Higher Order Large-Domain FEM Modeling of 3-D Multiport Waveguide Structures With Arbitrary Discontinuities

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***Abstract*—A highly efficient and accurate higher order large-domain finite-element technique is presented for three-dimensional (3-D) analysis of N-port waveguide structures with arbitrary metallic and dielectric discontinuities on standard PCs. The technique implements hierarchical polynomial vector basis functions of arbitrarily high field-approximation orders on Lagrange-type curved hexahedral finite elements of arbitrary geometrical orders. Preprocessing is carried out by a semiautomatic higher order meshing procedure developed for waveguide discontinuity problems. The computational domain is truncated by coupling the 3-D finite-element method (FEM) with a two-dimensional (2-D) modal expansion technique across the waveguide ports. In cases where analytical solutions are not available, modal forms at the ports are obtained by a higher order 2-D FEM eigenvalue analysis technique. The examples demonstrate very effective higher order hexahedral meshes constructed from a very small number of large curved finite elements (large domains). When compared to the existing higher order (but small domain) finite-element solutions, the presented models require approximately 1/5 of the number of unknowns for the same (or higher) accuracy of the results.**

***Index Terms*—Computer-aided analysis, electromagnetic analysis, finite-element methods (FEMs), microwave devices, waveguide discontinuities.**

I. Introduction

RESEARCH and development of modern waveguide-based microwave devices is heavily dependent on full-wave three-dimensional (3-D) electromagnetic simulations, and the need for advanced electromagnetic analysis and design tools for predicting the performance and optimizing the parameters of these devices prior to costly prototype development is now greater than ever. Along with the mode-matching (MM) technique [1], [2] and the method of moments (MoM) [3], the finite-element method (FEM) has proven to be a very powerful computational frame for analysis and characterization of multiport waveguide structures of arbitrary shapes and with arbitrary material complexities [4]–[8]. In addition to traditionally used low-order small-domain FEM tools, which employ electrically small finite elements (on the order of a tenth of the wavelength in each dimension) and low-order (zeroth and first order) basis functions for the approximation of fields, several novel FEM techniques based on curved elements for geometrical modeling and/or higher order basis functions for field modeling have recently been employed in analysis of arbitrary multiport waveguide structures [6]–[8]. However, none of the proposed techniques exploits the full potential of the higher order FEM modeling and most of the actually reported results are limited to the utilization of elements of the second order. In addition, these techniques still implement small finite elements for field modeling, and the higher order meshes reported actually represent small-domain solutions to 3-D waveguide problems.

This paper presents a highly efficient and accurate higher order large-domain FEM technique for 3-D analysis of N-port waveguide structures with arbitrary metallic and dielectric discontinuities on standard PCs. The technique implements recently proposed hierarchical polynomial vector basis functions of arbitrarily high field-approximation orders in conjunction with Lagrange-type curved hexahedral finite elements of arbitrary geometrical orders [9]. Preprocessing is carried out by a semiautomatic higher order large-domain FEM meshing procedure developed for waveguide discontinuity problems. Based on the higher order connectivity matrix, a global FEM matrix is generated using the Galerkin testing procedure for discretizing the curl–curl electric-field vector wave equation for an arbitrary linear medium inside the waveguide structure. Finally, to close the waveguide problem, a 3-D FEM–modal expansion technique is implemented, according to which the projections of 3-D FEM field solutions within the waveguide sections are matched with two-dimensional (2-D) modal fields across the waveguide ports. The modal forms at the ports are obtained analytically when possible or numerically, employing a 2-D higher order eigenvalue FEM analysis technique, for waveguide cross sections of arbitrary shapes.

The accuracy and efficiency of the proposed technique are demonstrated in four characteristic realistic examples of multiport waveguide structures that include different discontinuities with both flat and curved surfaces, made from both perfectly conducting and penetrable dielectric materials. The results obtained by the higher order large-domain FEM are validated and evaluated in comparisons with the experiments and the numerical results obtained by the existing low-order and higher order FEM techniques, MoM, and MM. The examples demonstrate very effective large-domain meshes that consist of a very small number of large curved finite elements. When compared to the existing higher order (but still small-domain) FEM

