Second Order System Governing Equations

from
"Introduction to Mechatronics and Measurement Systems" (2nd edition)
by Alciatore and Histand

differential equation: $$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_{\text{ext}}(t)$$

natural frequency: $$\omega_n = \sqrt{\frac{k}{m}}$$

critical damping constant: $$b_c = 2\sqrt{km} = 2m\omega_n$$

damping ratio("zeta"): $$\zeta = \frac{b}{b_c} = \frac{b}{2\sqrt{km}}$$

damped natural frequency: $$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

Frequency Response

forcing function: $$F_{\text{ext}}(t) = F_o \sin(\omega t)$$

steady state frequency response: $$x(t) = X_o \sin(\omega t + \phi)$$

amplitude ratio: $$\frac{X_o}{F_o/k} = \frac{1}{\left\{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + 4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2\right\}^{1/2}}$$

phase angle: $$\phi = -\tan^{-1}\left\{\frac{2\zeta}{\frac{\omega_n}{\omega} - \frac{\omega}{\omega_n}}\right\}$$

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