QUANTIFICATION OF INCISED CHANNEL EVOLUTION AND EQUILIBRIUM¹

Brian P. Bledsoe, Chester C. Watson, and David S. Biedenharn²

ABSTRACT: Incised channels are caused by an imbalance between sediment transport capacity and sediment supply that alters channel morphology through bed and bank erosion. Consistent sequential changes in incised channel morphology may be quantified and used to develop relationships describing quasi-equilibrium conditions in these channels. We analyzed the hydraulic characteristics of streams in the Yazoo River Basin, Mississippi in various stages of incised channel evolution. The hydraulic characteristics of incising channels were observed to follow the sequence predicted by previous conceptual models of incised channel response. Multiple regression models of stable slopes in quasi-equilibrium channels that have completed a full evolutionary sequence were developed. These models compare favorably with analytical solutions based on the extremal hypothesis of minimum stream power and empirical relationships from other regions. Appropriate application of these empirical relationships may be useful in preliminary design of stream rehabilitation strategies.

(KEY TERMS: channel evolution; erosion; stream incision; sedimentation.)

INTRODUCTION

The beds of incised channels are lowered by degradation that sets in motion a period of considerable instability with potential for serious damage to aquatic and riparian habitat, adjacent and in-channel infrastructure, and adjacent agricultural improvements (Schumm *et al.*, 1984; Shields *et al.*, 1994). During the period of instability, which may persist for decades or longer depending on watershed size and materials, engineers and watershed managers may be

faced with an apparently chaotic assemblage of diverse channel morphologies. Researchers have noted that incising channels in different environments, destabilized by different natural and humaninduced disturbances, pass through a consistent, predictable, sequence of channel forms with time (Davis, 1902; Ireland et al., 1939; Schumm and Hadley, 1957; Daniels, 1960; Emerson, 1971; Keller, 1972; Elliott, 1979; Schumm et al., 1984; Watson et al., 1988; Simon, 1989). The observed systematic response of incising channels to changes in sediment transport and sediment supply is referred to as channel evolution. Recognizing specific stages of channel evolution permits the development of temporal and spacial relationships in the watershed. These relationships can then be used for prediction of future channel forms and processes.

The objectives of this paper are to describe efforts to predict quasi-equilibrium slopes of incised channels using empirical data, and to describe a computational procedure that is useful in the design of incised channel rehabilitation measures. Therefore, quantification of hydraulic and sediment transport characteristics for the quasi-equilibrium reaches is of critical importance. The new approach allows direct specification of a desired sediment transport capacity and is more flexible than previous approaches, allowing a wide range of stream rehabilitation strategies to be evaluated.

¹Paper No. 00110 of the Journal of the American Water Resources Association. Discussions are open until February 1, 2003.

²Respectively, Assistant Professor and Professor, Department of Civil Engineering, Colorado State University, Fort Collins, Colorado 80523; and Research Hydraulic Engineer, Water Experiment Station, U.S. Army Corps of Engineers, Vicksburg, Mississippi 39180 (E-Mail/Bledsoe: bbledsoe@engr.colostate.edu).

BACKGROUND

Study Area

In 1984, the U. S. Congress directed the initiation of the Demonstration Erosion Control (DEC) Project as a systems approach to sediment, erosion and flood control in six watersheds in north central Mississippi. Between 1985 and 1989 an additional nine watersheds were added to the Project. In 1996, the Yalobusha River watershed was added upstream from Grenada Dam. Figure 1 is a map of the 16 DEC watersheds, and the study sites are taken from the Yalobusha River, Abiaca Creek, Batupan Bogue, Burney Branch, and Long Creek watersheds.

Channel Evolution Model (CEM)

Conceptual incised channel evolution models (CEM) have been of value in developing an understanding of watershed and channel dynamics, and in characterizing stable reaches of these channels. Location-for-time substitution was used by Schumm *et al.* (1984) to generate a five-stage, incised channel evolution sequence that described the erosion evolution of Oaklimiter Creek, a tributary of Tippah River in northern Mississippi (Figure 2). This system is a useful, field-oriented model based primarily on empirical data.

