]. Particularly, this study seeks to verify the findings of the 1990 study using two-dimensional runoff simulations.

A Monte Carlo ensemble of 50 precipitation fields was generated using the precipitation model at the resolution of the watershed elevation data, which is 125 m for Macks Creek and 200 m for Taylor Arroyo. The precipitation model rainfall fields all have similar first- and second-order statistics and identical spatial covariance functions. The precipitation data were applied to the runoff model for different durations of rainfall t_r/t_e to find the relation between R_s and rainfall duration.

Experimental Method

The 50 precipitation fields for each watershed were adjusted to have the same average intensity of 30 mm/hour over the entire watershed by the addition of a constant offset. This adjustment scheme preserved the spatial gradients of rainfall intensity, while providing an equal average intensity. The other precipitation field statistic of interest is the coefficient of variation. The 50 precipitation fields have an average C_v of 1.069 on Macks Creek and 1.099 on Taylor Arroyo.

The testing scheme used the duration of rainfall, defined as a fraction of the time to equilibrium t_r/t_e, as the primary test variable. The values of t_r/t_e tested were 0.1, 0.2, 0.3, 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.8, 2.2, 2.5, 3.0, and 5.0. Hydrograph envelopes were constructed from the 50 outflow hydrographs, as previously described, and the relative sensitivity R_s was calculated.

Two-Dimensional Spatial Variability Results

The distribution of peak outflow magnitude and timing as a function of rainfall duration reveals some aspects of the influence of spatial variability on R_s. These distributions are plotted in Figures 2*a* and 2*b* for the Macks Creek and Taylor Arroyo watersheds, respectively. For all rainfall durations, in no case did the hydrograph peak occur at a time shorter than half the time to equilibrium of the basin. The data also indicate that the variability in hydrograph timing is larger for short ($t_r/t_e < 0.3$) rainfall duration events than the variability in the peak discharge. The variability in peak discharge increases and the variability in peak timing decreases as the rainfall duration increases from $t_r/t_e = 0.3$ to 0.6. As the duration of rainfall increases to $0.8t_e$, the variability in both peak discharge and time to peak decreases, and the data tend to converge on the points $Q_p/Q_e = 1.0$ and $t_p/t_e = 1.0$ as $t_r \rightarrow t_e$. For rainfall durations greater than or equal to the time to equilibrium, the variability in outflow hydrograph peak and time to peak is very small. The distributions of hydrograph Q_p/Q_e and t_p/t_e appear very similar for both study watersheds although they are of considerably different sizes and slopes.

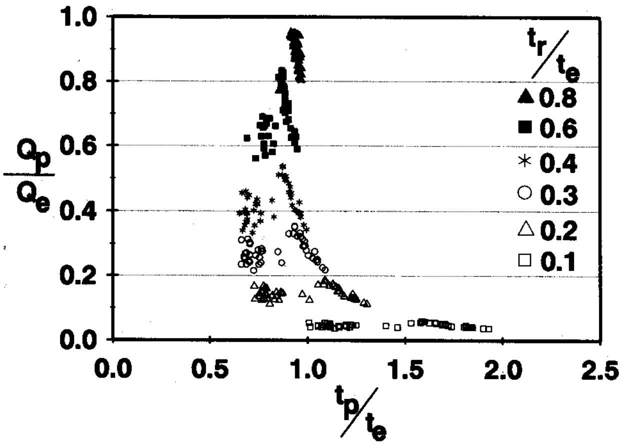

Fig. 2*a*. Dimensionless peak discharge versus dimensionless time to peak for Macks Creek due to spatial rainfall variability.

The relative sensitivity data show that the following runoff geometries are more sensitive to the spatial variability of precipitation in increasing order: 100 m runoff plane of *Julien and Moglen* [1990], Macks Creek watershed, and Taylor Arroyo watershed. The normalized relative sensitivities are plotted in Figure 3. These data provide evidence that the relative sensitivities of geometries as simple as a one-dimensional runoff plane and as complicated as a two-dimensional watershed are similar. The phenomena of runoff sensitivity to the spatial variability of precipitation intensity is therefore independent of scale under the conditions of this study. Of course, the absolute value of the sensitivity is surely geometry dependent; however, the phenomena itself is not. The only difference between the relative sensitivity of a one-dimensional runoff plane and a two-dimensional distributed basin to the variability in space of precipitation intensity appears to be a scaling factor. The scaling factor contains the effects of runoff geometry on the parameter R_s. The R_s scaling factors for the Macks Creek and Taylor Arroyo watersheds, on the basis of the one-dimensional runoff plane are 4.17 and 6.39, respectively.

