General entire-domain Galerkin method for analysis of wire antennas in the presence of dielectric bodies

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Abstract: A novel, entire-domain moment-method is proposed for the analysis of wire antennas and scatterers in the presence of lossy inhomogeneous dielectric bodies of finite extent. The wire part of the structure is approximated by arbitrarily positioned and interconnected straight-wire segments of lengths which can exceed one wavelength. The dielectric bodies are approximated by a system of trilinear hexahedrons, which can be electrically large (can also exceed one wavelength inside the dielectric, in any direction). The current along wires and inside dielectric bodies is approximated by one- and three-dimensional polynomials, respectively. The unknown current-distribution coefficients are obtained by a Galerkin-type solution of the system of coupled two-potential integral equations. The method is accurate, efficient and reliable. Its fundamental advantage over the (only existing) subdomain methods for the analysis of the same type of structures is a significantly reduced number of unknowns (as a rule, for an order of magnitude) and, consequently, greatly reduced computing time. The proposed method enables rapid analysis of wire/dielectric structures exceeding moderate electrical size with even standard personal computers.

1 Introduction

Numerical analysis of wire antennas and scatterers in the presence of finite-size dielectric bodies (possibly lossy and inhomogeneous), which may be termed wire/dielectric antennas and scatterers, is of great practical interest. There are two basic classes of general methods used for such analysis. The first class includes the moment-method solutions of the system of coupled integral equations consisting of a line integral equation for wires, and a volume integral equation for dielectric bodies [1–3]. The wire/dielectric antenna (or scatterer) is approximated by many electrically small geometrical elements, with low-order basis functions for the current/field approximation (the subdomain-type approach). The second class includes the FDTD (finite-difference time-domain) methods for the solution of the differential electromagnetic-field equations [4]. These methods also imply a kind of subdomain (small-domain) discretisation of space, with the additional discretisation in the time domain. An advantage over the integral-equation subdomain methods is that the process is sequential, not requiring a solution of the system of linear equations.

The basic problem with both existing classes of solution methods for arbitrary wire/dielectric structures is the need for a very large number of unknowns to obtain results of satisfactory accuracy. In the opinion of the present authors, this is solely the consequence of the subdomain philosophy that is used.

To the best knowledge of the authors, the present paper presents the first entire-domain (more precisely, large-domain) general method for the analysis of wire antennas and scatterers in the presence of dielectric bodies. A system of coupled integral two-potential equations in unknown current distribution along wires and inside dielectric bodies is solved by the Galerkin entire-domain method. The wires are approximated by straight segments that can be quite long (exceeding one wavelength), with a polynomial current aproximation which automatically satisfies the first Kirchhoff law at the wire ends and interconnections. The dielectric bodies are modelled by large, generally nonorthogonal, trilinear hexahedrons, with a three-dimensional polynomial approximation for current distribution that automatically satisfies the boundary condition for the normal component of the equivalent electric displacement vector on surfaces shared by two adjacent hexahedrons [5, 6]. One of the major problems, that has been solved successfully, was an efficient numerical procedure for the evaluation of a total of four different types of the Galerkin system-matrix elements (wire/wire, wire/dielectric, dielectric/wire and dielectric/dielectric).

Numerical results obtained by the proposed method (some of which are presented in this paper) are in excellent agreement with available experimental results and those obtained by other methods. However, when compared with available (subdomain) general methods for the analysis of the same type of structures, the proposed method requires significantly less unknowns (as a rule, for an order of magnitude). It therefore enables the analysis of wire/dielectric structures exceeding moderate electrical size on even standard personal computers, in a very reasonable amount of time.

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2 Theory

Consider a perfectly conducting wire antenna in the presence of an inhomogeneous, lossy dielectric body, situated in a vacuum. The wire segments can be curved, and positioned arbitrarily. The body may consist of several parts, possibly in contact with one another. Assume that the permittivity and conductivity of the dielectric, ε and σ, are known functions of the space co-ordinates, and that $\mu = \mu_0$. Finally, let the wire/dielectric structure be excited by a time harmonic incident field, of complex electric field intensity $\mathbf{E}_i$ and angular frequency ω, that may be a plane wave or the field of one or more concentrated generators.

