Alluvial Channel Geometry: Theory and Applications[a]

Discussion by Willi H. Hager,[3] Fellow, ASCE

The discusser would like to congratulate the authors for their interesting approach to the generalized regime formulas. The purpose of this discussion is (1) to generalize the authors' findings; and (2) to give an explicit approach for the flow depth h.

Eqs. (28)–(31) are used as the basis of this discussion. The common denominator of all exponents is $\gamma = 5 + 6m$, and one may show that

$$\frac{1}{g}\left(5\frac{h}{d_s}\right)^{\gamma} = q^2 \tag{32}$$

$$\frac{1}{g}\left(\frac{3}{4}\frac{W}{d_s}\right)^{\gamma/(1+2m)} = q^2 \tag{33}$$

$$\frac{g^{\gamma/(1+2m)}}{g}\left(\frac{\bar{U}^2}{3.76^2 gSd_s}\right)^{\gamma/(1+2m)} = q^2 \tag{34}$$

$$\frac{1}{g}\left(\frac{\tau_\theta^*}{0.121S}\right)^{\gamma} = q^2 \tag{35}$$

where $q^2 = Q^2/(gSd_s^5)$ is the square of a sediment Froude number. The scaling length of both the flow depth h and the river width W is thus the mean sediment diameter d_s. For the average velocity, the expression is somewhat more difficult because the dimensional coefficient $g^{[\gamma/(1+2m)-1]}$ should vanish in a correct system of equations, that is, $m = -1$. The dimensionless Shields number should be scaled with the bottom slope S, such that the dominant parameter is $\tau_\theta^* = Kh\rho/[(\rho_s - \rho)d_{50}]$.

Eq. (32) can be further simplified because γ is a function of h/d_s only, that is, from (4)

$$\gamma = 5 + 6/[\ln(12.2h/d_s)] \tag{36}$$

Using for $g = 9.81\ \mathrm{m/s^2}$ gives $q = f_1(h/d_s)$. Table 2 lists this function and one may note that q increases very rapidly with h/d_s.

TABLE 2. Variation of q with h/d_s according to (32) and (36)

h/d_s (1)	q (2)
10^0	1.23×10^2
10^1	6.5×10^4
10^2	2.46×10^7
10^3	8.53×10^9
10^4	2.85×10^{12}

The function under consideration may be approximated to ±20% in the interval of Table 2 and to ±5% for $1 < h/d_s < 10^2$ as

$$\log(h/d_s) = 0.375\log(q/123) \tag{37}$$

or

$$h/d_s = (q/123)^{0.375} \tag{38}$$

Accordingly, the highly implicit function f_1 is explicit, and one has not to follow the authors' computational procedure. From (38) it can be seen that h is mainly influenced by the grain diameter, slightly by the discharge and only to a small extent by the bottom slope.

With the example of the author, for instance, one knows $Q = 104\ \mathrm{m^3/s}$, $d_s = 0.056$ m, and $S = 2.87 \times 10^{-3}$, based on the value of τ_θ^*. Therefore, $q = 8.352 \times 10^5$. The "exact solution" obtains $h/d_s = 26.75$, and $h/d_s = 27.35$ (+2%) results according to (37). Therefore, $\gamma = 6.03$ and $W/d_s = 966$, or $W = 54.1$ m; and $\bar{U} = 1.28$ m/s.

It is necessary to add here that in the authors' example, the numbers for the width and the average velocity are wrong. The correct equations should read

$$W = 1.33Q^{(2+4m)/(5+6m)}d_s^{-4m/(5+6m)}S^{-(1+2m)/(5+6m)} \tag{29}$$

and

$$\bar{U} = 3.76Q^{(1+2m)/(5+6m)}d_s^{-2m/(5+6m)}S^{(2+2m)/(5+6m)} \tag{30}$$

[a]April, 1995, Vol. 121, No. 4, by Pierre Y. Julien and Jayamurni Wargadalam (Paper 7171).