In each reach of an idealized channel, at a given time, Types I through V occur in series and, at a given location, Types I through V will occur in the channel through time. The channel evolution model describes the systematic response of a channel to base level lowering, and encompasses conditions that range from disequilibrium to a new state of dynamic equilibrium. The following paragraphs characterize the conceptual types. It should be recognized that these categories are only conceptual and variation may be encountered in the field.

Type I reaches are characterized by a sediment transport capacity that exceeds sediment supply, bank height (h) that is less than the critical bank height (hc), a U-shaped cross section, small precursor knick-points in the bed of the channel providing that the bed material is sufficiently cohesive, and little or no bed material deposited. The critical bank height is the bank height at which the bank instability is beginning to develop for the existing soil conditions and channel morphology. Width-depth ratios at bankfull stage are highly variable. A knickpoint is an abrupt change in thalweg elevation, and may be visualized as a small waterfall.

Type II reaches are located immediately downstream of the primary knickpoint and are characterized by a sediment transport capacity that exceeds sediment supply, a bank height that is less than the critical bank height (h < h_c), little or no bed sediment deposits, a lower bed slope than the Type I reach, and a lower width-depth ratio value than the Type I reach because the depth has increased but the banks are not failing.

Type III reaches are located downstream of Type II reaches and are characterized by a sediment transport capacity that is highly variable with respect to the sediment supply, a bank height that is greater than the critical bank height (h > h_c), erosion that is due primarily to slab failure (Bradford and Piest, 1980), bank loss rates that are at a maximum, and variable bed sediment accumulation that may be present, absent, or locally significant due to local erosion sources. Channel depth is somewhat greater than in Type II. The channel is widening due to bank failure.

Type IV reaches are downstream of Type III reaches and are characterized by a sediment supply that exceeds sediment transport capacity resulting in aggradation of the channel bed, a bank height that approaches the critical bank height with a rate of bank failure lower than Type III reaches, a nearly trapezoidal cross-section shape, and a width-depth ratio higher than the Type II reaches. The Type IV reach is aggradational and bank height is less than Type III. Bank failure has increased channel width, and in some reaches the beginnings of berms along the margins of an effective discharge channel can be observed. These berms are the initiation of natural levee deposits that form in aggraded reaches that were over-widened during earlier degradational phases. Bradford and Piest (1980) observed that in the later phases of evolution (Type IV), the mode of bank failure changes from slab-type to circular arc failures.

Type V reaches are located downstream of Type IV reaches and are characterized by a dynamic balance between sediment transport capacity and sediment supply, a bank height that is less than the critical bank height for the existing bank angle, colonization by riparian vegetation, accumulated bed sediment, a top width-depth ratio that exceeds the Type IV reach, and generally a compound channel formed within a newly formed floodplain. Bank angles have been reduced by accumulation of failed bank materials at the toe of the slope and by accumulation of berm materials.

The sequence of channel evolution is based on the assumption that the observed changes in channel morphology are due to the passage of time in response to a single base level lowering. In actual field conditions, the sequence may be interrupted by changes in

