Velocity of Rolling Bed Load Particles

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Abstract: Experimental measurements of the reach-averaged bed-load particle velocity V_p on smooth and rough plane surfaces were analyzed for particles of different shape, size d_s, and density G. Particle types included natural quartz particles (1.2 mm < d_s < 13.6 mm at $G = 2.65$), spherical glass marbles (14.5 mm < d_s < 29.3 mm at $G = 2.6$), and spherical steel ball bearings (1.6 mm < d_s < 19 mm at $G = 8.02$). The velocity of 9,739 individual bed load particles continuously rolling on aluminum plates coated with a layer of sand/gravel (roughness 0 < k_s < 3.4 mm) was measured for 356 different conditions. For each flow condition, the reach-averaged particle velocity measurements over a 2 m test reach were repeated at least 15 times to provide mean values and standard deviations. For bed load particles rolling on smooth surfaces ($k_s = 0$), it was concluded that (1) V_p is within ±30% of the calculated flow velocity from the vertical velocity profile at the top of the particle; (2) V_p for spheres does not vary much with particle density; and (3) V_p increases slightly with particle size d_s, up to approximately $20u_*$. On rough surfaces ($k_s > 0$), for particles of diameter d_s continuously rolling on a stationary bed of roughness k_s, it was concluded that (1) bed load particles roll in continuous motion in the range $2.5 < V_p/u_* < 12.5$; (2) steel particles are much slower than spherical marbles ($G = 2.6$); and (3) the particle velocity increases primarily with a new parameter $\tau_{*ks} = R_h S_f/(G-1)\, k_s$ in the range $0.008 < \tau_{*ks} < 0.2$ up to a maximum $V_p \approx 12\, u_*$. Spherical particles roll slightly faster than natural particles. In a comparison with a large data set that included 1,018 measurements, the analysis of discrepancy ratios showed that the proposed formula was in good agreement with other measurements from the literature. **DOI:** **10.1061/(ASCE)HY.1943-7900.0000657.** *© 2013 American Society of Civil Engineers.*

CE Database subject headings: Bed loads; Particle size; Sediment loads; Sediment transport; Velocity.

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Introduction

Bed-load sediment transport is very important in the analysis of river morphology. Bed-load transport rates depend on the near-bed flow characteristics and sediment properties, such as shear stress, surface roughness, and particle size, density, and shape. The seminal contribution of Einstein (1950) defined the bed layer as twice the sediment size above the bed. Einstein's approach combined both the deterministic and stochastic properties of the flow and particle motion to define bed-load sediment transport. It is therefore important to experimentally examine the motion of bed load particles.

Meland and Norrman (1966) conducted experimental research on bed-load particle velocity leading to a significant analysis of single grain transport velocities. Several theoretical papers on sediment transport rates rely heavily on their data to validate their models. Their experiments used glass beads, rolling on top of a bed made from the same size beads. They found that the influence of size on transport velocity decreases with increasing shear velocity and with decreasing bed roughness size. In other words, at high shear velocities and small bed roughness, the particle velocity tends to be constant for all particle sizes. They proposed

$$\frac{V_p}{k_s^m} = 7.05 u_* \frac{d_s^n}{k_s} - 5.1 \qquad (1)$$

where u_* = shear velocity (m/s); V_p = particle velocity (m/s); k_s = roughness size (mm); $m = 0.75$; and $n = [0.014\, k_s/u_*]^{0.26}$. This empirical equation is not dimensionless, which limits its range of applicability beyond the range of experimental conditions.

Fernandez Luque and van Beek (1976) used a different approach with a loose bed for all their experiments. They measured particle velocities as a function of bed shear stress in a rectangular recirculating flume. The measured grains were scoured from the bed and then rolled on top of it. The average transport velocity of particles is given by

$$V_p = c_a(u_* - 0.7u_{*c}) \qquad (2)$$

where u_{*c} = critica shear velocity at the Shields condition for entrainment (m/s); V_p = particle velocity (m/s); u_* = shear velocity (m/s); and c_a = constant approximately 11.5. Eq. (2) is valid over a wide range of slopes. As in Francis (1973), and Meland and Norrman (1966), the particles were rolling on top of a bed made from the same material.

Bridge and Dominic (1984), and Bridge and Bennett (1992) analyzed bed-load grain transport velocity data to calibrate their proposed model to estimate sediment transport rates. On theoretical grounds, they proposed a relatively similar relationship:

$$V_p = c_b(u_* - u_{*c}) \qquad (3)$$

where $c_b u_{*c} = \omega[\tan\alpha]^{1/2}$; ω = the settling velocity of particles (m/s); $\tan\alpha$ = dynamic friction coefficient; $6 < c_b < 14.3$; u_* = shear velocity (m/s); and u_{*c} = critical shear velocity (m/s). Defining u_{*c} from the Shields parameter means that V_p reduces to zero when $u_* < u_{*c}$, or when the Shields parameter reaches critical value. The critical shear velocity can be defined from

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the Shields parameter for rolling and stationary bed particles of the same size. However, u_{*c} should reduce to zero on smooth surfaces and may thus depend on the ratio d_s/k_s of the rolling particle diameter d_s to the stationary boundary roughness k_s.

In a nutshell, other earlier studies include Kalinske (1942), Einstein (1950), Ippen and Verma (1955), Francis (1973), Steidtmann (1982), and Chien and Wan (1983). More recent articles about individual particle velocities include laboratory experiments by Meier (1995) and Bigillon (2001). There are numerous recent references to sediment transport equations, including Best et al. (1997), Dancey et al. (2002), Guo and Jin (2002), Chang and Yen (2002), Almedeij and Diplas (2003), Roarty and Bruno (2006), Recking et al. (2008), Wong et al. (2007), and Wang et al. (2008). Several studies have also focused on improved field measurements of bed load transport, mostly in gravel-bed streams, by Dixon and Ryan (2000), de Vries (2002), Habersack and Laronne (2002), Kleinhans and van Rijn (2002), Bunte et al. (2004), and Rennie et al. (2002, 2007). Gao (2008) proposed an approach based on the Shields parameter and high-speed video measurements.

Based on similar experiments in laboratory flumes, it was expected that the turbulence intensity atop the bed particles would increase as the bed roughness increased (e.g., Nezu and Nakagawa 1993). It is reasonable to observe an increased variability in turbulent intensity attributable to the presence of bed roughness thus resulting in higher variability in particle velocity (Nicholas 2001; Papanicolaou et al. 2001). Different bed characteristic roughness lengths can have a direct effect on the degree of intermittency and variability observed in the bed load movement of grains. For instance, Papanicolaou et al. (1999) showed that the movement of particles is sporadic and random at low values of the Shields boundary roughness parameter (approximately 0.008) and tends to become more general with lower variability when the parameter exceeds 0.03. Experimental research on instantaneous velocity measurements of individual particles on transitionally rough beds also includes Papanicolaou et al. (2002) and Ramesh et al. (2011). In these studies, the sediment particles and flow conditions were selected such that both rolling motion and saltation could occur. As opposed to instantaneous particle motion, further definition of reach-averaged bed load particle velocities as a function of local shear velocity, particle size d_s, and boundary roughness k_s could eventually become attractive to provide bed load formulations based on local values of shear velocity u^* determined from velocity profiles (e.g., Byrd et al. 2000; Julien et al. 2002).