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Discussion by H. Q. Huang[4]

In an excellent review of the existing methods for the determination of alluvial channel geometry, the authors identify the limitations of simplified one-dimensional analyses of flow and sediment transport in alluvial channels. They then proceed to develop an alternative method by introducing a two-dimensional flow equation, using the secondary flow equation, (11), and the Shields number τ_θ^* as the mobility index of noncohesive particles. The exponents of flow discharge in the derived hydraulic geometry equations vary well in the ranges of those empirically established relationships, and an acceptable agreement between a very large set of field and laboratory measurements and the corresponding calculations from the derived equations is achieved.

Of the derived channel geometry relations developed by the authors, their (28)–(30) are of particular interest to the writer. Because exponent m has a varying value of 0.0–0.5, these equations can then be written in the following numerical forms:

$$W = k_w Q^{0.4-0.5}d_s^{-(0.0-0.25)}S^{-(0.2-0.25)} \tag{39}$$

$$h = k_h Q^{0.4-0.25}d_s^{0.0-0.375}S^{-(0.2-0.125)} \tag{40}$$

$$\bar{U} = k_{\bar{U}} Q^{0.2-0.25}d_s^{-(0.0-0.125)}S^{0.4-0.375} \tag{41}$$

where coefficients k_w, k_h, and $k_{\bar{U}}$ were found by the authors to hold average values of 1.33, 0.2, and 3.76, respectively.

Eqs. (39)–(41) are very similar to those obtained by Huang and Warner (1995) in their investigation (from a very large set of field observations) of the applicability of an experimental relationship between channel shape and boundary shear distribution. The downstream hydraulic geometry relations established by Huang and Warner are

$$W = k'_w Q^{0.5}n^{0.355}S^{-0.156} \tag{42}$$

$$h = k'_h Q^{0.3}n^{0.383}S^{-0.206} \tag{43}$$

$$\bar{U} = k'_{\bar{U}} Q^{0.2}n^{-0.738}S^{0.362} \tag{44}$$

where coefficients k'_w, k'_h, and $k'_{\bar{U}}$ have average values of 4.059, 0.427, and 0.576, respectively, but are related well to bank material (Huang and Nanson 1995).

It is interesting to note that the above two sets of hydraulic geometry relationships are highly consistent in describing the

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effects of flow discharge and channel slope. Since the Manning's roughness n is in direct proportion to $d_s^{1/6}$ in certain circumstances [e.g., Chow (1959)], the above two sets of relationships are also consistent in interpreting the effects of channel roughness or sediment size on channel depth and velocity. The inverse effect of channel roughness or sediment size on channel width can be accounted for by the use of different flow resistance relations and a different additional flow regime relationship, such as the secondary flow equation, (11), used by the authors, but a relationship between channel shape and boundary shear distribution is used by Huang and Warner (1995).

Furthermore, it can also be found that the coefficients in the above two sets of hydraulic geometry relations actually possess consistent physical meanings. This is due to the finding by the authors that the coefficients C_h, C_w, and C_U defined by (20)–(22) vary in the comparatively small ranges. According to (20)–(22), and considering the effects of only major variables, the following relations can be derived:

$$C_h \propto \left[\left(\frac{h}{d_s}\right)^p \cdot \frac{h}{w}\right]^{1/(2+m+p)} \quad (45)$$

$$C_w \propto C_h^{-(1+m)}; \quad C_U \propto C_h^m \quad (46)$$

Furthermore, because the coefficients C_h, C_w, and C_U exhibit very little variation, factor ζ varies in a limited range, where ζ is defined by

$$\zeta = \left(\frac{h}{d_s}\right)^p \cdot \frac{h}{W} \quad (47)$$

Eq. (47) can also be written in the following form:

$$\frac{W^{1/(1+p)}}{h} = \zeta^{-1/(1+p)} \cdot d_s^{-p/(1+p)} = \zeta' \quad (48)$$

For $p = 2m$ and $0 < m < 0.5$, as suggested by the authors, (48) can then be expressed as

$$\frac{W^{1.0-0.5}}{h} = \zeta' \quad (49)$$

Eq. (49) is a very commonly observed relationship and factor ζ' can be related to channel sediment composition, bank vegetation, and, hence, bank strength. Typical examples of this aspect are the studies of Schumm (1969) and Hey and Thorne (1986).