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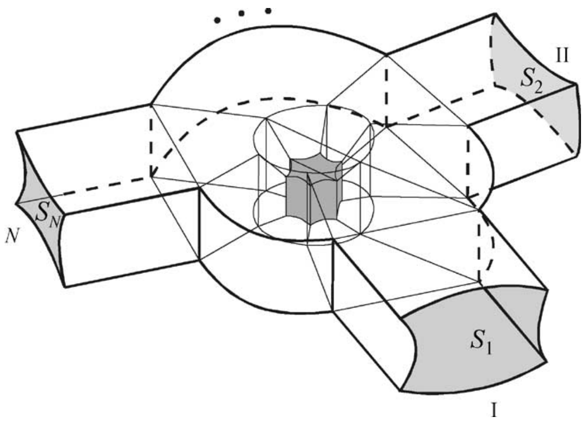

Fig. 1. Generic passive microwave device with N ports modeled by large generalized curved hexahedral finite elements.

solutions, the presented higher order large-domain hexahedral FEM models require approximately 1/5 of the number of unknowns for the same (or higher) accuracy of the results.

II. Theory and Implementation

Consider a generic waveguide problem (a multiport passive microwave device) shown in Fig. 1. In our analysis method, the computational domain is first truncated by introducing fictitious transversal surfaces at each of the ports. The closed structure thus obtained is then tessellated using generalized Lagrange-type curved parametric hexahedra of higher geometrical orders [9], as indicated in Fig. 1. The electric fields inside each of the hexahedra are expanded as

$$\begin{aligned}\boldsymbol{E} = &\sum_{i=0}^{N_u-1}\sum_{j=0}^{N_v}\sum_{k=0}^{N_w}\alpha_{uijk}\,\boldsymbol{f}_{uijk}(u,v,w)\\ &+\sum_{i=0}^{N_u}\sum_{j=0}^{N_v-1}\sum_{k=0}^{N_w}\alpha_{vijk}\,\boldsymbol{f}_{vijk}(u,v,w)\\ &+\sum_{i=0}^{N_u}\sum_{j=0}^{N_v}\sum_{k=0}^{N_w-1}\alpha_{wijk}\,\boldsymbol{f}_{wijk}(u,v,w)\end{aligned} \tag{1}$$

where $\boldsymbol{f}_{uijk}$, $\boldsymbol{f}_{vijk}$, and $\boldsymbol{f}_{wijk}$ are curl-conforming hierarchical-type vector basis functions of higher field-approximation orders [9] for modeling the reciprocal u-, v-, and w-components, respectively, of the field vector; u, v, and w being the parametric nonorthogonal curvilinear coordinates in the generalized hexahedron. With this, electrically large elements that are on the order of a wavelength in each dimension (large domains) can be used, thus fully exploiting the accuracy, efficiency, and convergence properties of the higher order FEM. Moreover, the hierarchical nature of the field-approximation basis functions and the geometrical flexibility of the curved parametric hexahedral elements enable different element shapes and sizes, geometrical orders, and field-approximation orders to be used at the same time in a single simulation model of a complex microwave device.

We recognize here that development and implementation of general fully automatic geometrical preprocessors (meshers) for an arbitrary geometry using generalized hexahedral volume elements of arbitrarily high geometrical orders may present a considerable difficulty and great research challenge, which is beyond the scope of this paper. To overcome this problem, a semiautomatic meshing procedure has been developed and utilized for the purpose of the higher order large-domain FEM analysis of a class of waveguide-related problems. It has been observed that all realistic discontinuities commonly found in waveguides in microwave practice can be decomposed into a number of predefined geometrical types of discontinuities (e.g., a cylindrical post in a waveguide section, ridge, slot, fin, etc.), all of which have simple topology. Hence, each discontinuity type is first automatically coarsely meshed (in its parent domain) using the predefined set of templates, i.e., using the smallest possible number of hexahedral elements with user-defined geometrical orders. Interpolation nodes are then assigned to the elements comprising thus obtained sub-meshes, where the number of nodes and their proper ordering depend on the geometrical orders of elements ("top-down" approach). Next, parts of the structure are mapped from the parametric space to the curved hexahedral elements in real 3-D space (geometrical embedding). These forms (sub-meshes) are then connected together appropriately into an optimal large-domain mesh. This step includes a prenumeration of the initially locally assigned nodes (within each of the sub-meshes) and a setup of the uniquely defined global nodes. Finally, if needed, the obtained coarse mesh can be automatically refined by internally subdividing the particular elements. The higher order meshes for all the examples discussed in Section III are generated using this semiautomatic meshing procedure.