This study is specifically focuses on the continuous rolling motion of bed load particles on smooth and rough rigid plane surfaces. This study therefore does not describe the instantaneous motion of sediment particles in suspension or in saltation. The specific objectives were to (1) present new laboratory experiments on reach-averaged velocity measurements for rolling bed load particles of different shape, size, and densities on both smooth and rough rigid surfaces; (2) define the particle reach-averaged velocity in relation to fluid velocity in smooth open channels; (3) define the particle velocity in relation to the mean flow velocity and the shear velocity in rough boundary channels; (4) define suitable particle velocity formulas for smooth and rough boundary channels; and (5) test velocity formulas with a large experimental database. The time required for individual particles to roll over a fixed distance without stopping defines reach-averaged particle velocity values, as opposed to instantaneous particle velocity fluctuations. The primary interest of this experimental research was to measure the reach-averaged velocity of individual particles of different size, shape, and density as a function of the boundary roughness of rigid plane bed surfaces.

Laboratory Experiments

In previous experiments, most investigations considered rolling particles of the same size, shape, and density as the bed material. Table 1 presents a summary of existing laboratory data sets. Most existing experiments (Meland and Norrman 1966; Fernandez Luque and van Beek 1976; Bridge and Dominic 1984) were performed with moving bed load particles of the same size as the boundary roughness. All existing experiments also had a relatively narrow range of particle densities ($2.0 < G < 4.6$) hence, the consideration for steel particles with a density of 8.02 in these experiments.

Colorado State University Laboratory Experiments

The primary purpose of the laboratory experiments was to broaden the range of experimental conditions and focus on measurements and comparisons of bed load velocities for particles of different size, density, and shape. A novel aspect of this experimental program was also to vary the surface roughness to induce differences between the size of the rolling particle and the size of the stationary boundary roughness. Also, to avoid any interference with possible bed form configuration in sand-bed channels (e.g., ripples and dunes), particles of a given diameter were glued to a plane rigid boundary for the experiments. Finally, these experiments were for particles in continuous rolling motion in contact with the bed without stopping and without step lengths. The experiments

Table 1. Range of Hydraulic and Bed-Load Particle Parameters

	Earlier experimental data				Recent experiments	
Variables	Meland and Norrman (1966)	Fernandez Luque and van Beek (1976)	Steidtmann (1982)	Bridge and Dominic (1984)	Julien et al. (1995) CSU data	Bigillon (2001)
Number	120	85	330	77	356	50
S_f	—	—	—	—	0.00073–0.011	0.02–0.05
$T°$ (°C)	20	22	18–22	20	17.25–21.5	20
$\nu \times 10^{-6}$ (m^2/s)	1.004	1.26	0.98–1.095	1.004	0.968–1.08	1.004
h (mm)	—	—	180	—	50.3–71.2	2.1–28.6
u_* (m/s)	0.0172–0.1108	0.0122–0.0641	0.0172–0.0277	0.0122–0.0641	0.0097–0.0641	0.018–0.038
V (m/s)	0.25–1.0	—	0.29–0.44	—	0.25–0.89	0.22–0.49
k_s (mm)	2.09–7.76	0.9–3.3	0.35	0.19–3.5	0–3.4	1.5–3
d_s (mm)	2.09–7.76	0.9–3.3	0.21–1.25	0.19–3.5	1.2–29.3	1.5–3
d_s/k_s	1.0	1.0	0.6–3.57	1.0	2.02–24.4	1.0
G	2.65	2.64–4.58	2.5–4.5	2.56	2.6–8.02	2.6–8.02
Particle shape	Spherical	Angular	Spherical	Spherical and angular	Spherical and angular	Spherical

on the reach-averaged velocities of bed load particles in smooth and rough open channel flows were conducted in the Hydraulics Laboratory at the Colorado State University (CSU) Engineering Research Center (Julien et al. 1995). This data set is referred to as the CSU data and includes a total of 49 runs carried out in the recirculating acrylic flume shown in Fig. 1. The trapezoidal flume was 9.77 m long at bed slopes S_0 less than 1%. The side walls of the flume were kept fixed at a 1V:3H ratio to minimize the influence of sidewall effects on particle velocity.

Experiments were carried out on smooth aluminum plates (bed roughness $k_s = 0$) as well as on four rough plane surfaces. Uniform natural sand/gravel particles were glued onto smooth aluminum plates to control the surface roughness. Altogether, five sets of plates were prepared with surface roughness values of $k_s = 1.2$ mm, 1.7 mm, 2.4 mm, and 3.4 mm. Detailed views of the 1.2 mm and 3.4 mm plates are presented in Figs. 2(a and b), respectively. Two 3.65-m (12-ft) long plates were connected for each roughness condition. The upstream plate was used to develop uniform flow conditions for the downstream test reach. A 2-m long test reach near the midpoint of the downstream plate was used to measure reach-average particle velocities. For each run, steady-uniform flow conditions were reached within approximately 10 minutes. The discharge measurements from the precalibrated flow meter were repeated during each run to ensure steady flow condition. The hydraulic parameters were measured from point gauge water surface elevation measurements at different locations along the test reach to ensure that the flow remained uniform.

The particles used in the experiments included stainless steel ball bearings, glass marbles, and natural quartz particles. Fig. 3 shows the array of different particles used in the experiments:

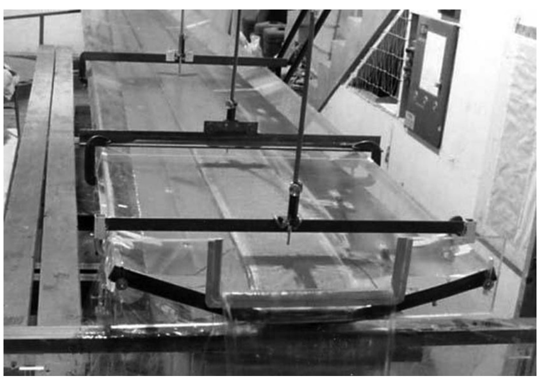


Fig. 1. Trapezoidal flume

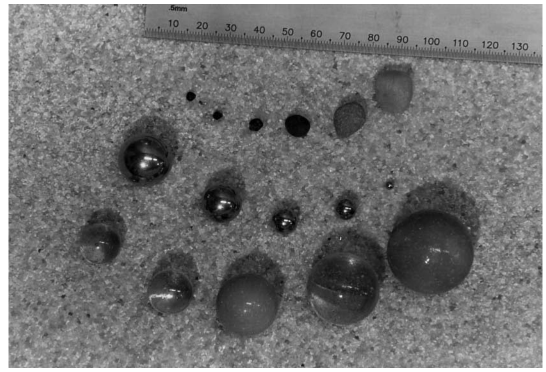
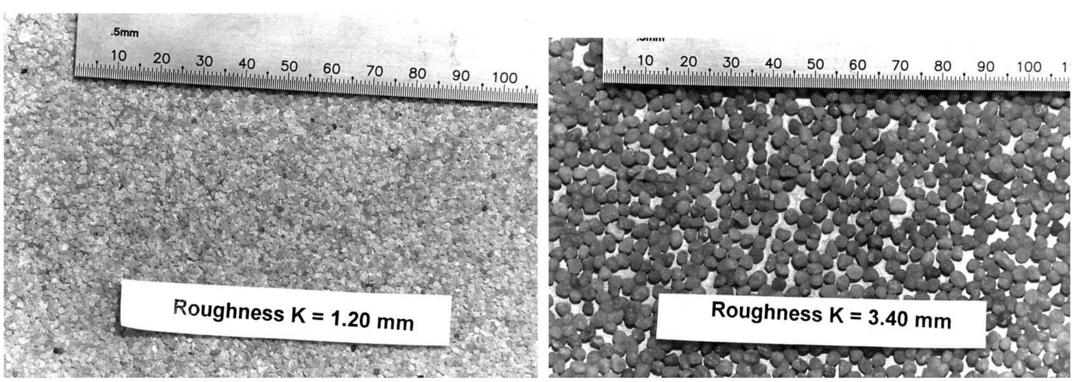


Fig. 3. Particles used for the experiments

(1) steel spherical particles at a specific gravity $G = 8.02$ and diameter $d_{\mathrm{steel}} = 1.57$, 3.14, 4.75, 6.34, 7.9, 9.5, 14.28, 15.88, and 19.04 mm; (2) spherical glass marbles at $G = 2.6$ and diameter $d_{\mathrm{glass}} = 14.48$, 15.97, 21.7, 25.17, and 29.3 mm; and (3) natural particles at $G = 2.65$ and size $d_{\mathrm{natural}} = 1.2$, 1.7, 2.4, 3.4, 4.8, 6.8, 9.6, and 13.6 mm. The steel ball bearing and glass marbles were used because of their precise spherical shape. Some similar sizes were available for some steel and glass particles (e.g., 15.9 mm) for an indication of the particle density effects. The natural "non-spherical" quartz particles were used to emulate natural conditions and also to examine the effects of particle angularity. For example, the 3.4-mm diameter "natural particle" was, in fact, a set of particles, all passing though the 4.0 mm sieve and retained in the 2.8 mm sieve. A greater variability in the test results for natural particles was thus expected.

The reach-averaged velocity of rolling bed load particles was measured over the 2-m long test reach. At a given steady-uniform flow condition, all particle sizes were tested. The glass marbles showed consistent rolling motion for all different hydraulic conditions. The particle travel time over the test reach typically varied between 5–15 s, which could easily be measured with a stop watch with an accuracy of ± 0.1 second. In higher flow conditions, some of the smaller natural particles simply disappeared when submerged, presumably whisked away in suspension, and such experimental results were not recorded. At low flows, most steel particles did not move at all or did so only for short distances before halting. Some natural particles also frequently stopped at low stages, and tended to sit on their flatter sides. The results of these experiments in which particles moved randomly, erratically, or did not move at all, were discarded from the analysis. It was considered that the database was representative of continuous bed load motion, and



(a) (b)

Fig. 2. (a) plate with $k_s = 1.2$ mm roughness; (b) plate with $k_s = 3.4$ mm roughness

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that it excluded intermittent bed load motion, saltation, and sediment suspension.

The individual particle velocity measurements were repeated at least 15 times for each particle in continuous and uninterrupted bed load motion. This experimental program produced a total of 9,739 individual reach-average bed load particle velocity measurements for a total of 49 runs representing different flow conditions with 22 different particles. From the repeated velocity measurements, only the mean velocity and the standard deviation of the measurements were included in the database. When the variability in the particle velocity measurements was too large (due to particle stoppage, saltation, or suspension), the measurements were discarded. Consequently, only the measurements of bed load velocities for particles that were steadily rolling through the test reach were included in this database. This resulted in 356 complete new measurements, including both the average particle velocity and the standard deviation of the repeated measurements. A detailed compilation of particle velocity measurements on rough surfaces can be found in Julien et al. (1995).

Trends in Laboratory Measurements

A sample of the experimental data set is presented in Table 2. The hydraulic parameters for each run include the flow depth h, the friction slope S_f, the mean flow velocity V, the kinematic viscosity ν, the shear velocity $u_* = [gR_hS_f]^{0.5}$ calculated from the gravitational acceleration g, the hydraulic radius R_h, and the laminar sublayer thickness $\delta = 11.6\,\nu/u_*$. For the fixed hydraulic parameters of each run, Table 2 lists the type of rolling bed load particle (steel, glass, or natural), the rolling bed-load particle diameter d_s, the rolling particle specific gravity G, the plate roughness k_s, the number N of individual particle velocity measurements, the mean value of the reach-average bed-load particle velocity V_p, the standard deviation of particle velocity measurements σ, the dimensionless grain diameter $d_* = d_s[(G-1)g/\nu^2]^{1/3}$, and the grain shear Reynolds number $\mathsf{R}_* = u_*d_s/\nu$. Additionally, two values of the Shields parameter were calculated: (1) the bed-load particle Shields parameter $\tau_{*ds} = R_hS_f/(G-1)d_s$, describing the mobility of the moving bed load particle; and (2) the boundary roughness Shields parameter $\tau_{*ks} = R_hS_f/(G-1)k_s$, describing the roughness of the stationary bed particles. The boundary roughness Shields parameter, τ_{*ks}, is a hybrid parameter combining the bed-load particle Shields parameter and the relative roughness ratio, d_s/k_s, such that $\tau_{*ks} = \tau_{*ds}(d_s/k_s)$. These experiments defined the range of conditions for which rolling particles reached continuous motion. The values of the shear to fall velocity ratio remained in the range $0.009 < u_*/\omega < 0.32$, which was less than 0.4 for bed load transport, as suggested by Julien (2010). The lower values in that range corresponded to very smooth surfaces.

The data set in Table 2 is representative of smooth and rough surface experiments, respectively, and some definite trends are noticeable. For instance, Run 3 shows the following overall characteristics on smooth surfaces: (1) coarse particles roll faster than fine particles; (2) at a given grain diameter of approximately 15 mm, quartz spherical particles moved only slightly faster than steel spherical particles; and (3) natural particles moved slower than steel particles of the same size, because of their angularity. Run 47 showed typical observations of rolling bed-load particle velocities on rough plates: (1) some of the particle sizes did not move at all on rough surfaces; (2) spherical particles showed a very slight increase in particle velocity with particle diameter; and (3) at a given diameter, the steel particles moved a lot slower than the glass spheres.