The incident field induces surface conduction currents, of density $\mathbf{J}_s$, over the surfaces of the wires, and volume polarisation and conduction currents, of total density $\mathbf{J}$, in the volume of the dielectric body. These currents can be considered in a vacuum, so that the scattered electric field, $\mathbf{E}$, due to them can be expressed in terms of the retarded Lorentz potentials, $\mathbf{A}$ and Φ. In calculating the potentials, we approximate the actual current distribution $\mathbf{J}_s$ over the wire surface by a line current of intensity I along a generatrix of the wire (the reduced-kernel approximation). The line, surface and volume charge densities in the expression for the scalar-potential are expressed in terms of I and $\mathbf{J}$ by the continuity equation. We thus obtain

$$\mathbf{E} = \mathbf{E}(\mathbf{J}, I) = -j\omega\mathbf{A} - \mathrm{grad}\Phi \tag{1}$$

$$\mathbf{A} = \mu_0 \left(\int_l I g dl + \int_V \mathbf{J} g dV \right) \tag{2}$$

$$\Phi = \frac{j}{\omega\epsilon_0} \left[\int_l \frac{dI}{dl} g dl + \int_V \mathrm{div}\mathbf{J} g dV + \int_{S_d} \mathbf{n}\cdot(\mathbf{J}_1 - \mathbf{J}_2) g dS \right] \tag{3}$$

where l is the wire generatrix, and V is the domain of the dielectric body. S_d is the surface of discontinuity in the dielectric properties, i.e. the boundary surface of dielectrics 1 (with current density $\mathbf{J}_1$) and 2 (with current density $\mathbf{J}_2$). The unit vector $\mathbf{n}$, normal to the surface S_d, is directed into dielectric 1. (In air, of course, $\mathbf{J} = 0$.) The free-space Green's function, g, is given by

$$g = \frac{e^{-j\beta_0 R}}{4\pi R}, \quad \beta_0 = \omega\sqrt{\epsilon_0\mu_0} = \frac{2\pi}{\lambda_0} \tag{4}$$

R is the distance of the field point, P, from the source point, P′, while β_0 and λ_0 are the free-space propagation coefficient and wavelength, respectively.

On wire surfaces, the locally axial tangential component of the total (incident plus scattered) electric field vector is zero. By the theorem on extended boundary conditions [7], this can also be assumed to hold along the local wire axis. On the other hand, the total electric field vector inside the dielectric is connected with the vector $\mathbf{J}$ by the generalised local Ohm's law. So we obtain

$$\begin{cases} -E_a(\mathbf{J}, I) = E_{ia} & \text{(along axes of wires)} \\ \mathbf{J}/\sigma_e - \mathbf{E}(\mathbf{J}, I) = \mathbf{E}_i & \text{(inside dielectric bodies)} \end{cases} \tag{5}$$

where E_a stands for the axial electric-field component, and $\sigma_e = \sigma + j\omega(\varepsilon - \varepsilon_0)$ is the dielectric equivalent complex conductivity. The integral equations in eqn. 5, which include eqns. 1–4, represent a system of coupled, simultaneous two-potential integral equations, with $\mathbf{J}$ and I as unknowns.