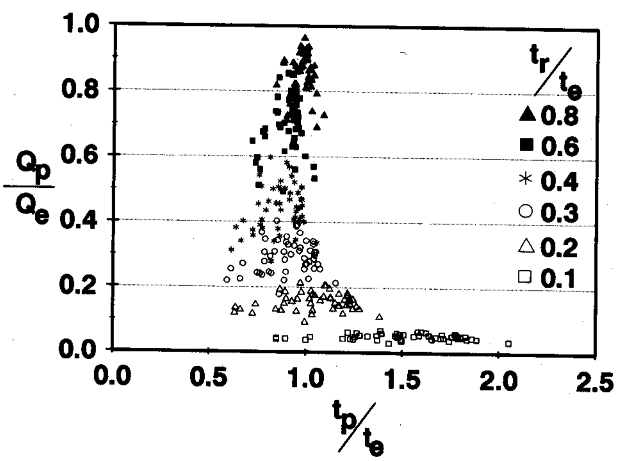

Fig. 2*b*. Dimensionless peak discharge versus dimensionless time to peak for Taylor Arroyo due to spatial rainfall variability.

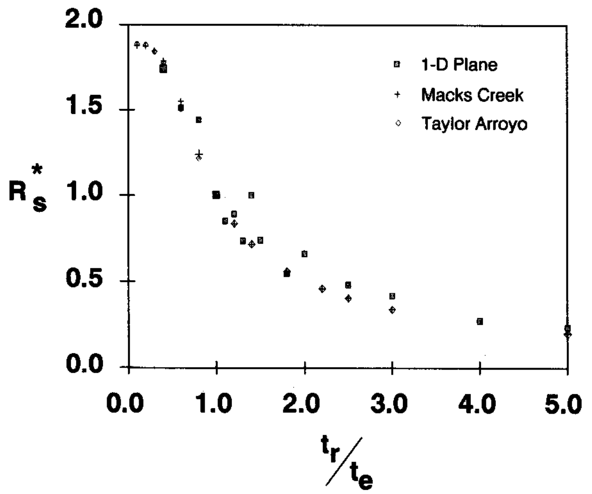

Fig. 3. Normalized R_s due to spatially varied rainfall on Macks Creek, Taylor Arroyo, and one-dimensional runoff plane.

One-Dimensional Runoff Sensitivity to Rainfall Temporal Variability

The one-dimensional overland flow model CASC was used to determine the effect of the temporal resolution of precipitation on computed outflow hydrographs. The specific parameters of interest were the storm duration t_r, the temporal resolution of precipitation δt, and the time to equilibrium of the overland flow plane t_e. The average rainfall rate over the duration of the rainfall was used to calculate t_e. The testing methodology imposes systematic variability in rainfall duration t_r and temporal resolution of rainfall δt, while all other parameters are held constant. Both of these variables are nondimensionalized by the time to equilibrium t_e of the runoff plane.

The computational time step Δt must always be smaller than the temporal resolution δt of the precipitation data. The temporal resolution must also be evenly divisible by the time step. The computational time step $\Delta t = 0.1 t_e$ when $\delta t > 0.1 t_e$ or $0.04 t_e$ for finer values δt. This choice has been previously shown [*Julien and Moglen* 1990] to more than adequately define the rising limb of the discharge hydrograph. Eleven computational nodes are used to define all systems. Preliminary analyses by *Julien and Moglen* [1990] showed that discharge was relatively unaffected by the number of nodes used, provided there are a minimum of seven present. The length, width, slope, and Manning n values for the overland flow plane are assigned constant values of 100 m, 10 m, 0.1%, and 0.1, respectively. The precipitation intensity varies only in time, not in space. The mean rainfall intensity is 1×10^{-5} m/s (36 mm/hour).