Table 2. Sample of CSU Bed-Load Particle Velocity Experiments

Run	Type	d_s (mm)	G	k_s (mm)	N	V_p (m/s)	σ (m/s)	d_*	R_*	τ_{*ds}	τ_{*ks}
1	2	3	4	5	6	7	8	9	10	11	12
Run 3	Steel	19.04	8.02	0	15	0.2	0.0059	769	298	0.0002	—
Smooth surface	Steel	15.88	8.02	0	15	0.19	0.0042	641	249	0.0002	—
$h = 60.9$ mm	Steel	14.28	8.02	0	15	0.185	0.0069	577	223	0.0003	—
$S_f = 0.0005$	Steel	9.5	8.02	0	15	0.164	0.0036	384	149	0.0004	—
$V = 0.253$ m/s	Steel	7.9	8.02	0	15	0.156	0.0083	319	124	0.0005	—
$v = 0.000001$ m^2/s	Steel	6.34	8.02	0	15	0.145	0.0072	256	99	0.0006	—
$u_* = 0.016$ m/s	Steel	4.75	8.02	0	15	0.128	0.0095	192	74	0.0008	—
$\delta = 0.00074$ m	Steel	3.14	8.02	0	15	0.106	0.0041	127	49	0.0012	—
	Steel	1.57	8.02	0	15	0.071	0.002	63	25	0.0024	—
	Glass	29.3	2.6	0	15	0.226	0.0062	723	459	0.0006	—
	Glass	25.17	2.6	0	15	0.221	0.0059	621	394	0.0007	—
	Glass	21.7	2.6	0	15	0.211	0.0098	535	340	0.0008	—
	Glass	15.97	2.6	0	15	0.201	0.0068	394	250	0.001	—
	Glass	14.48	2.6	0	15	0.194	0.0066	357	227	0.0011	—
	Natural	13.6	2.65	0	21	0.145	0.0088	339	213	0.0012	—
	Natural	9.6	2.65	0	21	0.148	0.0085	239	150	0.0017	—
Run 47	Steel	19.04	8.02	1.2	15	0.134	0.0038	778	679	0.001	0.016
Rough surface	Steel	15.88	8.02	1.2	15	0.121	0.006	649	566	0.0012	0.016
$h = 55.57$ mm	Steel	14.28	8.02	1.2	15	0.113	0.0039	584	509	0.0013	0.016
$S_f = 0.0033$											
$V = 0.468$ m/s	Glass	29.3	2.6	1.2	15	0.344	0.0166	732	1045	0.0028	0.068
$\nu = 0.000001$ m^2/s	Glass	25.17	2.6	1.2	15	0.336	0.0161	629	897	0.0033	0.068
$u_* = 0.036$ m/s	Glass	21.7	2.6	1.2	15	0.328	0.0136	542	774	0.0038	0.068
$\delta = 0.00033$ m	Glass	15.97	2.6	1.2	15	0.314	0.0095	399	569	0.0051	0.068
	Glass	14.48	2.6	1.2	15	0.309	0.0227	362	516	0.0057	0.068
	Natural	13.6	2.65	1.2	15	0.224	0.0163	343	485	0.0058	0.066
	Natural	9.6	2.65	1.2	15	0.237	0.0257	242	342	0.0083	0.066
	Natural	6.8	2.65	1.2	15	0.247	0.028	172	242	0.0116	0.066
	Natural	4.8	2.65	1.2	15	0.273	0.0218	121	171	0.0165	0.066

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Variability in Particle Velocity Measurements

The variability in bed-load particle velocity measurements could be assessed from the repeated particle velocity measurements at given hydraulic and surface roughness conditions. The rolling bed-load particle velocity measurements were repeated 15 times for spherical particles and 21 times for natural particles. The variability was measured in terms of the ratio of the standard deviation σ of individual measurements to the mean rolling bed-load particle velocity V_p. Results of the ratio σ/V_p are plotted as a function of V_p/u_* for both smooth and rough boundaries in Fig. 4(a). The results showed that the variability in the measurements was smaller for smooth boundaries than for rough boundaries. The values of σ/V_p for these experiments were less than 9% on smooth surfaces compared to less than 15% on rough surfaces. In comparison with other experiments, the variability in the velocity measurements of Bigillon (2001) was usually less than 20%, but could be as high as 65%, as shown in Fig. 4(b). This was primarily due to those experimental conditions on very rough boundaries and lower values of V_p/u_*.

The fact that these experiments could be replicated numerous times ($N \geq 15$) with a relatively small standard deviation indicates that the mean rolling bed-load particle velocity can likely be determined from mean flow parameters, such as shear velocity and surface roughness, and particle characteristics, such as diameter, density and shape. More detailed discussions of the data and the analysis of bed-load particle motion on rough surfaces can be found in Julien et al. (1995) and Bounvilay (2003).

Rolling Bed-Load Particle Velocity on Smooth Surfaces

This section first focuses on smooth surfaces. The reach-averaged velocity of rolling bed load particles is compared with the mean and shear flow velocities. The velocity of rolling bed load particles is then compared with the fluid velocity on smooth surfaces.

Comparisons with Mean and Shear Flow Velocities

In comparison with the mean, or depth-averaged, fluid flow velocity V in Fig. 5, the velocity of natural particles ranged from $0.3 < V_p/V < 0.8$. In the experiments of Kalinske (1942), 0.9–1.0 was suggested. As expected, bed-load particle velocities were less than the depth-averaged fluid flow velocity.

The ratio of particle velocity V_p to the shear velocity u_* is shown in Fig. 6. Three cases are presented: (1) steel spheres in the range $2.5 < V_p/u_* < 6$ are shown in Fig. 6(a). The largest steel spheres had the slowest velocities at approximately three times the shear velocity; (2) in Fig. 6(b), the velocity ratios for glass spheres were significantly larger and could move as fast as 20 times the shear velocity on smooth surfaces; and (3) the velocity ratio for natural particles was comparatively lower on smooth surfaces and ranged from $4 < V_p/u_* < 11$, as shown in Fig. 6(c).