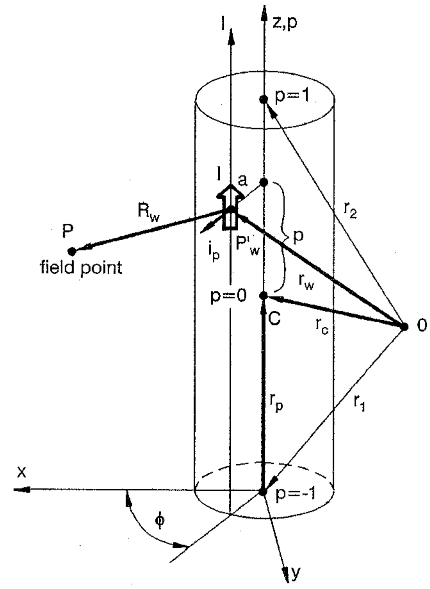

Fig. 1 *Straight wire segment*

3 Numerical solution

As the basic element for the approximation of wire structure we adopt a straight wire segment (Fig. 1). Let the field point, P, be in the x–z plane of the local rectangular co-ordinate system indicated in the figure. The position vector of a point P'_w at the segment surface (the source point) can be expressed as

$$\mathbf{r}_w(p, \phi) = \mathbf{r}_c + \mathbf{r}_p p + a\mathbf{i}_\rho(\phi)$$
$$-1 \le p \le 1, \quad -\pi \le \phi \le \pi \tag{6}$$

where p and ϕ are local parametric co-ordinates along the wire axis and around the local wire contour, respectively, and $\mathbf{i}_\rho(\phi)$ is the corresponding local radial unit vector. The symbol a denotes the wire radius, and $\mathbf{r}_c$ and $\mathbf{r}_p$ are constant vectors shown in Fig. 1. With the adopted reduced-kernel approximation, we need to define the angle ϕ, i.e. the wire generatrix with the line current I. In this paper we adopt $\phi = \pi/2$ for all wire segments.

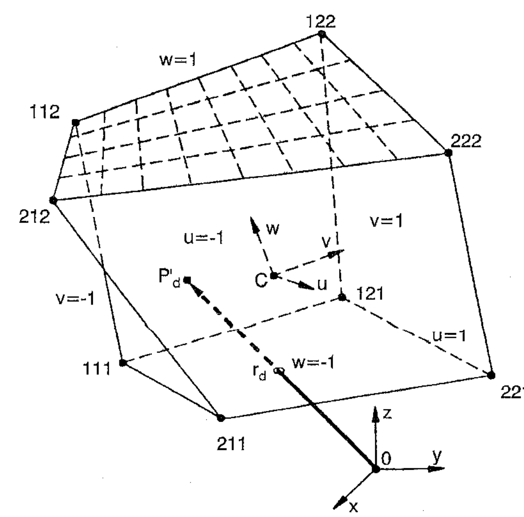

Fig. 2 *Trilinear hexahedron*

As the basic volume element for the approximation of dielectric bodies we adopt a trilinear hexahedron [5, 6], sketched in Fig. 2. This is a body defined uniquely by its eight vertices, which can be positioned in space arbitrarily. The parametric equation of the hexahedron in a local (generally nonorthogonal) *u-v-w* co-ordinate system in the figure reads

$$\mathbf{r}_d(u,v,w) = \mathbf{r}_c + \mathbf{r}_u u + \mathbf{r}_v v + \mathbf{r}_w w + \mathbf{r}_{uv} uv + \mathbf{r}_{uw} uw + \mathbf{r}_{vw} vw + \mathbf{r}_{uvw} uvw$$
$$-1 \le u,v,w \le 1 \qquad (7)$$

where $\mathbf{r}_d$ is the position vector of a hexahedron point, P'_d, and $\mathbf{r}_c$, $\mathbf{r}_u$, $\mathbf{r}_v$, $\mathbf{r}_w$, $\mathbf{r}_{uv}$, $\mathbf{r}_{uw}$, $\mathbf{r}_{vw}$ and $\mathbf{r}_{uvw}$ are constant vectors that can be expressed in terms of the position vectors of the hexahedron vertices, $\mathbf{r}_{111}$, ..., $\mathbf{r}_{222}$. The hexahedron edges and all co-ordinate lines are straight, but its sides in the general case are curved.