A standard Galerkin-type weak-form discretization of the curl–curl electric-field vector wave equation [9] for an inhomogeneous (possibly lossy) medium of complex relative permittivity ε_r and permeability μ_r inside the structure yields

$$\int_V \mu_{\mathrm{r}}^{-1}(\nabla\times\boldsymbol{f}_{\hat{i}\hat{j}\hat{k}})\cdot(\nabla\times\boldsymbol{E})\,\mathrm{d}V - k_0^2\int_V \varepsilon_{\mathrm{r}}\boldsymbol{f}_{\hat{i}\hat{j}\hat{k}}\cdot\boldsymbol{E}\,\mathrm{d}V = \mathrm{j}k_0Z_0\oint_S \boldsymbol{f}_{\hat{i}\hat{j}\hat{k}}\cdot\boldsymbol{n}\times\boldsymbol{H}\,\mathrm{d}S \tag{2}$$

where V is the volume of a generalized hexahedron in the mesh, $\boldsymbol{f}_{\hat{i}\hat{j}\hat{k}}$ stands for any of the testing functions $\boldsymbol{f}_{u\hat{i}\hat{j}\hat{k}}$, $\boldsymbol{f}_{v\hat{i}\hat{j}\hat{k}}$, or $\boldsymbol{f}_{w\hat{i}\hat{j}\hat{k}}$, S is the boundary surface of the hexahedron, $\boldsymbol{n}$ is the outward unit normal, Z_0 and $k_0 = \omega\sqrt{\varepsilon_0\mu_0}$ are the free-space intrinsic impedance and wave number, respectively, and ω is the angular frequency of the implied time–harmonic variation. Due to the continuity of the tangential magnetic field across the interface between any two finite elements in the FEM model, the right-hand-side term in (2) contains the surface integral over the overall boundary surface of the entire FEM domain, and not over the internal boundary surfaces between the individual hexahedra in the model, which, for the waveguide problem in Fig. 1, reduces to the surface integral across the artificially introduced transversal surfaces (waveguide ports).

In order to introduce the proper boundary condition and/or excitation at the waveguide ports, a 3-D FEM–modal expansion method [5], [8] is invoked, according to which the projection of the field solution within a waveguide section is matched with the desired modal field at the port. Specifically, the tangential electric and magnetic field at each of the ports are expanded as linear combinations of the incoming and outgoing waveguide modes

$$\boldsymbol{E}_{\mathrm{t}} = \sum_{m=1}^{N_{\mathrm{m}}} (a_m + b_m)\boldsymbol{e}_m(u,v) \quad \boldsymbol{H}_{\mathrm{t}} = \sum_{m=1}^{N_{\mathrm{m}}} (b_m - a_m)\boldsymbol{h}_m(u,v) \tag{3}$$

where $\boldsymbol{e}_m$ and $\boldsymbol{h}_m$ represent the transversal (tangential to the port surface) electric- and magnetic-field components of the mth mode, while a_m and b_m stand for the amplitudes of the incident and reflected waves, respectively. The modal forms at the ports are obtained analytically when possible (first three examples discussed in Section III) or numerically employing a 2-D higher order eigenvalue FEM analysis technique for waveguide cross sections of arbitrary shapes (last example in Section III). The 2-D FEM is developed as the 2-D version of the eigenvalue analysis technique described in [9]. Substituting the 3-D electric-field expansion (1) and 2-D magnetic-field expansion from (3) into (2) and enforcing the continuity of the tangential electric fields in (1) and (3) over the port surfaces yield, respectively, the following two matrix systems of equations:

$$\begin{aligned} \left([A] - k_0^2[B]\right) \cdot \{\alpha\} &= [P]\{a\} - [P]\{b\} \\ [C]\{\alpha\} &= [D]\{a\} + [D]\{b\}. \end{aligned} \tag{4}$$