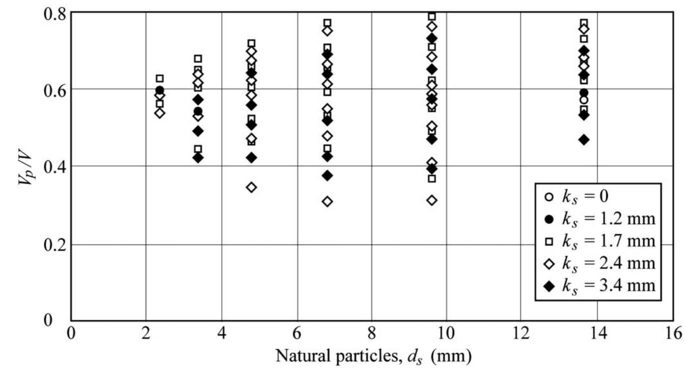

Fig. 5. V_p/V vs d_s for natural particles

Particle Velocity on a Smooth Boundary

The analysis of turbulent flow over a smooth plane boundary provided knowledge of the fluid velocity profile in the turbulent boundary layer. Because some bed load particles had diameters d_s relatively close to the laminar sublayer thickness $\delta = 11.6\nu/u_*$, two regions of the boundary layers were considered: the inner region and the outer region. The inner region was influenced by viscous shear, whereas the outer region was influenced by turbulent shear (Guo et al. 2005). The inner region was further divided into three layers: the laminar sublayer, the buffer layer, and the log layer. Because the bed load movement always occurred near the bed without saltation, it related to the velocity profile in the inner region. In the laminar sublayer, the velocity profile is described as

$$\frac{u}{u_*} = \frac{u_* y}{\nu} \tag{4}$$

where u = fluid velocity at distance y from the smooth surface; u_* = shear velocity; and ν = kinematic viscosity of water. In the log layer, the velocity distribution is

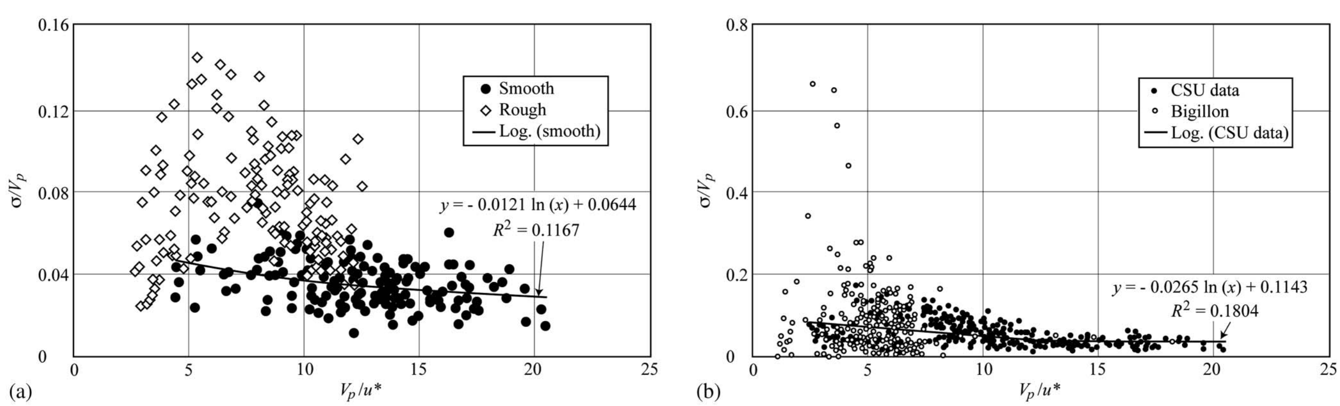

Fig. 4. Variability of the CSU velocity measurements σ/Vp vs Vp/u^* for: (a) smooth and rough boundaries; and (b) comparison with rough surface data from Bigillon (2001)

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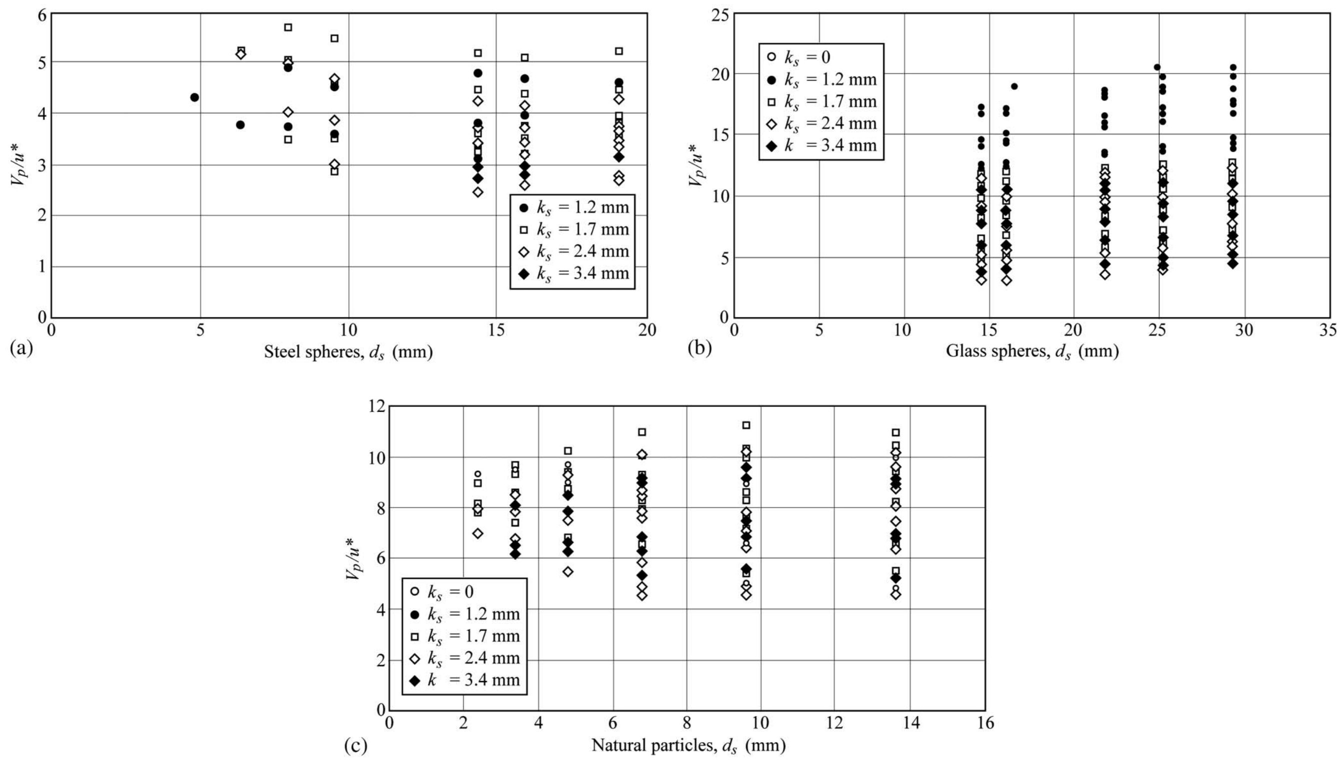

Fig. 6. (a) V_p/u_* vs d_s for steel spheres, (b) V_p/u_* vs d_s for glass spheres and (c) V_p/u_* vs d_s for natural particles

$$\frac{u}{u_*} = \frac{1}{\kappa}\ln\left(\frac{u_* y}{\nu}\right) + 5.5 \tag{5}$$

where $\kappa = 0.4$.