The current intensity, $I(p)$, along the adopted wire generatrix in Fig. 1 is approximated as

$$I(p) = \sum_{i=0}^{N_p} b_i \xi_i(p)$$
$$\xi_i(p) = \begin{Bmatrix} 1-p, & i=0 \\ p+1, & i=1 \\ p^i - 1 & i = 2,4,\ldots,N_p(2) \\ p^i - p & i = 3,5,\ldots,N_p(2) \end{Bmatrix}$$
$$-1 \le p \le 1 \qquad (8)$$

where b_i are unknown complex coefficients, and $N_p + 1$ is the adopted number of terms. The first two terms, $b_0(1 - p)$ and $b_1(p + 1)$, serve to automatically satisfy the first Kirchhoff law at the wire-segment interconnection and free ends. The higher-order terms are zero at the segment ends, $\xi_i(\pm 1) = 0$, for $i = 3, 4, ..., N_p$, and serve to improve the current approximation along the segment.

The local *u*-component of the current-density vector, $\mathbf{J}$, in the hexahedron in Fig. 2 is approximated by the following three-dimensional polynomial:

$$J_u = \frac{\sigma_e}{\epsilon_e}[Q_s(u,v,w)\cos\gamma(u,v,w)]^{-1} \times \sum_{i=0}^{N_u}\sum_{j=0}^{N_v-1}\sum_{k=0}^{N_w-1} c_{uijk}\xi_i(u)v^j w^k$$
$$-1 \le u,v,w \le 1 \qquad (9)$$

where the function ξ is defined in eqn. 8, and $\varepsilon_e = \varepsilon - j\sigma/\omega$ is the dielectric equivalent complex permittivity. N_u, N_v and N_w are the adopted degress of the polynomials, c_{uijk} are unknown complex coefficients, and γ is the angle between the *u*-co-ordinate line and the normal to the *v-w* co-ordinate surface at the point (u, v, w). In particular, for $u = \pm 1$, $(\varepsilon_e/\sigma_e)J_u \cos\gamma$ represents the component of the equivalent electric displacement vector, $\mathbf{D}_e = \varepsilon_e(\mathbf{E}_i + \mathbf{E})$, normal to the hexahedron surface, which enters the corresponding continuity boundary condition. In analogy with wires, the terms $c_{u0jk}(1 - u)v^j w^k$ and $c_{u1jk}(u + 1)v^j w^k$ serve for the automatical adjustment of this condition at the hexahedron interconnections. Finally, the function Q_s, defined as the ratio of the differential surface element dS_{vw} (in the form of an infinitely small parallelogram with sides along the *v*- and *w*-co-ordinate lines) at a point (u, v, w) and the parametric differential element $dv\,dw$, enables significant simplification of the integral expressions for the potentials. Analogous expansions are defined for the current-density vector components J_v and J_w, with the same degrees of the polynomials, N_u, N_v and N_w.

If the wire/dielectric structure considered represents a transmitting antenna, the incident field, $\mathbf{E}_i$, is computed as the field of the TEM magnetic-current frill.

The unknown current-distribution parameters, $\{b\}$ and $\{c\}$, defined in eqns. 8 and 9, are obtained by solving the system of coupled integral equations in eqn. 5 by means of the Galerkin method [8]. After a partial integration, the resulting generalised impedances (the elements of the system matrix) can be given in the following form:

$$Z_{mn} = \begin{cases} j\omega \int\limits_{l_m} I_m A_{na} dl_m - \int\limits_{l_m} \Phi_n \frac{dI_m}{dl_m} dl_m \\ \quad \text{if the } m\text{th element is a wire segment} \\ \int\limits_{V_m} \mathbf{J}_m \cdot \mathbf{J}_n/\sigma_e dV_m + j\omega \int\limits_{V_m} \mathbf{J}_m \cdot \mathbf{A}_n dV_m \\ - \int\limits_{V_m} \Phi_n \mathrm{div}\mathbf{J}_m dV_m + \oint\limits_{S_m} \Phi_n \mathbf{J}_m \cdot d\mathbf{S}_m \\ \quad \text{if the } m\text{th element is a hexahedron} \end{cases} \qquad (10)$$

In this equation, I_m and $\mathbf{J}_m$ are testing functions defined in the *m*th element of the geometrical model of the system, which may be a wire segment, of length l_m, or a trilinear hexahedron, of volume V_m and boundary surface S_m. $\mathbf{A}_n$ and Φ_n are the potentials due to basis functions I_n or J_n, defined in the *n*th geometrical element.