The elements of matrices $[A]$ and $[B]$ are given in [9]. The elements of matrices $[P]$, $[C]$, and $[D]$ are obtained as

$$\begin{aligned} p_{\hat{i}\hat{j}\hat{k}m} &= -\mathrm{j}k_0 Z_0 \int_S \boldsymbol{f}_{\hat{i}\hat{j}\hat{k}} \cdot (\boldsymbol{n} \times \boldsymbol{h}_m)\,\mathrm{d}S \\ c_{\hat{m}ijk} &= \int_S \boldsymbol{e}_{\hat{m}} \cdot \boldsymbol{f}_{ijk}\mathrm{d}S \\ d_{\hat{m}m} &= \int_S \boldsymbol{e}_{\hat{m}} \cdot \boldsymbol{e}_m\,\mathrm{d}S \end{aligned} \tag{5}$$

where the domain of integration (S) either coincides with a side of the generalized hexahedron belonging to the port surface (in the first two integrals) or encompasses the entire port surface (in the third integral). The two coupled systems of (4) are solved simultaneously for the unknown FEM field-distribution coefficients $\{\alpha\}$ over the volume of the multiport structure and scattering wave coefficients $\{b\}$ at the waveguide ports. By post-processing of these coefficients, all quantities of interest for the multiport structure in Fig. 1 are obtained in a straightforward manner.

In this paper, the waveguides in Fig. 1 are assumed to operate in the single-mode (dominant-mode) regime, which is a standard assumption for practical microwave applications, so that the number of modes in the described FEM–MM procedure is set to one (for each of the ports). This implies that the ports are far enough from all discontinuities (to ensure the relaxation of the higher order modes). The single-mode condition has frequently been found to be impractical and computationally costly in traditional small-domain FEM models due to the fact that placing the ports far from discontinuities requires a considerable number of additional elements to be employed, which significantly enlarges the computational domain and introduces a large number of new unknowns to be determined. However, this major drawback can be very effectively overcome in the higher order large-domain waveguide modeling proposed in this paper by placing only a few large elements (most frequently, a single one) with a high field-approximation order in the longitudinal direction as a buffer zone between each port and the domain with discontinuities. The sufficient length of the buffer elements allows for the higher modes excited at the discontinuity to relax before they reach the port, while the high-order field expansion in the longitudinal direction ensures the accurate approximation of the fields throughout the elements, without introducing an unnecessarily large number of new unknowns. The efficiency of this technique is demonstrated in all of the examples in Section III. On the other hand, note that the overall number of unknowns in the higher order solution can additionally be reduced by carrying out the full multimode analysis in (4) at the ports placed close to the discontinuity region in Fig. 1.

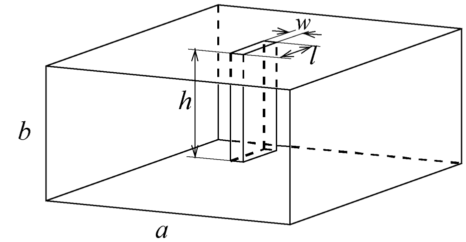

Fig. 2. E-plane ridge waveguide discontinuity ($a = 19.05$ mm, $b = 9.524$ mm, $w = 1.016$ mm, $h = 7.619$ mm, and $l = 5.08$ mm).

Finally, it is worth mentioning that, for dispersionless media, integrals appearing in all of the matrices in (4) are frequency independent. Therefore, for a multifrequency analysis of the same microwave structure, they can be calculated only once, conveniently stored, and then recalled during the problem solution for different excitation frequencies since the only change in the global system is that of the wavenumber. This procedure significantly reduces the overall computational time by allowing the global FEM matrix to be filled only once at the expense of a considerably larger storage space that needs to be allocated since matrices $[A]$, $[B]$, and $[P]$ have to be stored separately. However, as will be demonstrated in Section III, higher order large-domain FEM models of practical multiport waveguide discontinuity structures require very small numbers of unknowns (even with buffer elements at the ports included in the FEM region) for high levels of accuracy, which makes them perfect for implementing the described multifrequency solution acceleration procedure, and this procedure is used in all of the examples in Section III.

III. Results and Discussion

As the first example, consider an E-plane ridge waveguide discontinuity, shown in Fig. 2. The structure is analyzed using a simple 11-element mesh depicted in Fig. 3(a). All elements in the mesh are of the first geometrical order (trilinear hexahedra), whereas the polynomial orders of the field approximations are varied from 2 to 5 for different elements and in different directions, as shown in Fig. 3(b). The total number of unknowns is 1636. Fig. 4 shows the transmission coefficient of the struc-