Temporal Variation of Rainfall Intensity

The rainfall time series generation scheme uses a variation factor ϕ and a uniform probability density function. Strictly speaking, a methodology which ignores the time correlation structure in the timewise generation of rain series may not produce realistic rainfall time series. However, this scheme does capture some general behavior of rainfall variability, while maintaining consistency with the previous one-dimensional spatial variability study [*Julien and Moglen*, 1990]. Further refinements to this methodology would include a point rainfall model, which includes a temporal covariance function.

The parameter ϕ describes the maximum allowable variation from the mean value using the following relation:

$$\mu_x\left(1 - \frac{\phi - 1}{\phi + 1}\right) \le X_j \le \mu_x\left(1 + \frac{\phi - 1}{\phi + 1}\right) \qquad (11)$$

where μ_x is the mean value of parameter X, and X_j is the value of parameter X at the jth time. The parameter ϕ should always be chosen to be greater than or equal to one, where $\phi = 1$ indicates no perturbations. The value of ϕ used in this study was 4 and was selected so that the results of this temporal resolution study could be compared with the study of *Julien and Moglen* [1990], which used the same ϕ value, regarding spatial variability of the overland flow parameters. The value $\phi = 4$ produces precipitation intensities ranging from a maximum of 57.6 to a minimum of 14.4 mm/hour.

The above temporal variation scheme was used to generate 50 unique realizations with the same average precipitation intensity, coefficient of variation of the precipitation time series, duration of rainfall, and temporal sampling resolution of the precipitation. Simulations were performed for 17 different durations of rainfall: t_r/t_e = 0.4, 0.6, 0.8, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.8, 2.0, 2.5, 3.0, 4.0, 5.0, 7.0, and 10.0. For the temporal sampling resolution of the precipitation data, 14 different values were tested. These values of $\delta t/t_e$ were 0.04, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 2.0, and 3.0. The storm duration values cover the range from partial to complete equilibrium. Similarly, the temporal resolution values range from much smaller ($0.04 t_e$) than the time to equilibrium to three times the time to equilibrium. For each permutation of the above conditions, 50 simulations were performed. This number was found to yield fairly smooth results in the bounds of all 50 hydrographs when plotted together. A total of 11,900 hydrographs were simulated over all possible permutations of the above conditions.

One-Dimensional Temporal Variability Results

Several observations can be made regarding the sensitivity of the overland flow plane to the temporal resolution of the input data from the four hydrograph envelopes shown in Figures 1*a*–1*d*.

1. When t_r/t_e is less than 1.0, the result is a partial equilibrium hydrograph, with the highest simulated discharge occurring at the last time interval experiencing rainfall, followed by a long recession limb (see Figures 1*a* and 1*b*). The region enclosed by the hydrograph envelope in Figure 1*a* is much smaller than that for t_r/t_e = 4.0 (Figure 1*c*). It is therefore apparent that the sensitivity of overland flow to temporal resolution of the precipitation data is significantly higher for $t_r > t_e$.

2. There is no difference between the maximum and minimum hydrograph envelope in Figure 1*b*. This occurs because the temporal sampling resolution is longer than the time of rainfall. Therefore variation in the outflow hydrograph due to sampling resolution effects cannot be observed. This is, in essence, equal to the constant temporal intensity assumption which is used in many runoff models.

3. When $t_r/t_e \gg 1$ (e.g., t_r/t_e = 4.0), the result is a

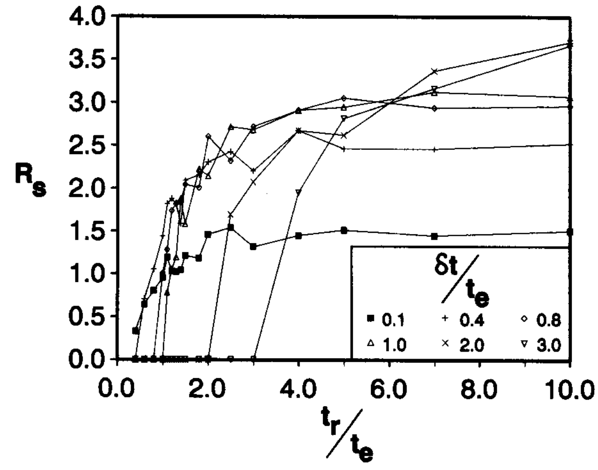

Fig. 4. Relative sensitivity to temporal variability for a one-dimensional runoff plane.