It is not difficult to conclude that the generalised impedances defined in eqn. 10 can be represented as linear combinations of nine basic types of Galerkin integrals, depending on the domain of the outer (test) and inner integration, which may be a wire segment (L), a side of the trilinear hexahedron (S), or the volume of a trilinear hexahedron (V). Thus we have the Galerkin integrals of the type L/L, L/S, ..., S/V and V/V. They all contain power functions of parametric co-ordinates *p*, *u*, *v* and *w*.

The rapid and accurate numerical/analytical integration methods are developed for the basic potential integrals, contained in the expressions for the potentials $\mathbf{A}_n$ and Φ_n. Of course, these integrals can be of the L, S or V type, depending on the domain of integration. When the distance R in the Green's function, given in eqn. 4, is relatively small, the procedure of extracting the (quasi) singularity is performed. As could be expected, the problems with the (quasi) singular integrals are most pronounced in cases when wires are in contact with dielectric bodies. According to the procedure, the function $f \cos \beta_0 R$, where $f = p^i$ for the L integrals, $f = u^i v^j$ for the S integrals, and $f = u^i v^j w^k$ for the V integrals, is first expanded into Taylor series about the (quasi) singular point of the Green's function. Several principal, (quasi) singular, terms of these expansions, divided by $4\pi R$, are then extracted from the integrands and integrated analytically, so that the remainder, which is well behaved for $R \approx 0$, can be integrated numerically with high accuracy, even with relatively low orders of the integration formulae.

The algorithm for multiple numerical integration, based on the Gauss–Legendre integration formula, is optimised, to avoid redundant operations relating to the summation indices in the integration formulae, as well as the indices *i*, *j* and *k* of the power testing and basis functions.

Finally, the algorithm for efficient, nonredundant recursive construction of the Galerkin impedance matrix is used, in which, for any pair of geometrical elements, first and only once, the basic Galerkin integrals for all values of the indices i, j and k are recursively evaluated, and then simultaneously introduced into all impedances containing them. It should be noted that, if the V/V integrals are considered, the evaluation of which is obviously the most time-consuming, there are nine combinations for the corresponding impedances, relating to the u-, v- and w-components of the vector $\mathbf{J}$ in the two hexahedrons, and in all of them it is necessary to evaluate both $\mathbf{A}$ and Φ.

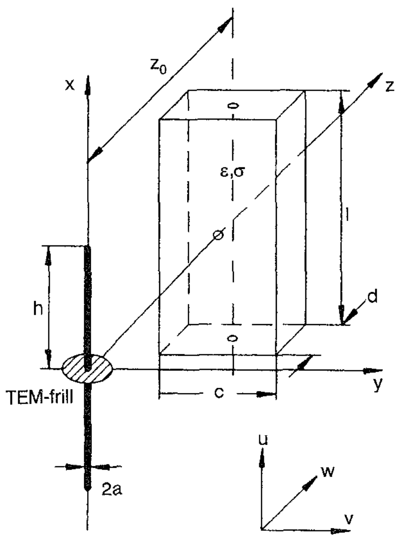

Fig. 3 *Dipole wire antenna near a parallelepiped of biological tissue*

With the lowest degrees of current approximation, $N_p = 1$ for wires (piecewise linear basis functions) and $N_u = N_v = N_w = 1$ for dielectric hexahedrons (3D rooftop basis functions [9]), the subdomain (small-domain) version of the proposed method is obtained. When compared with the entire-domain (large-domain) version of the method, it requires much more unknowns for a given problem. This is, of course, the result of the flexibility of the polynomial current approximation. On the other hand, as a consequence of the efficiency of the procedure for the evaluation of the Galerkin generalised impedances in eqn. 10, briefly described above, the impedance-matrix fill time per unknown in the entire-domain version of the method is, on average, not larger than the corresponding time with the method in its subdomain version. So the overall computation time needed for solving the same problem with approximately equal accuracy strongly favours the entire-domain method for the analysis of wire/dielectric antennas proposed in this paper over the subdomain solutions, at least as far as the subdomain version of the proposed method is considered as the reference subdomain solution.