Concerning the influence of sediment composition on channel geometry, Schumm (1969) obtained

$$W = 2.3\,\frac{Q^{0.38}}{M^{0.39}}; \quad h = 0.6Q^{0.29}M^{0.34} \quad (50)$$

where M = average percentage of silt-clay in the channel boundary.

Hey and Thorne (1986) considered the effect of bank vegetation on channel geometry and found

$$W = k_{\text{veg}}Q^{0.50}; \quad h = 0.22Q^{0.37}d_{50}^{-0.11} \quad (51)$$

where k_{veg} has values of 4.33, 3.33, 2.73, and 2.34 for grassy banks with no trees or bushes and the banks covered with 1–5%, 5–50%, and >50% tree/shrub, respectively.

Eliminating discharge Q from (50) and (51) results in

$$\frac{W^{0.763}}{h} = 3.147M^{-0.638} \quad (52)$$

$$\frac{W^{0.74}}{h} = (k_{\text{veg}}^{0.74}/0.22)d_{50}^{0.11} \quad (53)$$

In terms of (45)–(49) and (52)–(53), it can be inferred

$$C_h(C_U) \sim (M, k_{\text{veg}})^+; \quad C_w \sim (M, k_{\text{veg}})^- \quad (54)$$

According to the authors' method, k_w, k_h, and k_U in (39)–(41) have the relationships with C_w, C_h, and C_U as

$$k_h \propto C_h^{6m/(6m+5)}; \quad k_w \propto C_h^{-(2m+1)(3m+1)/(6m+5)}; \quad k_U \propto C_h^{[m(6m-1)+1]/(6m+5)} \quad (55)$$

Hence, for $0 < m < 0.5$ as suggested by the authors, it can be inferred from (54) and (55)

$$k_h(k_U) \sim (M, k_{\text{veg}})^+; \quad k_w \sim (M, k_{\text{veg}})^- \quad (56)$$

Eq. (56) means that channels with a high silt-clay content or with banks covered with trees or shrubs and hence having a high bank strength have a smaller k_w but a larger k_h and k_U, and vice versa. Obviously, the physical meanings of k_w, k_h, and k_U in (39)–(41) are consistent with those of k'_w, k'_h, and k'_U in (42)–(44) as studied by Huang and Warner (1995) and by Huang and Nanson (1995).

In summary, the two sets of downstream hydraulic geometry relations proposed by the authors and by Huang and Warner (1995) are essentially consistent, although two different approaches were followed in each study. Since the two studies each involves a very large set of field observations from different sources, both sets of hydraulic geometry relations are of general applicability and of practical use.

However, in investigating the applicability of their dejrived hydraulic geometry relations, the authors actually used equation (47) without giving a physical explanation of why their proposed secondary flow equation, (11), can be simplified into (47). Although this discussion has provided a physical rational basis for the use of (47), it is based on purely empirical observations; hence, the physical mechanisms that result in this simplification need to be justified.

Furthermore, the authors used the Einstein-Chien flow resistance equation, even though it only emphasizes the influence of the relative submergence of bed sediment on flow resistance. In reality, the flow resistance of alluvial channels is determined by many factors, such as channel irregularity, channel boundary sediment composition, vegetation, and the conditions of sediment transport [e.g., Chow (1959)]. Because of this, the discusser prefers to use Manning's flow resistance equation, for this procedure for evaluating Manning's coefficient has been well established for many practical circumstances [e.g., Barnes (1967)]. In addition, use of Manning's flow resistance equation means a constant m, and thus leaves only exponent p in (11) to be determined. This can be easily solved by applying a multivariate regression technique to the large set of field observations collected by the authors.