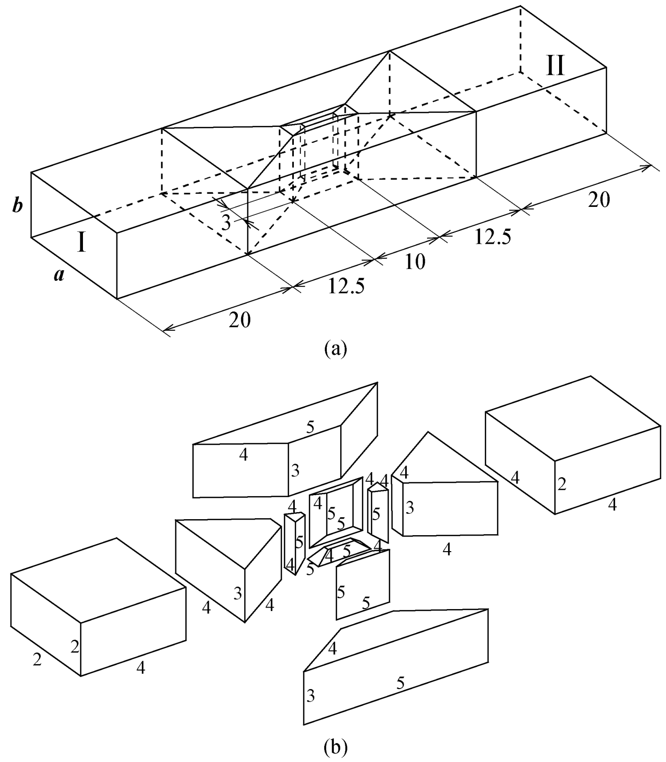

Fig. 3. Large-domain meshing for the higher order FEM analysis of the microwave structure in Fig. 2. (a) Mesh constructed from 11 trilinear hexahedral elements with dimensions given in millimeters. (b) Orders of the polynomial field approximation adopted for each of the elements in the mesh.

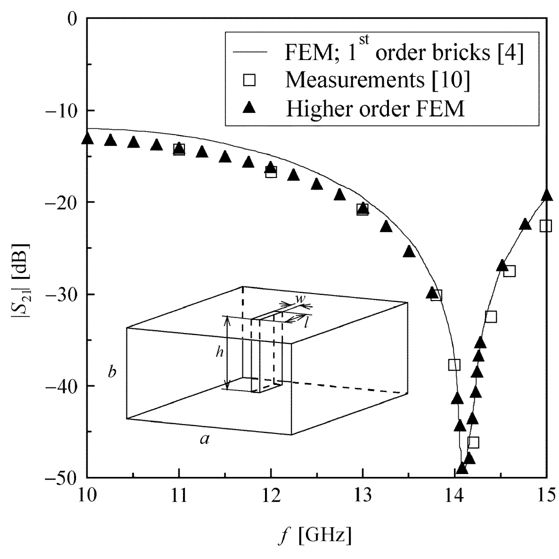

Fig. 4. Transmission coefficient of the microwave structure in Fig. 2: comparison of the higher order FEM solution using the mesh in Fig. 3, first-order FEM solution with brick elements [4], and experiment [10].

ture. The higher order FEM results are compared with the solution obtained by an alternative FEM approach using a first-order brick mesh [4] (the number of unknowns is not reported in [4]) and with experimental results [10]. It can be observed that the solution obtained by means of the higher order FEM is in excellent agreement with both reference sets of results.

The second example is a WR-62 waveguide with two crossed cylindrical posts, shown in Fig. 5. The large-domain mesh of

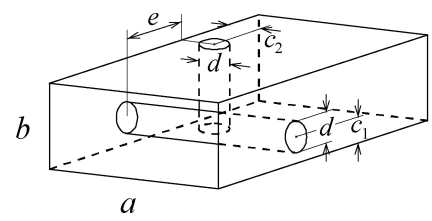

Fig. 5. Two crossed posts in a WR-62 waveguide ($a = 15.7988$ mm, $b = 7.8994$ mm, $c_1 = 2.5$ mm, $c_2 = 4$ mm, $d = 3$ mm, and $e = 11.51$ mm).

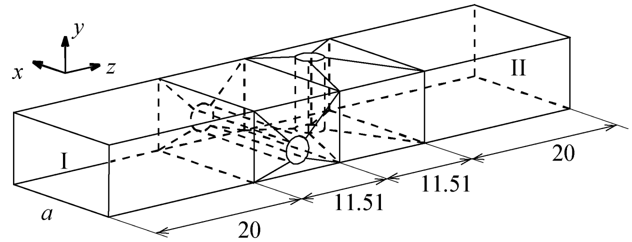

Fig. 6. Large-domain ten-element mesh generated for the microwave structure in Fig. 5 with dimensions given in millimeters.