The resulting system of linear algebraic equations with complex unknowns $\{b\}$ and $\{c\}$ is solved classically, by the Gaussian elimination.

4 Results

Consider first a symmetrical wire dipole-antenna near a homogeneous lossy-dielectric parallelepiped (Fig. 3). The dielectric parameters are $\varepsilon_r = 71$ and $\sigma = 4.4$ S/m (a biological tissue), the frequency of the generator $f = 600$ MHz, and the dimensions of the structure $h = 12.5$ cm, $a = 0.3125$ cm, $l = 25$ cm, $c = 6.25$ cm and $d = 1.56$ cm.

Each antenna arm was represented as a single segment with the degree $N_p = 4$ of the current approximation, and the parallelepiped as a single trilinear hexahedron with adopted degrees of current approximation $N_u = 4$, $N_v = 2$ and $N_w = 1$ (the total number of geometrical elements was $N_{el} = 3$). This resulted in $(N_{un})_{wires} = 7$ unknowns for the wire antenna plus $(N_{un})_{diel} = 38$ unknowns for the dielectric body (a total of $N_{un} = 45$ unknowns) and $T_{CPU} = 5.7$ s of CPU time on a PC-486/66MHz. The existing symmetry was not utilised.

Fig. 4 shows the difference in the antenna impedance when the parallelepiped is present, and when the antenna is isolated, $\Delta Z_a = Z_a - (Z_a)_0$, against the distance z_0 shown in Fig. 3. The results obtained by the proposed method are compared with those obtained with the subdomain moment-method [2], where the number of unknowns for the field approximation in the dielectric body was 192 (the number of unknowns