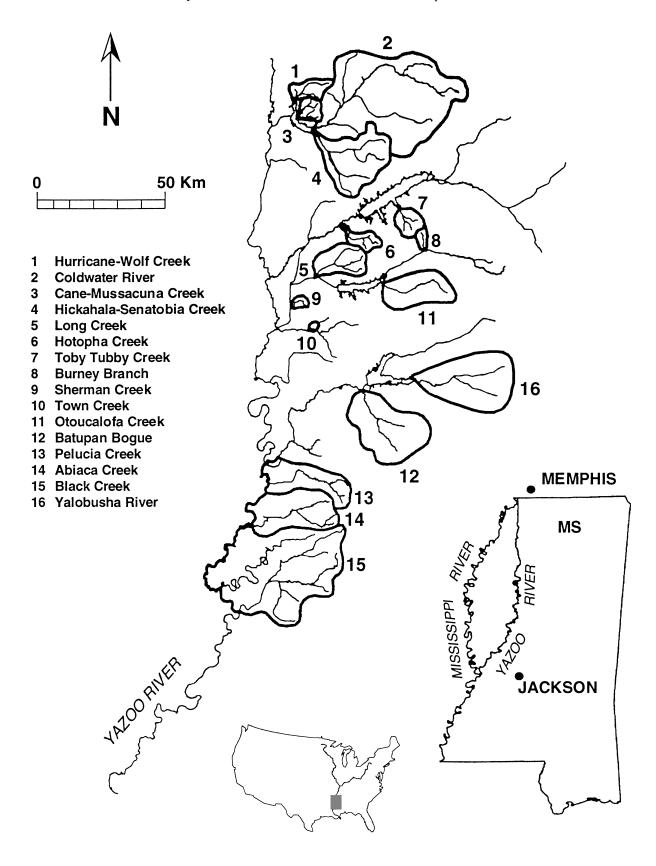
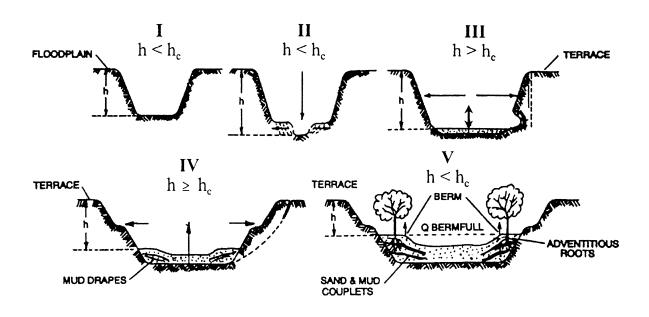
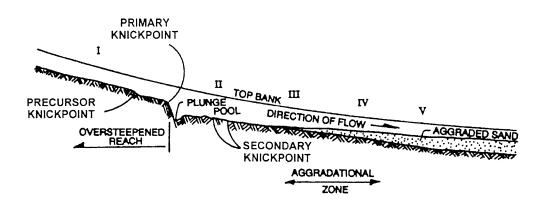


Figure 1. Map of the 16 DEC Watersheds.

INCISED CHANNEL EVOLUTION PHASES





h e= CRITICAL BANK HEIGHT

Figure 2. A Conceptual Sketch of the Incised Channel Evolution Sequence (after Schumn $et\ al.,\ 1984$). Drawings are not to scale.

the upstream land use and sediment supply from the watershed. Application of the sequence assumes that the materials forming the channel perimeter are erodible and all degrees of the channel adjustment are possible. The sequence is primarily applicable in a system context for straight channels, and local erosion such as in bends or caused by deflection of flow by

debris may cause difficulty in recognition of the sequence.

The primary value of the CEM is to determine the evolutionary state of the channel from field reconnaissance. The morphometric characteristics of the channel reach types can also be correlated with hydraulic, geotechnical, and sediment transport parameters

(Harvey and Watson, 1986; Watson *et al.*, 1988). Quantification of hydraulic and sediment transport characteristics, in particular for the CEM Type IV and V reaches, is a primary topic of this paper.

Slope-Area Relationships

In an effort to identify stable slopes for incising channels in the Yazoo River Basin, the U.S. Army Corps of Engineers (USACE) has developed empirical slope-area diagrams that are analogous to the traditional Q-S diagrams used in discriminating meandering and braided channels (e.g. USACE, 1990; Figure 3). Discriminators derived from such a diagram also correspond to stream power since drainage area, within a region, is a surrogate for dominant discharge. This approach was proposed by Schumm et al. (1984) who defined the "Area-Gradient Index" (AGI) as a measure of total stream power. The AGI is simply the product of slope and drainage area. Slope-area diagrams use space for time substitution and the characteristics of stable channels that are presumably near equilibrium prior to or following a full evolutionary sequence.