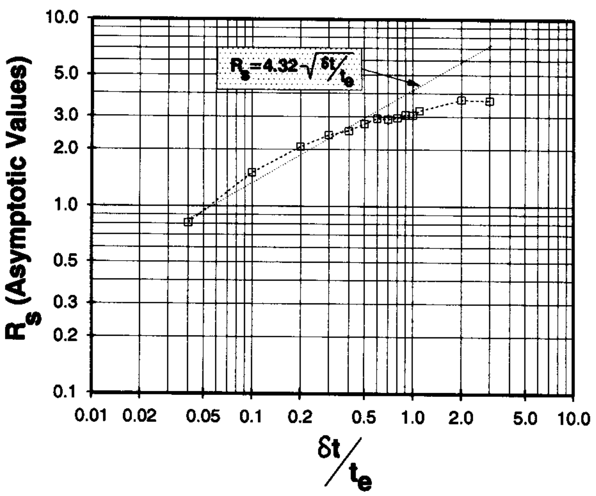

Fig. 5. Asymptotic relative sensitivity values for a one-dimensional runoff plane at $t_r = 10.0t_e$ and various values of $\delta t/t_e$.

complete equilibrium hydrograph, characterized by a non-equilibrium plateau region where the dimensionless discharge varies around 1.0 from $t/t_e \approx 1.0$ to $t = t_r$ (see Figures 1*c* and 1*d*). Temporal variation in the rainfall rate causes the flow from the runoff plane to seek a new equilibrium state after each change in rainfall intensity. This is the reason for the extreme outflow variability around the dimensionless discharge of 1.0.

4. Examination of the hydrograph envelopes indicates that the value of $\delta t/t_e$ has a large effect on the width of the hydrograph envelope when $t_e < t < t_r$. When the temporal resolution is smaller than the time to equilibrium, the hydrograph envelopes tend to be smoother (Figure 1*c*). Values of δt near or greater than t_e produce highly variable hydrograph envelopes where the variability in outflow volume is considerably greater (Figure 1*d*).

The values of R_s for various temporal resolutions are plotted in Figure 4 as a function of t_r/t_e. It is apparent that the magnitude of relative sensitivity increases rapidly for smaller values of t_r/t_e and then changes slowly for larger values of t_r/t_e. In other words, the change in variability in discharge is largest under partial equilibrium conditions and smallest under complete equilibrium conditions. The data in Figure 4 also indicate that the relative sensitivity of the overland flow plane approaches an asymptote as t_r/t_e increases. This can be explained by considering that as t_r/t_e increases considerably beyond 1.0, the upper and lower bounds of the hydrograph envelope becomes roughly parallel. Because the relative sensitivity is an integrated measure of sensitivity, the irregular envelope volumes which occur during the rising and falling limbs of the hydrograph become negligible as $t_r \gg t_e$. This tends to cause R_s to approach an asymptote.

For a given coefficient of variation of the rainfall time series, the asymptotic R_s is thus a function of $\delta t/t_e$, as is shown on Figure 5. Figure 5 shows the dependence of the maximum value of the relative sensitivity on the temporal resolution of the input data. One possible inference from this figure is that the dependence of R_s on $\delta t/t_e$ is not simply a power function. The curvature exhibited in the data on the log-log plot indicates that other variables may play a role. Another, more likely, explanation is that the relative sensitivity reaches a maximum at a value of $\delta t/t_e = 1.0$. This results from the testing methodology employed, which forces the rainfall intensity to vary between two bounds. As $\delta t/t_e$ increases beyond 1.0, the upper and lower limits of the hydrograph become approximately parallel, as in the case of $t_r \gg t_e$.

Two-Dimensional Runoff Sensitivity to Rainfall Temporal Variability

In this portion of the study, the CASC2D runoff model [*Julien and Saghafian* 1991] is used to examine the effects of temporal resolution of rainfall data on two-dimensional surface runoff from a natural watershed. Macks Creek watershed elevation data at 152 m resolution was used as input to the square grid CASC2D runoff model without infiltration or channel routing, as was previously discussed.