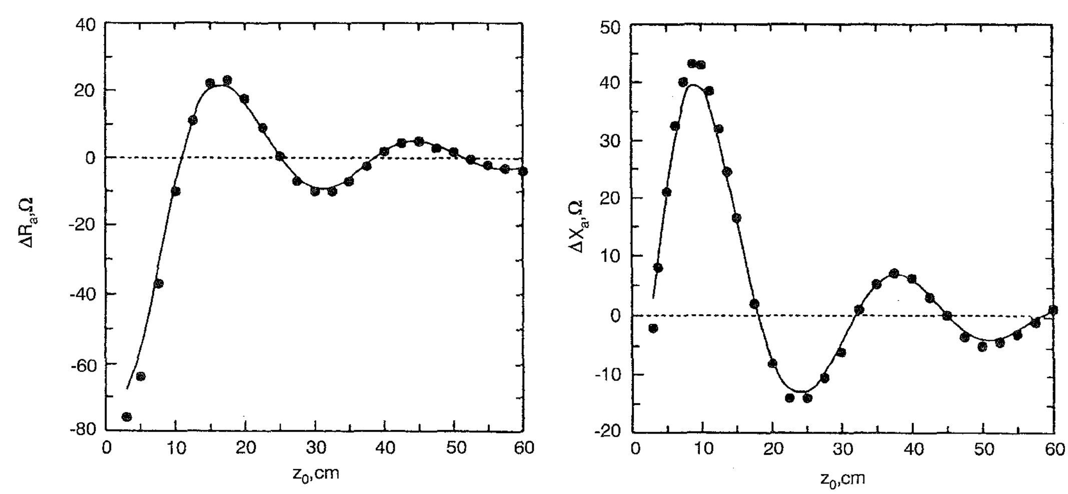

Fig. 4 *Difference of the impedance of a dipole in presence of a dielectric body in Fig. 3, Z_a, and impedance of dipole when isolated, $(Z_a)_0$, plotted against distance z_0*
a Difference in resistance
$(R_a)_0 = 97.50\ \Omega$ (EDGM); $(R_a)_0 = 103.62\ \Omega$ ([2])
b Difference in reactance
$(X_a)_0 = 43.22\ \Omega$ (EDGM); $(X_a)_0 = 49.31\ \Omega$ ([2])
$f = 600$ MHz, $h = 12.5$ cm, $a = 0.3125$ cm, $l = 25$ cm, $c = 6.25$ cm, $d = 1.56$ cm, $\varepsilon_r = 71$, $\sigma = 4.4$ S/m
——— entire-domain Galerkin method (EDGM), $(N_{un})_{wires} = 7$, $(N_{un})_{diel} = 38$
● ● ● method [2], 192 unknowns for dielectric body

relating to the wires was not given). Excellent agreement between the two sets of results is observed. Note that the number of unknowns for the dielectric body required in [2] is five times that required by the present method.

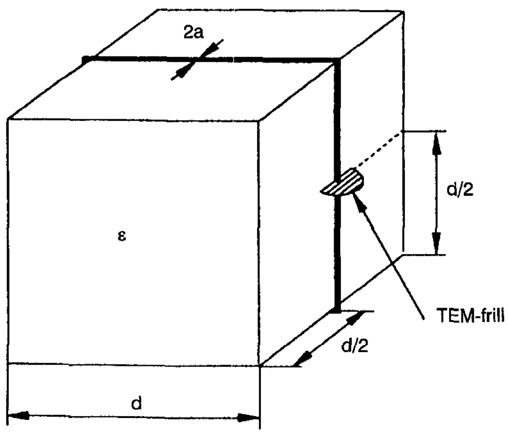

Fig. 5 *Square-loop wire antenna wound onto a dielectric cube*

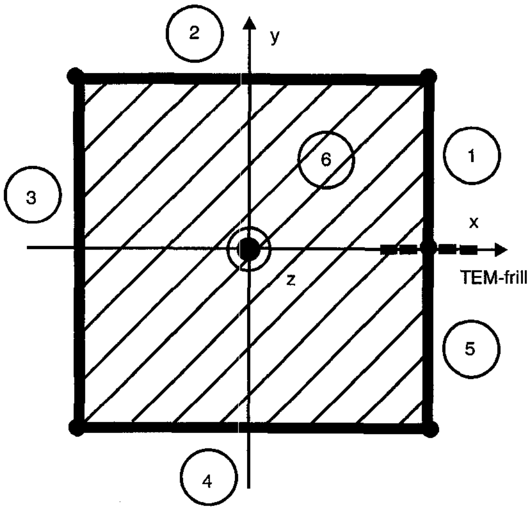

Fig. 6 *Cross-section of geometrical model of structure with six elements*

Consider now a square-loop wire antenna wound onto a perfect-dielectric core in the form of a cube, of relative permittivity $\varepsilon_r = 2.1$ ($\sigma = 0$), as sketched in Fig. 5. Let the cube side length be $d = 2$ in (5.08cm), and the wire radius $a = 0.02$in (0.508mm). The wire loop was divided into five segments, and the cube was considered as a single hexahedron ($N_{el} = 6$), as in Fig. 6. The adopted degrees of the polynomial approximation were $N_p = 4$ for all wire segments [$(N_{un})_{wires} = 20$], and $N_u = N_v = N_w = 4$ for the cube [$(N_{un})_{diel} = 240$]. This resulted in $T_{CPU} = 67.2$s with PC-486/66MHz (45.5s for the evaluation of the impedance-matrix plus 21.7s for solving the system of linear equations). Symmetry was not utilised.