APPENDIX. REFERENCES

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Closure by Pierre Y. Julien,[5] Member, ASCE, and Jayamurni Wargadalam[6]

It is with delight that the interesting contributions of discussers Hager and Huang were received. Their content is substantial, and careful consideration called for a lengthy closure. Both lend considerable support to the proposed downstream hydraulic geometry relationships, and progress has certainly been achieved in better understanding alluvial river mechanics in support of empirical regime equations. Originating from antipodes, Hager emphasizes theoretical and computational aspects while Huang stresses field observations and practical use. We may have generated slight confusion in presenting two sets of equations in the paper. It is judged appropriate to explicitly outline our calculation procedure prior to formulating responses to individual discussions.

RECOMMENDED PROCEDURE

Our recommended procedure for the calculation of the downstream hydraulic geometry starts with the user selection of three independent variables. To include the effects of sediment transport, the user may want to calculate four dependent variables: average flow depth h, surface width W, average flow velocity $\bar{U}$, and equilibrium slope S as a function of three known independent variables in discharge Q in m^3/s, median grain size d_s in m, and dimensionless Shields number τ_θ^*. The equations (24)–(27) in the paper are solved with the five-step procedure outlined on pp. 321 and 322 of the paper. The calculation example in the paper is indeed correct. Values of the coefficients and exponents for values of $0 < m < 0.5$ are summarized in Table 3.

Another example of the procedure is given here for calculating the downstream hydraulic geometry given $Q = 104$ m^3/s, $d_{50} = 0.056$ m, and $\tau_\theta^* = 0.047$ at the beginning of motion:

1. Roughly estimate the flow depth; e.g., $h = 1$ m.
2. From the flow depth and grain size, calculate m from $m = 1/\ln(12.2h/d_s) = 0.186$.
3. Calculate the exponents b, c, and d for flow depth from Table 3, given $m = 0.186$: $h = aQ^b d_s^c \tau_\theta^{*d} = 0.133(104)^{0.39} (0.056)^{0.023} (0.047)^{-0.195} = 1.38$ m.
4. Repeat steps 2 and 3 with calculated flow depth until convergence: $m = 0.175$ gives $h = 0.149$ m, and $m = 0.172$ gives $h = 1.51$ m.
5. Calculate the channel width W, flow velocity $\bar{U}$, and slope S using the last value of m and the exponents of Q, d_s, and τ_θ^* in Table 3; e.g., with $m = 0.172$

$$W = 0.512(104)^{0.534}(0.056)^{-0.335}(0.047)^{-0.267} = 36.4 \text{ m}$$

$$\bar{U} = 14.7(104)^{0.068}(0.056)^{0.329}(0.047)^{0.466} = 1.87 \text{ m/s}$$

$$S = 12.4(104)^{-0.397}(0.056)^{0.994}(0.047)^{1.199} = 2.86 \times 10^{-3}$$

The user may prefer to use a different set of known independent variables. Eqs. (24)–(27) must then be rearranged through algebraic transformation isolating each dependent variable on the left-hand side as a function of power functions of the three user-selected independent variables. For instance, geomorphologists may prefer to calculate flow depth h, width W, mean velocity $\bar{U}$, and Shields number τ_θ^* as explicit functions of discharge Q in m^3/s, median grain size d_s in m, and channel slope S. Notice that the exponents of discharge and grain size in Table 4 for values of $0 < m < 0.5$ are not identical to those in Table 3 because exponents depend on the arbitrary selection of the third independent variable. The recalibrated (28)–(31) can be solved with the following procedure. To calculate the downstream hydraulic geometry given $Q = 104$ m^3/s, $d_{50} = 0.056$ m, and $S = 2.87 \times 10^{-3}$