In the buffer layer, Spalding's equation can be used (White 1991, p. 415), but this formulation requires an iterative solution to determine the flow velocity at a given elevation. Julien and Guo (1997) proposed the following empirical formulation for the fluid velocity profile near a smooth boundary. It is comparable to Spalding's equation except that the mean velocity u can be explicitly determined at any elevation y:

$$\frac{u}{u_*} = 6.5\tan^{-1}\left(\frac{u_* y}{26\nu}\right) + \frac{1}{2\kappa}\ln\left[1 + \left(\frac{u_* y}{26\nu}\right)^2\right] \tag{6}$$

A plot of Eq. (6) is shown in Fig. 7 comparing the particle velocity measurements at $y = d_s$. In this figure, the mean rolling bed-load particle velocities are shown with circles. The tick marks, or whiskers, represent the standard deviation σ of the measured bed-load particle velocities measurements around the mean value V_p. The variability in velocity measurements σ/V_p is shown to be small, particularly at low values of R^*. Also in Fig. 7, the mean particle velocity measurements are within an envelope of $\pm 30\%$ of the fluid velocity measurements, as determined by Eq. (6). As a first approximation, the bed-load particle velocity V_p on a smooth surface can be estimated from the fluid velocity V_f obtained from Eq. (6) at the top of the particle, i.e., $y = d_s$, which yields

$$V_p = \left\{6.5\tan^{-1}\left(\frac{u_* d_s}{26\nu}\right) + \frac{1}{2\kappa}\ln\left[1 + \left(\frac{u_* d_s}{26\nu}\right)^2\right]\right\} \times u_* \tag{7}$$

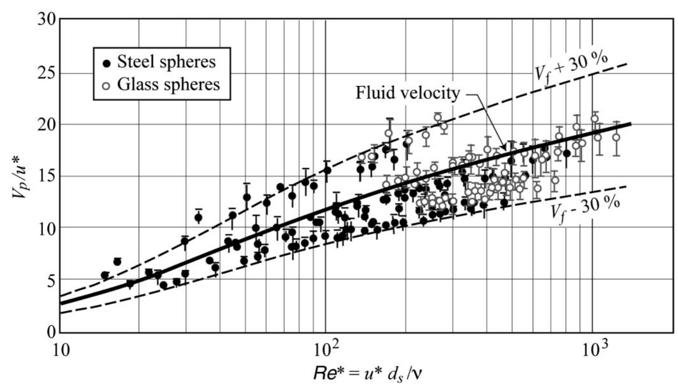

Fig. 7. V_p/u_* vs R^* for steel and glass spheres on smooth surfaces, fluid velocity at $y = d_s$

The agreement shown in Fig. 7 is sufficiently good to conclude that the particle velocity was comparable to the fluid velocity at the top of the particle, i.e., $y = d_s$. This figure also clearly shows that at given flow conditions, the rolling bed-load particle velocity on a smooth surface slightly increased with particle diameter. Indeed, at a constant shear velocity u_*, the rolling particle velocity increased with particle diameter d_s, or $\mathsf{R}^* = u_* d_s/\nu$. On a smooth boundary, the bed-load particle velocity could be up to 20 times the shear velocity u_*. This maximum value of V_p on smooth surfaces was observed experimentally, as shown in Fig. 7, and corresponds to the theoretical velocity obtained from Eq. (7) in the experimental range for $\mathsf{R}^* < 1{,}500$.

Rolling Bed-Load Particle Velocity on Rough Surfaces

The method of dimensional analysis was first applied to the parameters governing the motion of rolling bed load particles on rough plane boundaries. An empirical relationship was then obtained by regression based on laboratory experiments. A simplified formulation is suggested at the end of this section.

Dimensional and Regression Analysis

A formal dimensional analysis was performed to find the framework of an empirical relationship between the particle velocity V_p and six independent kinematic variables:

$$V_p = f[u_*, (G-1), \nu, d_s, k_s, g] \quad (8)$$

where f = unspecified function of shear velocity u_*, kinematic viscosity of the fluid ν, bed-load particle diameter d_s and its relative specific gravity $(G-1)$, stationary bed roughness k_s, and gravitational acceleration g. The results of this dimensional analysis using g and d_s as repeating variables indicated that the dimensionless bed-load particle velocity was a function of four dimensionless parameters: (1) the Shields parameter τ_{*ds}, describing the flow conditions in dimensionless form; (2) the dimensionless diameter of the rolling particle d_*, representing the particle size; (3) the specific gravity $(G-1)$ of the rolling particle, representing its density; and (4) the relative boundary roughness k_s/d_s. Particle shape was described by plotting natural particles separately from spherical particles.

The bed-load particle velocity was assumed to be proportional to the product of the powers of the following dimensionless parameters:

$$\frac{V_p}{\sqrt{(G-1)gd_s}} = a\left[\frac{u_*}{\sqrt{(G-1)gd_s}}\right]^b d_*^c \left(\frac{d_s}{k_s}\right)^d (G-1)^e \quad (9)$$

where $d_* = d_s[(G-1)g/\nu^2]^{1/3}$. The empirical coefficients a, b, c, d, and e were determined for the CSU database by using the Microsoft Excel multiple regression analysis toolbox.

$$V_p = 11.5\tau_{*ds}^{0.95} d_*^{0.21} \left(\frac{d_s}{k_s}\right)^{0.36} (G-1)^{-0.28}[(G-1)gd_s]^{0.5} \quad (10a)$$

An alternative formulation that involves only two parameters (the Shields parameter τ_{*ds} and the relative boundary roughness k_s/d_s was proposed by Bounvilay (2003).

$$V_p = 30.5\tau_{*ds}^{1.0} \left(\frac{d_s}{k_s}\right)^{0.583} [(G-1)gd_s]^{0.5} \quad (10b)$$

The good agreement between Eq. (10*a*) and the CSU database is shown in Fig. 8. This regression equation was tested against other data sets, as discussed in the last section of this paper.

Simple Bed-Load Particle Velocity Approximation

A close examination of the terms in Eqs. (10*a*) and (10*b*) shows that the exponents of several parameters, such as the moving particle diameter d_s, are rather small. Many terms cancel out and point in the direction of a very simple approximation for the bed-load particle velocity as a function of a single parameter τ_{*ds} that is equivalent to $\tau_{*ds} = \tau_{*ds}(d_s/k_s)$. As shown in Fig. 9, as the moving particle diameter d_s became much larger than the boundary roughness k_s, the surface appeared very smooth and the value of the Shields parameter τ_{*ds} became vanishingly small. It was thus found that the parameter τ_{*ks} looked promising as a predictor for the bed-load particle velocity on rough surfaces. A plot of V_p/u_* as a function of τ_{*ks} is thus presented in Fig. 10(a) for different boundary roughness values and in Fig. 10(b) for different particle types. A logarithmic relationship was fitted through the data, and