A series of simulations were performed to determine the relationship between rainfall intensity and time to equilibrium. Rainfall intensities from 1 to 100 mm/hour were used. The actual equilibrium discharge is an asymptotic value that the outflow discharge approaches as simulation time increases indefinitely. For this reason, 98.5% of the equilibrium discharge was selected to determine the time to equilibrium. The following time to equilibrium relation (12) was determined, with t_e in minutes and i in millimeters per hour:

$$t_e = 497.3i^{-0.428} \tag{12}$$

The two-dimensional methodology is very similar to that used in the one-dimensional analysis. The duration of rainfall t_r was predetermined as a fraction of the time to equilibrium t_e of the basin. As in the one-dimensional study, the precipitation intensity was applied uniformly in space to the entire watershed, while varied in time using a uniform distribution. The same coefficient of variation was chosen for this study on Macks Creek as was applied in the one-dimensional study. Values of t_r/t_e tested were 0.2, 0.4, 0.6, 0.8, 1.0, 1.2, 1.5, 2.0, and 5.0. Values of $\delta t/t_e$ tested included 0.005, 0.01, 0.04, 0.08, 0.16, 0.25, and 0.50.

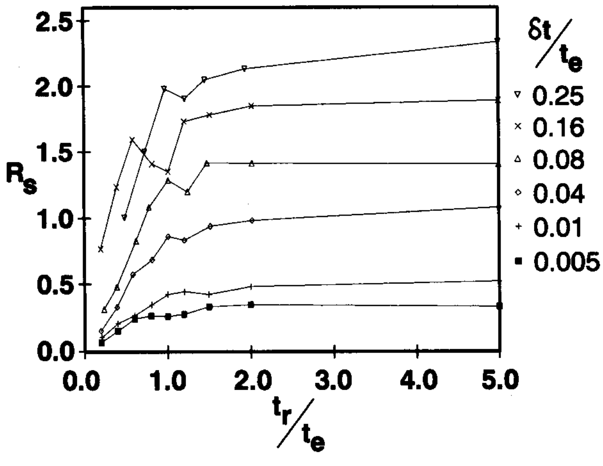

Fig. 6. Relative sensitivity to temporal variability for Macks Creek watershed.

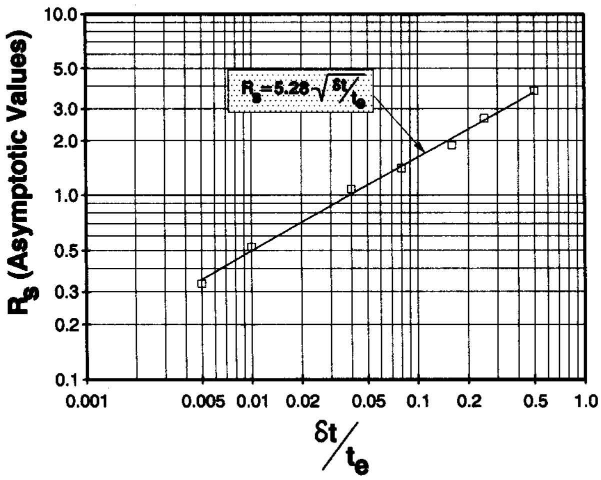

Fig. 7. Asymptotic relative sensitivity values for Macks Creek watershed at $t_r = 5.0t_e$ and various values of $\delta t/t_e$.

Two-Dimensional Temporal Variability Results

The results of this study indicate that the two-dimensional runoff geometry is sensitive to the temporal resolution of precipitation intensity much like the one-dimensional runoff plane in the previous study. The primary difference is the magnitude of the sensitivity. Because of the relatively large time to equilibrium of the two-dimensional system, smaller values of $\delta t/t_e$ were tested on Macks Creek than in the one-dimensional simulations. The hydrograph envelopes produced are very similar to those shown in Figures 1*a* through 1*d*. The relative sensitivity, as shown in Figure 6, increases with both rainfall duration and temporal sampling interval.

From the data in Figure 6 it is apparent that the relative sensitivity to the precipitation data temporal resolution increases substantially as the storm duration increases. This increase is, however, limited. The greatest portion of the increase in relative sensitivity occurs before $t_r/t_e = 2.0$. Beyond this rainfall duration, the increase in R_s is minor. In fact, it appears as though an asymptote is reached as in the one-dimensional study. These asymptotic values represent the maximum relative sensitivity of the model for a given temporal resolution. The asymptotic values from all simulations with $t_r/t_e = 5.0$ are plotted in Figure 7. A regression of the data in Figure 7 show that the relative sensitivity of the Macks Creek watershed to the temporal sampling resolution of the precipitation input data for storms with duration at least 5 times the time to equilibrium is a power function. This function is shown on Figure 7.