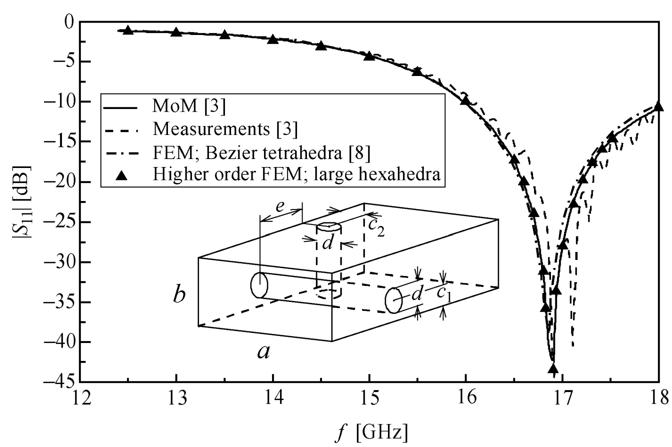

Fig. 7. Reflection coefficient for the microwave structure in Fig. 5. Comparison of the higher order large-domain FEM solution using the hexahedral mesh in Fig. 6, MoM solution using RWG basis functions on triangular patches and eigenvectors of the waveguide port sections [3], second-order FEM solution with Bezier-type curved tetrahedral elements [8], and experimental results.

this structure (Fig. 6) consists of ten curvilinear hexahedra; two trilinear elements are positioned near ports I and II and eight triquadratic (second geometrical order) elements are placed around the posts to account for their cylindrical curvature. The orders of the polynomial field-approximation are $N_u = N_v = N_w = 4$ in all triquadratic elements, whereas $N_u = N_v = 2$ and $N_w = 5$ in the trilinear element near port I and $N_u = 4$, $N_v = 2$, and $N_w = 5$ in the element near port II (directions of axes in local u–v–w coordinate systems in the elements correspond to the directions of axes in the global x–y–z coordinate system). This arrangement results in 1184 FEM unknowns. Fig. 7. shows the reflection coefficient S_{11} of the structure. The higher order large-domain hexahedral FEM solution is compared with the experiment [3] and with results obtained by two alternative numerical techniques, a low-order MoM using triangular surface meshing with Rao–Wilton–Glisson (RWG) basis functions and eigenvectors of the waveguide port sections [3] and a higher order

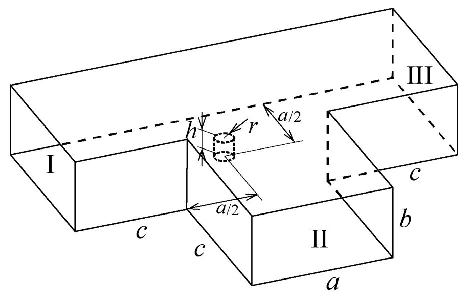

Fig. 8. H-plane waveguide T-junction loaded with a "partial-height" metallic post ($a = 19.05$ mm, $b = 9.525$ mm, $c = 30$ mm, $r = 1.27$ mm, and $h = 2.54$ mm).

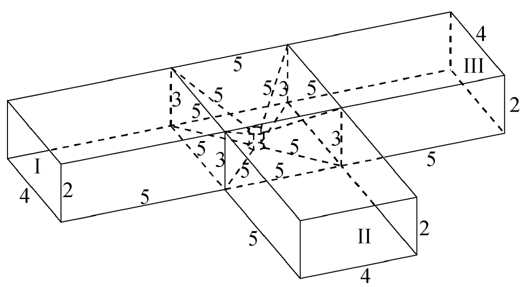

Fig. 9. Higher order mesh of the microwave structure in Fig. 8 constructed from eight (three trilinear and five triquadratic) large curvilinear hexahedral finite elements with designated field approximation orders in different directions.