1. Roughly estimate the flow depth, e.g., $h = 1$ m.
2. From the flow depth and grain size calculate m from $m = 1/\ln(12.2h/d_s) = 0.186$.
3. Calculate the exponents b, c, and d for flow depth from Table 4, given $m = 0.186$: $h = 0.2(104)^{0.327}(0.056)^{0.182} (0.00287)^{-0.163} = 1.40$ m.
4. Repeat steps 2 and 3 with calculated flow depth until convergence: $m = 0.175$ gives $h = 1.48$ m, and $m = 0.173$ gives $h = 1.50$ m.
5. Calculate the channel width W, flow velocity $\bar{U}$, and Shields number τ_θ^* using the last value of m and the exponents of Q, d_{50}, and S in Table 4; e.g., with $m = 0.173$

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TABLE 3. Downstream Hydraulic Geometry as Function of Q (m^3/s), d_s (m), and τ_θ^*

(1)	Coefficient a (2)	Discharge exponent b (3)	Grain size exponent c (4)	Shields number exponent d (5)
Flow depth h (m)	0.133	$1/(2 + 3m)$ $0.28 < b < 0.5$	$(-1 + 6)/(4 + 6m)$ $-0.25 < c < 0.28$	$-1/(4 + 6m)$ $-0.25 < d < -0.14$
Top width W (m)	0.512	$(1 + 2m)/(2 + 3m)$ $0.5 < b < 0.57$	$(-1 - 4m)/(4 + 6m)$ $-0.42 < c < -0.25$	$(-1 - 2m)/(4 + 6m)$ $-0.28 < d < -0.25$
Mean flow velocity $\bar{U}$ (m/s)	14.7	$m/(2 + 3m)$ $0 < b < 0.14$	$(2 - 2m)/(4 + 6m)$ $0.14 < c < 0.5$	$(2 + 2m)/(4 + 6m)$ $0.43 < d < 0.5$
Slope S	12.4	$-1/(2 + 3m)$ $-0.5 < b < -0.28$	$5/(4 + 6m)$ $0.71 < c < 1.25$	$(5 + 6m)/(4 + 6m)$ $1.14 < d < 1.25$

TABLE 4. Downstream Hydraulic Geometry as Function of Q (m^3/s), d_s (m), and Slope S

(1)	Coefficient a (2)	Discharge exponent b (3)	Grain size exponent c (4)	Slope exponent d (5)
Flow depth h (m)	0.2	$2/(5 + 6m)$ $0.25 < b < 0.4$	$6m/(5 + 6m)$ $0 < c < 0.375$	$-1/(5 + 6m)$ $-0.2 < d < -0.125$
Top width W (m)	1.33	$(2 + 4m)/(5 + 6m)$ $0.4 < b < 0.5$	$-4m/(5 + 6m)$ $-0.25 < c < 0$	$(-1 - 2m)/(5 + 6m)$ $-0.25 < d < -0.2$
Mean flow velocity $\bar{U}$ (m/s)	3.76	$(1 + 2m)/(5 + 6m)$ $0.20 < b < 0.25$	$-2m/(5 + 6m)$ $-0.125 < c < 0$	$(2 + 2m)/(5 + 6m)$ $0.375 < d < 0.4$
Shields number τ_θ^*	0.121	$2/(5 + 6m)$ $0.25 < b < 0.4$	$-5/(5 + 6m)$ $-1 < c < -0.625$	$(4 + 6m)/(5 + 6m)$ $0.8 < d < 0.875$

$$W = 1.33(104)^{0.446}(0.056)^{-0.115}(0.00287)^{-0.223} = 54.2 \text{ m}$$

$$\bar{U} = 3.76(104)^{0.223}(0.056)^{-0.057}(0.00287)^{0.388} = 1.29 \text{ m/s}$$

$$\tau_\theta^* = 0.121(104)^{0.331}(0.056)^{-0.828}(0.00287)^{0.834} = 0.047$$

Hydraulic geometry calculations from (28)–(31) will give slightly different results from those of (24)–(27) simply because the coefficients in (28)–(31) were recalibrated to average values to avoid coefficients with variable exponents. The results from both sets of equations differ within the variability of the coefficients shown in Fig. 5.