Conclusions

Two-dimensional watershed sensitivity to rainfall spatial variability was explored using the CASC2D runoff model and elevation data from two watersheds. Spatially variable precipitation fields of identical statistics were generated for each watershed using a precipitation model [*Rodriguez-Iturbe and Eagleson*, 1987]. A total of 50 spatially variable rainfall fields were applied to each watershed using a Monte Carlo methodology, for increasing values of rainfall duration t_r/t_e. Results from two-dimensional simulations are compared with one-dimensional results published by *Julien and Moglen* [1990].

The sensitivities of one- and two-dimensional runoff surfaces to temporally varying rainfall were explored. A uniform probability density function was applied to simulate rainfall rate time series in both sets of simulations. The relative sensitivity R_s of both overland flow geometries was examined with respect to the temporal resolution of the rainfall data $\delta t/t_e$ and the duration of rainfall t_r/t_e. The relative sensitivity R_s in both one- and two-dimensional simulations was found to increase with t_r/t_e and $\delta t/t_e$. As t_r exceeds t_e, an asymptotic value of R_s is approached. Asymptotic relative sensitivity values increase approximately with $(\delta t/t_e)^{1/2}$ in both one- and two-dimensional simulations.

With regard to the results, the following conclusions are stated.

1. The duration of spatially varied rainfall on two impervious watersheds affects the magnitude of the relative sensitivity R_s. Simulations using CASC2D under statistically equivalent rainfall fields indicate that R_s decreases with increasing t_r. This finding confirms the one-dimensional simulation results by *Julien and Moglen* [1990]. Curves shown in Figure 3 of R_s versus t_r/t_e normalized by the R_s value at $t_r/t_e = 1.0$ show similarity in the effect of rainfall duration on R_s, independent of watershed size.

2. The effect of temporally variable rainfall on the relative sensitivity R_s of a one-dimensional overland flow plane increases as the temporal sampling interval δt and rainfall duration t_r increase. Asymptotic values of R_s for $t_r \gg t_e$ increase approximately with the square root of $\delta t/t_e$ as shown in Figure 5, for $\delta t/t_e < 1.0$.

3. The effect of the temporal resolution of rainfall data on the R_s for two-dimensional runoff is similar to that for the one-dimensional overland flow plane. Results in Figure 7 show that the asymptotic R_s increases proportionally to the square root of $\delta t/t_e$. The magnitude of R_s also increases with t_r and δt as in the one-dimensional case.

4. The duration of rainfall has opposite effects on the relative sensitivity of runoff to spatial and temporal rainfall variability. The relative sensitivity to spatial variability was found to decrease as the duration of rainfall increases, while

the relative sensitivity to temporal variability increases. Specifically, as the duration of rainfall increases beyond t_e, the relative spatial sensitivity becomes quite small, while the relative temporal sensitivity approaches a large asymptotic value, for all values of $\delta t/t_e \geq 0.01$.

NOTATION

C_v coefficient of variation, dimensionless.
L overland flow plane length, L.
Q total flow rate, L^3/t.
Q_e equilibrium total flow rate, L^3/t.
Q_p peak total flow rate, L^3/t.
R_h hydraulic radius, L.
R_s relative sensitivity, dimensionless.
S_f slope of energy grade line, dimensionless.
S_{fx} slope of energy grade line in the x direction, dimensionless.
S_{fy} slope of energy grade line in the y direction, dimensionless.
S_o slope of overland flow plane, dimensionless.
U two-dimensional velocity magnitude, L/t.
V^* dimensionless hydrograph envelope volume, dimensionless.
V_r volume of rainfall, L^3.
e excess rainfall rate, L/t.
f infiltration rate, L/t.
g acceleration due to gravity, L^2/t.
h overland flow depth, L.
i rainfall intensity, L/t.
$\bar{i}$ rainfall intensity averaged over time, L/t.
n Manning's roughness coefficient.
q unit discharge, L^2/t.
t_e time to equilibrium, t.
t_p time of hydrograph peak discharge, t.
t_r duration of rainfall, t.
u flow velocity in x direction, L/t.
$\bar{u}$ average flow velocity, L/t.
v flow velocity in y direction, L/t.
α empirical coefficient for stage-discharge relation.
β empirical exponent for stage-discharge relation.
δt temporal sampling resolution, t.
μ mean value.
σ standard deviation.
Δt computational time step, t.
ΔV hydrograph envelope volume, L^3.
ϕ variation factor, dimensionless.