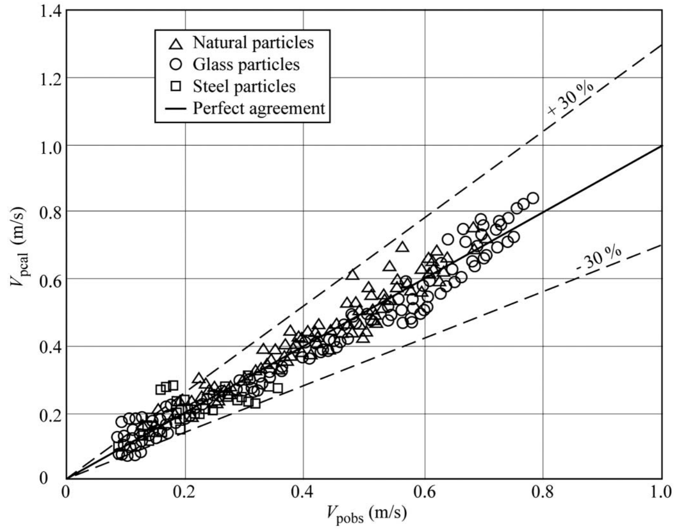

Fig. 8. Calculated versus observed particle velocity V_p on rough surfaces from Eq. (10*a*) for the CSU data

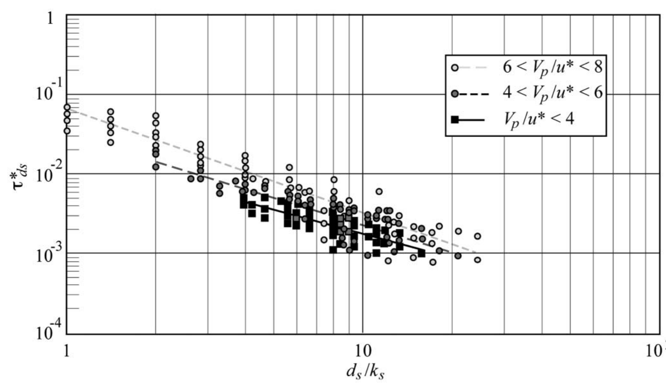

Fig. 9. Shields parameter τ^*_{ds} vs d_s/k_s for different values of ratio V_p/u_*

the bed-load particle velocity was approximated by the simple one-parameter relationship

$$Vp \approx u_*[3.3 \ln \tau_{*ks} + 17.7] \quad (11)$$

In this simple form, the rolling bed-load particle velocity depends primarily on the applied shear stress, or shear velocity u_*, the specific gravity of the moving bed load particle G, and the stationary boundary roughness k_s. Fig. 10(b) clearly shows the effects of particle shape and density. Indeed, bed-load particle velocities were slightly higher for glass spheres than for natural particles. Also, the steel particles were much slower than the glass particles on rough surfaces. These reach-average bed-load particle velocity relationships were essentially applicable for continuous rolling motion of bed load particles on rough surfaces at values of $0.01 < \tau_{*ds} < 0.2$. The maximum rolling bed-load particle velocity measurement on rough surfaces was approximately $V_{pmax} \approx 12u_*$, as shown in Figs. 10(b) and 11. Outside this range, the bed load particles did not roll in continuous motion on the bed surface.

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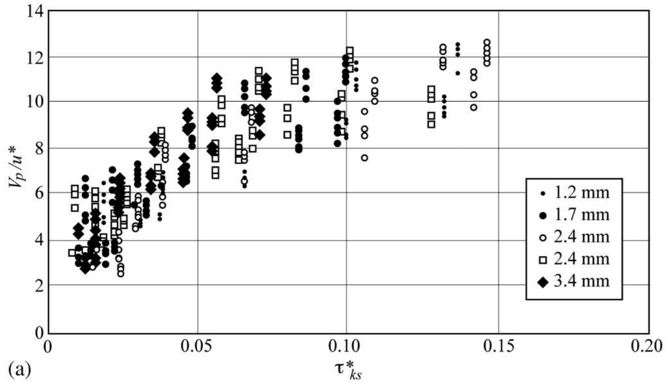

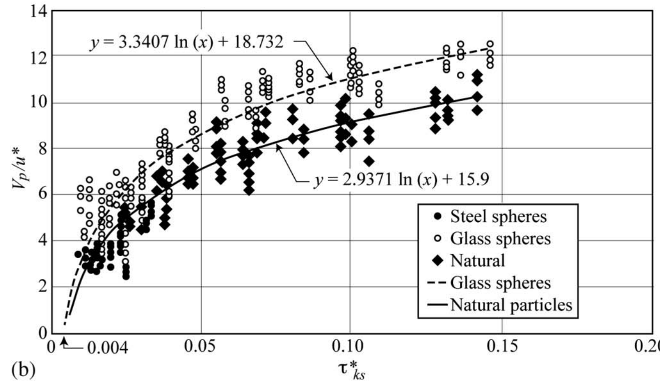

Fig. 10. (a) V_p/u_* versus $\tau^*_{ks} = u_*^2/[(G-1)g\,k_s]$ for different boundary roughness k_s, (b) V_p/u_* versus $\tau^*_{ks} = u_*^2/[(G-1)g\,k_s]$ for different particle types

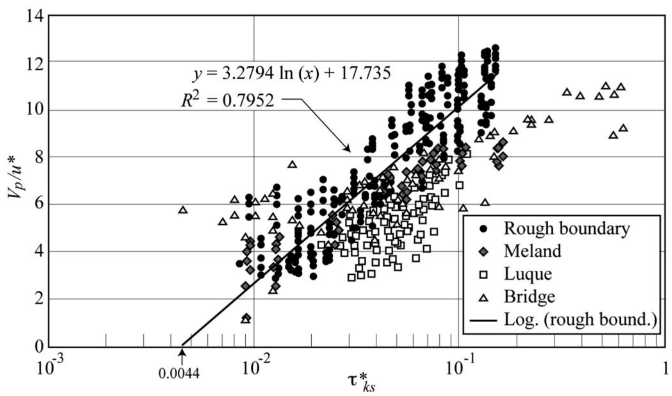

Fig. 11. Comparison of the bed-load particle velocity relationships with other data sets

Testing Rolling Particle Velocity Relationships with a Large Database

Various bed-load particle velocity equations were tested against two databases: (1) the CSU database (356 data points); and (2) the total database (1,018 data points), consisting of the entire database summarized in Table 1. The entire database includes the laboratory measurements of Meland and Norrman (1966), Fernandez Luque and van Beek (1976), Steidtmann (1982), Bridge and Dominic (1984), Julien et al. (1995), and Bigillon (2001). Three additional formulas (Meland and Norrman, Fernandez Luque and van Beek, and Bridge and Dominic) described by Eqs. (1)–(3), respectively, were also tested for comparison with their own data sets and comparison with the entire database. The discrepancy ratio Ri measures the ratio of the calculated particle velocity to the measured velocity. The results, in terms of values of the discrepancy ratio Ri and the coefficient of determination R^2, are presented in Table 3. All equations were found to predict 100% of the laboratory measurements with discrepancy ratios between $0 < Ri < 2.0$. In this sense, all equations are quite good to start. As expected, existing equations were found to perform particularly well when compared with their own data sets. However, their performance level dropped when compared with the entire database. For instance, the highest coefficient of determination of these three equations (Meland and Norrman, Fernandez Luque and van Beek, and Bridge and Dominic) compared with the entire database was only $R^2 = 0.39$. Comparatively, the two equations proposed for rough boundaries in Eqs. (11) and (10*a*), had coefficients of determination of $R^2 = 0.57$ and $R^2 = 0.70$, respectively. The percentages of data within $0.75 < Ri < 1.25$ for the equations of Meland and Norrman (1966), Fernandez Luque and van Beek (1976), and Bridge and Dominic (1984) were, at most, 12% for the total database. In contrast, the proposed formulas [Eqs. (11) and (10*a*)] predicted at least 50% of the entire database within a discrepancy ratio of $0.75 < Ri < 1.25$. Fig. 11 shows a comparison of Eq. (11) with the entire database. At least 84% of the particle velocity predictions ranged between 0.5 and 1.5 times the values calculated with Eq. (11). On this basis, the proposed Eqs. (10*a*) and (11) performed better than the existing formulas.