RESPONSE TO HAGER'S DISCUSSION

Hager astutely abbreviates the original formulation of (28)–(31) after defining a sediment Froude number. The formulations of (32)–(35) offer dimensional homogeneity, and the right-hand side of each equation nicely reduces to a single repeating dimensionless parameter. This was expected from a dimensional perspective because if the dependent variables on the left-hand side of (28)–(31) can be normalized using a combination of independent variables from the right-hand side of the same equations, the remaining independent variables on the right-hand side must combine into a single dimensionless parameter. Perhaps the advantage of (32)–(35) pertains to the identification of dimensionless terms on each side of the equation. In counterpart, each dependent variable is cluttered with independent variables, coefficients, and exponents. For specific evaluation of the unknown dependent variables, h, W, $\bar{U}$, and τ_θ^*, the dimensionless (32)–(35) must be rewritten in the dimensional form initially proposed in the paper, that is, (28)–(31). There is a false sense of generality in attributing any physical significance to the sediment Froude number. The matter is that the sediment Froude number simply emerges because the variables Q, d_s, and S were arbitrarily selected as independent variables. It should be clearly understood that a different combination of independent variables will yield a different repeating dimensionless parameter. For instance, when the independent variables are discharge Q, grain size d_s, and Shields parameter τ_θ^*, the repeating dimensionless parameter on the right-hand side of (24)–(27) corresponds to $Q^2/g\tau_\theta^* d_s^5$, which is different from the sediment Froude number. One should thus be careful in claiming generality because the final dimensionless parameter depends on an arbitrary selection of independent variables. Hager nevertheless found equivalent dimensionless formulations to the explicit formulations of (28)–(31).

Hager's second comment relates to a direct evaluation of flow depth. It is clear that the dimensionless form (32) does not yield an explicit evaluation of flow depth because flow depth h is found in both the argument and the exponent on the left-hand side of (32). The approximate relationship in (38) is interesting; however, one may guard against misleading interpretations thereof. For instance, the discusser erroneously concludes that "from (38) it can be seen that h is mainly influenced by the grain diameter, slightly by the discharge and only to a small extent by the bottom slope." Actually, one must explicitly rewrite h as a function of the independent variables, that is, $h \sim d_s(Q^2/gSd_s^5)^{0.1875}$ from which one finds that flow depth depends primarily on discharge $h \sim Q^{0.375}$, then slope $h \sim S^{-0.1875}$, and depends very little on grain diameter $h \sim d_s^{0.0625}$. This is in agreement with the authors' results from (28), also found in terms of the exponents of discharge, grain size, and slope for flow depth in Table 4. The need for explicit downstream hydraulic geometry relationships can be satisfied with regression equations. For instance, the following approximate equations have been obtained by simple regression with the data from the original paper:

$$h = 0.374Q^{0.341}d_{50}^{-0.004}\tau_\theta^{*0.074} \tag{57}$$

$$W = 0.761Q^{0.505}d_{50}^{-0.282}\tau_\theta^{*-0.329} \tag{58}$$

$$\bar{U} = 6.012Q^{0.120}d_{50}^{0.298}\tau_\theta^{*0.284} \tag{59}$$

$$S = 6.713Q^{-0.373}d_{50}^{0.954}\tau_\theta^{*1.075} \tag{60}$$

where the discharge Q is in cubic meters per second; d_{50} is the median grain diameter in meters; and τ_θ^* is the Shields number. These equations offer simplicity and rapid evaluations of the dependent variables. Their drawback lies in the fixed values of their exponents and their lack of theoretical support. They are thus considered far inferior to the equations proposed in the paper. They can nevertheless serve for rapid estimates as wall as educated guesses to start the iterative procedure outlined in Tables 3 and 4.