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REFERENCES

De St. Venant, B., Theory of unsteady water flow, with application to river floods and to propagation of tides in river channels, English Transl., *Rep. 49-g*, Waterw. Exp. Stat., U.S. Army Corps of Eng., Vicksburg, Miss., 1949.

Diskin, M. H., G. Wyseure, and J. Feyen, Application of a cell model to the Bellebeek watershed, *Nord. Hydrol., 15*, 25–38, 1984.

Erichsen, B., and K. Nordseth, Runoff models—Do they tell what actually happens!, *Nord. Hydrol., 15*, 273–282, 1984.

Green, W. H., and G. A. Ampt, Studies on soil physics, 1, The flow of air and water through soils, *J. Agric. Sci., 4*, 1–24, 1911.

Hjelmfelt, A. T., Jr., Overland flow from time-distributed rainfall, *J. Hydraul. Div. Am. Soc. Civ. Eng., HY2*, 227–238, 1981.

James, W. P., and K. W. Kim, A distributed dynamic watershed model, *Water Resour. Bull., 26*(4), 587–596, 1990.

Jensen, M., Runoff patterns and peak flows from moving block rains based on a linear time-area curve, *Nord. Hydrol., 15*, 155–168, 1984.

Julien, P. Y., and G. E. Moglen, Similarity and length scale for spatially varied overland flow, *Water Resour. Res., 26*(8), 1819–1832, 1990.

Julien, P. Y., and B. Saghafian, CASC2D users manual—A two-dimensional watershed rainfall-runoff model, *Civ. Eng. Rep. CER90-91PYJ-BS-12*, 66 pp., Colo. State Univ., Fort Collins, 1991.

Julien, P. Y., G. E. Moglen, and G. U. Sunada, CASC users manual-A finite element model for spatially varied overland flow simulation, *Cent. for Geosci. Rep. CER 87-88 GEO7*, Colo. State Univ., Fort Collins, September 1988.

Krajewski, W. F., V. Lakshmi, K. P. Georgakakos, and S. C. Jain, A Monte Carlo study of rainfall sampling effect on a distributed catchment model, *Water Resour. Res., 27*(1), 119–128, 1991.

Niemczynowicz, J., Investigation of the influence of rainfall movement on runoff hydrographs, *Nord. Hydrol., 15*, 57–84, 1984.

Ogden, F. L., Two-dimensional runoff modeling with weather radar data, Ph.D. dissertation, 211 pp., Colo. State Univ., Fort Collins, 1992.

Ponce, M. V., and R. M. Li, Overview of flood routing methods, in *Analysis Of Watershed and River Systems*, edited by D. B. Simons and R. M. Li, Colorado State University, Fort Collins, 1979.

Ponce, M. V., R. M. Li, and D. B. Simons, Applicability of the kinematic and diffusion models, *J. Hydaul. Div. Am. Soc. Civ. Eng., 104*(HY3), 353–360, 1978.

Richardson, J. R., The effect of moving rainstorms on overland flow using one-dimensional finite elements, Ph.D. dissertation, 239 pp., Colo. State Univ., Fort Collins, 1989.

Rodriguez-Iturbe, I., and P. S. Eagleson, Mathematical models of rainstorm events in space and time, *Water Resour. Res., 23*(1), 181–190, 1987.

Storm, B., and K. H. Jensen, Experience with field testings of SHE on research catchments, *Nord. Hydrol., 15*, 283–294, 1984.

Woo, D., and E. F. Brater, Spatially varied flow from controlled rainfall, *J. Hydraul. Div. Am. Soc. Civ. Eng., 88*(HY6), 31–56, 1962.

Wood, E. F., M. Sivapalan, K. Bevin, and L. Band, Effects of spatial variability and scale with implications to hydrologic modeling, *J. Hydrol., 102*, 29–48, 1988.

Yen, B. C., and V. T. Chow, A study of surface runoff due to moving rainstorms, *Hydrol. Eng. Ser. Rep. 17*, Dep. of Civ. Eng., Univ. of Ill., Urbana, June 1968.

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