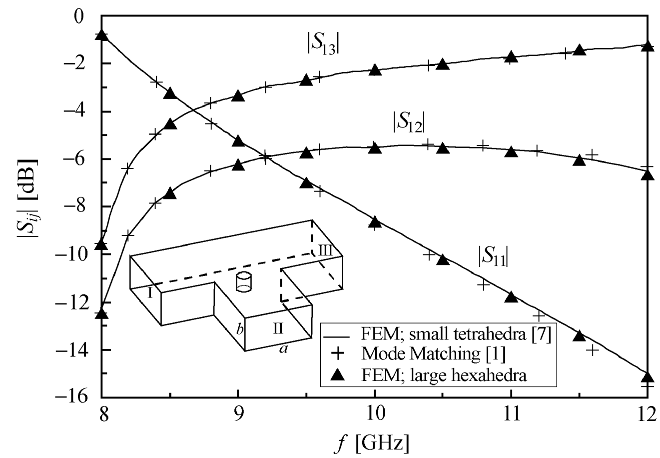

Fig. 10. S-parameters of the microwave structure in Fig. 8. Comparison of the higher order FEM solution using the large-domain mesh in Fig. 9, an alternative higher order FEM solution with tetrahedral elements [7], and an MM solution [1].

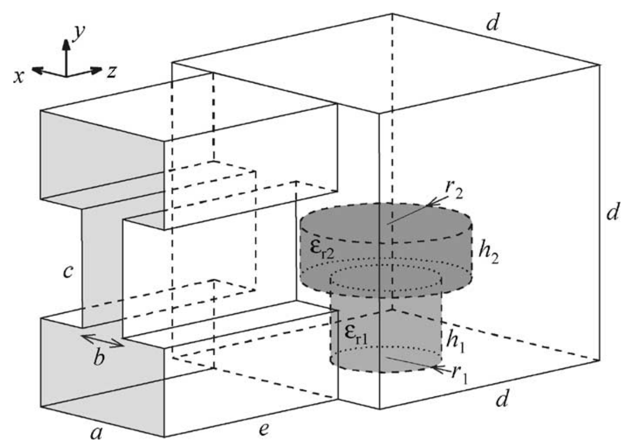

Fig. 11. Double-ridge waveguide–DR junction ($a = 15.24$ mm, $b = 5.08$ mm, $c = 10.16$ mm, $d = 25.4$ mm, $e = 30$ mm, $r_1 = h_2 = 4.7244$ mm, $r_2 = 7.1628$ mm, $h_1 = 7.0104$ mm, $\varepsilon_{r1} = 4.5$, and $\varepsilon_{r2} = 38$).

tetrahedral FEM [8]. An excellent agreement between the large-domain FEM and all other presented results is observed. Note that the number of unknowns with the large-domain FEM is approximately a fifth of that with the second-order FEM solution [8], where the use of 273 Bezier-type curved tetrahedral elements and 5709 FEM unknowns is reported (the number of MoM unknowns in [3] is not reported). Note also that the ten-element hexahedral FEM model used in the current higher order approach is much simpler (and easier to generate) than the Bezier-type tetrahedral model used in [8].

Consider next an H-plane waveguide T-junction loaded with a "partial-height" cylindrical metallic post, shown in Fig. 8. An eight-element large-domain mesh suitable for the higher order FEM analysis of the junction is shown in Fig. 9. Three trilinear hexahedral elements are used near the ports (I–III) and five triquadratic ones near the post, conformal to its curved cylindrical surface. The orders of the polynomial field-approximation in individual elements, ranging from 2 to 5 in different directions, are also shown in this figure. They yield a total of 1245 FEM unknowns. In Fig. 10, the results for S-parameters of the junction obtained by the higher order large-domain FEM are compared with those obtained by an alternative higher order FEM technique [7] and with results of an MM technique [1]. We observe an excellent agreement between the three sets of results. However, the higher order large-domain hexahedral FEM model requires, again, about 1/5 of the number of unknowns (6471) used in the FEM solution with higher order tetrahedral elements [7].

As the final example, Fig. 11 shows a dielectric-loaded resonator (DR) fed by a double-ridge waveguide. The dielectric resonator has a "mushroom" shape and consists of two dielectric cylinders (pill boxes) with different relative permittivities and no losses. Although rather complex, this structure is modeled using a total of only 48 hexahedral elements (seven bricks for the double-ridge waveguide section, and 25 trilinear and 16 triquadratic hexahedra for the DR cavity) with the polynomial field approximation orders ranging from 2 to 5, which results in a total of 3326 FEM unknowns. The dominant mode parameters of the waveguide are obtained from a seven-element 2-D FEM model of the double-ridge port geometry with sixth-order polynomial field approximations in both the u- and v-directions and 661 unknowns for the eigenvalue analysis. Shown in Fig. 12 is the phase of the reflection coefficient S_{11} of the structure computed at the waveguide–DR cavity junction. The results of the higher order large-domain FEM analysis are compared with the numerical results obtained by an MM method [2] and those using the commercial FEM software HFSS, reported in [2]. We see an excellent agreement between the two FEM solutions (the number of unknowns using HFSS is not given in [2]) and their good agreement with the MM solution.