RESPONSE TO HUANG'S DISCUSSION

Huang's contribution is also commendable as it can be seen that after assuming the Manning-Strickler relationship, $n \sim d_s^{1/6}$, the exponents of the regression equations are indeed quite comparable to the range of values compiled in Table 4. The system of equations proposed by Huang and coworkers should yield satisfactory results as long as Manning's equation is applicable. Like our empirical equations (57)–(60), they offer simplicity and rapid calculations at the expense of a lack of theoretical support. Since Huang highlights the importance of additional effects such as channel irregularity, sediment composition, vegetation, and sediment transport, it is noteworthy that both sediment transport and sediment diameter are included in (24)–(27) in terms of grain size and Shields number. The additional effects of bank vegetation and channel irregularity are very difficult to quantify. The key aspect of Huang's presentation relates to (56), where coefficients for depth C_h, width C_W, and velocity $C_{\bar{U}}$ are described in terms of average silt-clay percentage M and vegetation coefficient k_{veg}. Inferences based on the work of Schumm, Hey, and Thorne are useful and should guide further developments. Empirical relationships like (50) and (51) are interesting but site specific, and to quantify (56) remains speculative at this time. For instance, the effects of root vegetation are quite difficult to formulate mathematically, and it may always be difficult to assess whether any given type of vegetation exerts the same influence on small as well as large channels. Specifically, one could perceive that grassy banks without trees or bushes, $k_{veg} = 4.33$, can be effective in reducing the width of small channels, but the grass root zone may be ineffective against undercutting in large streams. Attempts at better defining relationships or coefficients for vegetation and cohesive material are definitely worth pursuing. Future refinements in the evaluation of the coefficients of the hydraulic geometry relationships would certainly be most welcome.

Two particular issues requesting a response are raised at the end of Huang's discussion: (1) the use of channel width instead of radius of curvature in (11); and (2) the use of Einstein-Chien's flow resistance instead of Manning's. Regarding the first point, our derivation in (11) is firmly based on the radius of curvature, as cited in the literature. Notice below (16)–(19) that the ratio R_w is defined as the ratio of radius of curvature to channel width. This ratio implies that the radius of curvature is carried throughout our derivation and included in the definition of the four coefficients in (20)–(23). The empirical variability in R_w is contained within the variability of C_W and $C_{\bar{U}}$ shown in Fig. 5. Actually, it is not clear from Huang's discussion why the radius of curvature has been substituted by the channel width in (47). Regarding the second point, our preference for the Einstein-Chien resistance equation is based on the mathematical simplicity of power relationships. Using the logarithmic relationship for the Darcy-Weisbach f would have been counterproductive because one cannot extract simple explicit formulations for the hydraulic geometry from a mixture of logarithmic and power functions. The reasons for not using Manning n are twofold: (1) There are many instances where Manning's equation is not applicable in that Manning n varies with flow depth, for example, in very steep and rough channels as well as very large rivers; and (2) Manning's resistance equation is the particular case of the Einstein-Chien power relationship where $m = 1/6$ and can thus be predicted from our equations whenever Manning's equation is applicable. Moreover, our method also enables calculations using the Chezy or the Darcy-Weisbach resistance equation with $m = 0$ or calculations for very coarse-grained and steep channels with $m > 1/6$.

In summary, both Hager and Huang should be commended for their enlightening contributions to the theoretical and practical aspects of our system of equations for calculating downstream hydraulic geometry. Hager's dimensionless form is to be remembered because one can now determine the single repeating dimensionless parameter once the independent variables have been selected. Huang's search for improved methods to account for bank material cohesion and vegetation will certainly yield refinements in the practical evaluation of the hydraulic geometry coefficients.