Fig. 7 shows the impedance, Z_a, of the antenna, plotted against $4d/\lambda_0$ in the range 0.3–1.4, which corresponds to a frequency range 442.91–2066.93MHz. The results obtained by the proposed method are compared with experimental and numerical results from [2]. Excellent agreement between the three sets of results is observed, the number of unknowns for the dielectric body requred by the subdomain method in [2] without using symmetry (3000) being 12.5 times that requred by the present method.

5 Conclusions

A Galerkin-type method is proposed for the analysis of arbitrary wire antennas and scatterers in the presence of inhomogeneous lossy dielectric bodies of arbitrary shape and finite extent. It is based on the solution of simultaneous two-potential integral equations and an entire-domain (or, more precisely, large-domain) philosophy. Basic elements for geometrical modelling are straight wire segments (for wire antennas) and trilinear hexahedrons (for dielectric bodies). These elements (domains) may be electrically large. Current distribution along the wires and in the dielectrics is approximated by one- and three-dimensional polynomials in local parametric co-ordinates, respectively, which satisfy automatically the continuity equation at wire-segment ends and interconnections, and over hexahedron sides.

Numerical results obtained by the proposed method (some of which have been presented in the paper) have demonstrated that it is accurate, reliable and very effi-

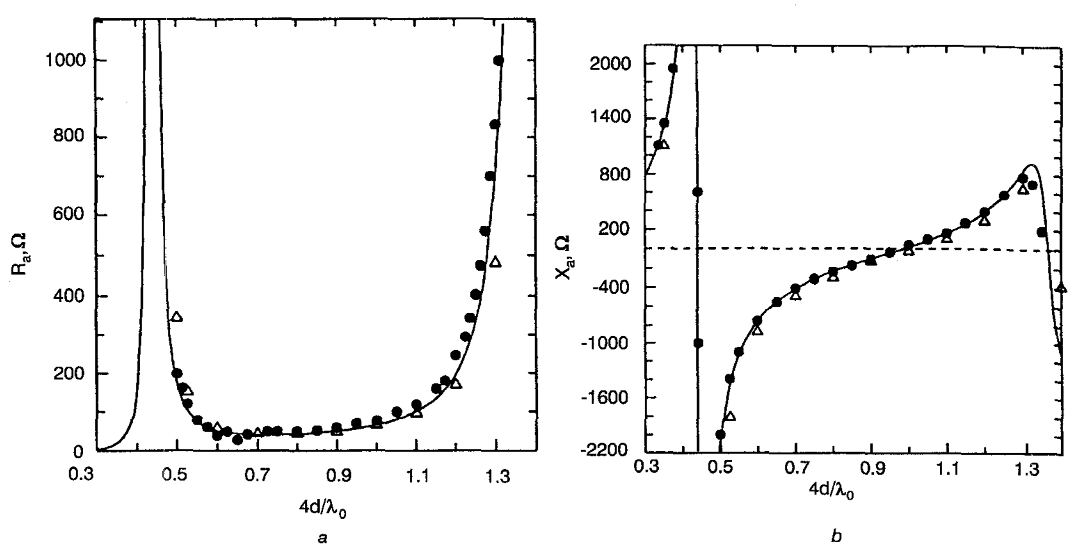

Fig. 7 *Impedance of loop antenna with a dielectric core from Fig. 5, for $\varepsilon_r = 2.1$, $\sigma = 0$, $d = 2$ in and $a = 0.02$ in, plotted against $4d/\lambda_0$*
a Resistance
b Reactance
——— entire-domain Galerkin method, $(N_{un})_{wires} = 20$, $(N_{un})_{diel} = 240$
● ● ● experimental results [2]
△ △ △ method [2], 3000 unknowns for dielectric body

cient. Perhaps the most important advantage of the proposed method over the existing general methods for the analysis of wire/dielectric structures is that it requires fewer unknowns (on average for about an order of magnitude), which enables systems exceeding medium electrical size to be analysed on standard personal computers, in a reasonable amount of time. This appears to be the unique feature of the proposed method when compared with the existing methods.

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