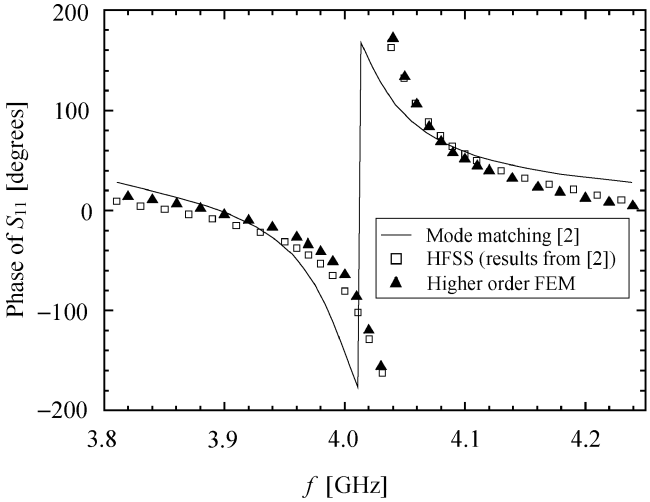

Fig. 12. Phase of the reflection coefficient of the microwave structure in Fig. 11. Comparison of the higher order FEM solution using a 48-element large-domain hexahedral mesh, a low-order FEM solution with tetrahedral elements (HFSS) [2], and an MM solution [2].

IV. Conclusions

This paper has presented a highly efficient and accurate higher order large-domain finite-element technique for analysis of 3-D N-port waveguide structures with arbitrary metallic and dielectric discontinuities on standard PCs. The technique implements hierarchical polynomial vector basis functions of arbitrarily high field-approximation orders on Lagrange-type curved hexahedral finite elements of arbitrary geometrical orders. It allows electrically large elements, on the order of a wavelength in each dimension, to be used, thus fully exploiting the accuracy, efficiency, and convergence properties of the higher order FEM. Preprocessing is carried out by a semiautomatic higher order meshing procedure developed for waveguide discontinuity problems. The Galerkin testing procedure is employed for discretization of the curl–curl electric-field vector wave equation for an arbitrary linear medium inside the waveguide structure. The computational domain is truncated combining the 3-D FEM with a 2-D modal expansion and matching technique across the waveguide ports. In cases where analytical solutions are not available, modal forms at the ports are obtained by a higher order 2-D FEM eigenvalue analysis technique.

The accuracy and efficiency of the presented technique have been validated and evaluated in four characteristic realistic examples of one-, two-, and three-port waveguide structures with metallic and dielectric discontinuities of different shapes. The input/output waveguides in all of the examples are assumed to operate in the dominant-mode regime, which enables the number of modes in the FEM–MM procedure to be set to one, but requires a buffer (relaxation) zone between each waveguide port and the domain with discontinuities. These buffer waveguide sections are conveniently modeled by only a few large elements (most frequently, a single one) with a high field-approximation order in the longitudinal direction, which results in a very accurate and efficient overall analysis. The results obtained by the higher order large-domain FEM have been found to be in excellent agreement with measurements and numerical results obtained by the existing low-order and higher order FEM, MoM, and MM techniques. The examples have demonstrated very effective higher order hexahedral meshes constructed from a very small number of large curved finite elements (large domains). It has been observed that, for the same (or higher) accuracy, the presented higher order large-domain hexahedral FEM models require approximately 1/5 of the number of unknowns used by the existing higher order (but small domain) FEM solutions.

Our future work will include applications of segmentation techniques for analysis of complex microwave waveguide networks where generalized scattering matrices are computed for all the building blocks (segments) of the network (which requires the full multimode analysis) and the whole structure is then characterized by cascading the individual matrices. Our FEM technique will also be extended to include nonreciprocal chiral materials as material discontinuities in multiport waveguide structures.

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