Table 3. Discrepancy Ratios of Calculated/Observed V_p for Several Formulas

Equation	Data sources	Data in range of discrepancy ratio, R_i (%) 0.75–1.25	0.5–1.5	0.25–1.75	0–2.0	Number of data points	R^2
Simple Eq. (11)	CSU	57	93	97.5	100	356	0.89
	Total[a]	50	84	96.5	100	1,018	0.57
Regression Eq. (10*a*)	CSU	60	92	98.5	100	356	0.96
	Total[a]	56	85	97.5	100	1,018	0.70
Meland Eq. (1)	Meland	83	95	100	100	120	0.95
	Total[a]	12	17	28	100	1,018	0.39
Luque Eq. (2)	Luque	98	100	100	100	85	0.93
	Total[a]	12	20	28	100	1,018	0.12
Bridge Eq. (3)	Bridge	68	96.5	100	100	77	0.93
	Total[a]	8.5	16	22.5	100	1,018	0.07

[a]Total data set includes Meland and Norrman (1966), Luque and van Beek (1976), Steidtmann (1982), Bridge and Dominic (1984), CSU (this paper, also Julien et al. 1995), and Bigillon (2001).

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Summary and Conclusions

This experimental study focused on the reach-averaged velocity of bed load particles in continuous rolling motion on smooth and rough rigid plane surfaces. This study therefore did not describe the instantaneous motion of sediment particles in suspension or in saltation. A total of 9,739 individual rolling bed-load particle velocity measurements were collected at CSU on a set of plates with surface roughness values of k_s equal to 0 mm, 1.2 mm, 1.7 mm, 2.4 mm, and 3.4 mm. Based on at least 15 measurements for each set of experimental conditions, this data set reduced to 356 measurements with average particle velocities and standard deviations. For all measurements, the standard deviation was less than 15% of the average particle velocities.

On smooth surfaces ($k_s = 0$), it was found that (1) the rolling bed-load particle velocity V_p increased gradually with particle sizes d_s and could be as high as $V_p = 20u_*$; (2) the bed-load particle velocity on smooth surfaces was comparable to the fluid velocity at the top of the particle and could be calculated by Eq. (7). The agreement with the calculated flow velocity at the top of the particle (i.e., $y = d_s$) was within ±30%, as shown in Fig. 7. The reach-averaged velocity of rolling particles on smooth surfaces was representative of asymptotic conditions when the ratio of the rolling particle to the bed roughness d_s/k_s became very large.

On rough surfaces, the analysis of bed-load particle velocities in continuous rolling motion led to the following conclusions: (1) bed load particles move slower than the mean flow velocity in the range $0.2 < V_p/V < 0.9$, the fastest were the glass marbles and the slowest were the steel ball bearings; (2) the ratio of particle velocity to shear velocity was in the range $2.5 < V_p/u_* < 14.3$, which compared well with values cited in the literature, i.e., $6.0 < V_p/u_* < 14.3$; (3) the values of the shear to fall velocity ratio remained in the range $0.009 < u_*/\omega < 0.32$, which was in agreement with $u_* < 0.4\omega$ (e.g., Julien 2010); (4) bed load particles moved in continuous motion at values of the Shields boundary roughness parameter $0.008 < \tau_{*ks} < 0.2$; and (5) the velocity of bed load particles increased with τ_{*ks}, as predicted by Eq. (11).

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Notation

The following symbols are used in this paper:

a, b, c, d, e = coefficient and exponents of Eq. (9);
c_a = constant (about 11.5) in Eq. (2);
c_b = constant ($6 < c_b < 14.3$) in Eq. (3);
d_s = diameter of the rolling bed load particle [L];
d_* = dimensionless diameter of the rolling particle, $d_* = d_s[(G-1)g/\nu^2]^{1/3}$;
G = specific gravity of the rolling particle;
g = gravitational acceleration [L/T^2];
h = flow depth (average measurement from three locations) [L];
k_s = diameter of the stationary particles describing the boundary roughness [L];
m, n = empirical coefficients of the Meland and Norrman Eq. (1);
N = number of particle velocity measurements ($N = 15$ for spheres and $N = 21$ for angular particles) in the CSU database;
R_h = hydraulic radius from the ratio of cross section area to wetted perimeter [L];
R_* = grain shear Reynolds number of the rolling particle, $\mathsf{R}_* = u_* d_s/\nu$;
R^2 = coefficient of determination;
Ri = discrepancy ratio defined as the ratio of calculated to measured particle velocity;
S_o, S_f = bed and friction slope, respectively;
T^o = average temperature reading in °C (measured at the beginning and at the end of each run);
u = flow velocity at elevation y above a smooth plane surface L/T;
u_* = shear velocity in m/s, $u_* = [gR_hS_f]^{1/2}$ [L/T];
u_{*c} = critical shear velocity for incipient condition of motion in Eqs. (2) and (3) [L/T];
V = depth-averaged stream flow velocity [L/T];
V_p = average rolling bed-load particle velocity [L/T];
y = elevation above a smooth plane surface at which the fluid velocity is u [L];
$\tan\alpha$ = dynamic friction coefficient in Eq. (3);
δ = laminar sublayer thickness, $\delta = 11.6\nu/u_*$ [L];
κ = von Kármán constant $\kappa \approx 0.4$;
ν = kinematic viscosity of water, $\nu \approx 1 \times 10^{-6}$ m^2/s [L^2/T]; $\nu = 0.0003625(T^o)^2 - 0.038775(T^o) + 1.6345$;
σ = standard deviation of N rolling bed-load particle velocity measurements;
τ_{*ds} = Shields parameter of the rolling bed load particle $\tau_{*ds} = R_hS_f/(G-1)d_s$;
τ_{*ks} = Shields parameter of the boundary roughness $\tau_{*ks} = R_hS_f/(G-1)k_s$; and
ω = settling velocity of the rolling particles, $\omega = [(1 + d_*^3/72)^{0.5} - 1]8\nu/d_s$ [L/T].

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