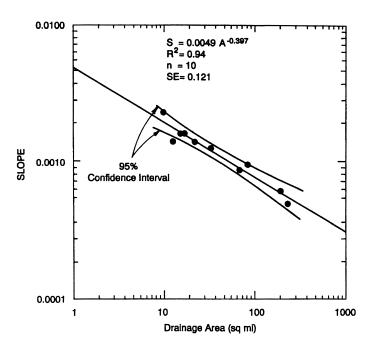


Figure 3. Slope-Drainage Area Relationship (from USACE, 1990).

A slope-area relationship based on 26 CEM Type IV and V streams in the Yazoo Basin of Mississippi was developed

$$S = 0.0031 \text{ A}^{-0.400} \tag{1}$$

where A is basin area in square miles and S is the dimensionless stream slope (ft/ft) (Watson and Bledsoe, 1999). Care must be taken in the application of this relationship since data from basin areas outside the range of 10 to 30 square miles are still relatively sparse.

The slope-area diagram is strictly empirical, providing a relationship between drainage area and channel slope for quasi-equilibrium reaches (CEM Types IV and V) in a given watershed and for the conditions encountered in that watershed. Implicitly included in the slope-area diagram are relationships between stream sediment supply and transport capacity, bed material size, and variation in water discharge. If strategies for stream rehabilitation include changing sediment supply or improvements that may affect the channel forming discharge, the slope-area diagram technique is poorly suited for use, and a more useful design technique would specifically include sediment supply, sediment transport and water discharge.

Other Empirical Relationships

Relationships among basin geometry, annual flood discharge, slope, and sedimentary characteristics were used in developing the new design procedure. Hack (1957) proposed an empirical relationship of mainstream length in miles (L) to drainage basin area (A) in square miles:

$$L = 1.4A^{0.6} \tag{2}$$

This relationship has subsequently been shown to hold for a wide range of conditions (Ijjasz-Vasquez *et al.*, 1993) and Figure 4 illustrates a similar relationship for the Yalobusha Basin.

Hack (1957) also proposed the following relationship for relating slope, S, (ft/mi) to drainage area (sq mi) and bed material (mm) in streams in Virginia and Maryland

$$S = a \left(\frac{d_{50}}{A}\right)^b \tag{3}$$

The d_{50} is the bed material particle size of which 50 percent are finer. Schröder (1991) used local slope data from Hack (1957) and reported that a=0.0076 and b=0.4 for units of mm and km². If Hack's units are converted to units of mm and square miles, the value of a becomes 0.00517 and the value of b remains

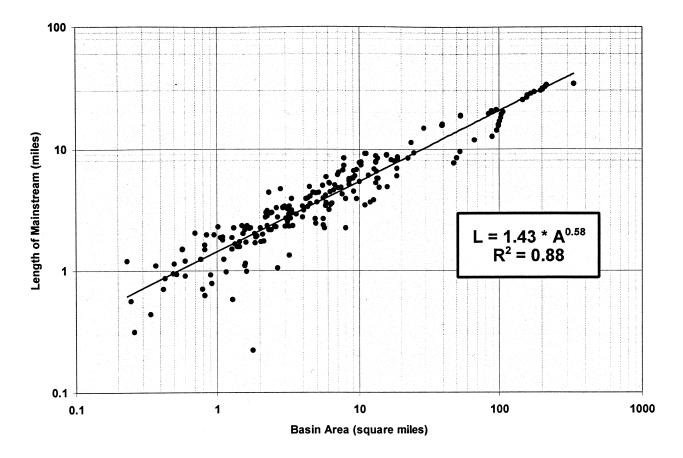


Figure 4. Relationship Between Mainstream Length and Basin Area in the Yalobusha Basin.

equal to 0.4. The general form of Hack's equation was also verified for streams in West Germany by Schröder (1991) which resulted in values of a = 0.00449 and b = 0.4 when converted to units of mm and square miles.

The more familiar value of b=0.6 originally reported by Hack (1957) was developed using mapmeasured slope data, not the field-measured local slope values. He also reported that map-measured slope values depart erratically from the field-measured values on gentle slopes; therefore, field-measured data were used in the present investigation.

Landers and Wilson (1991) developed empirical relationships for estimating peak discharges having recurrence intervals that range from 2 to 500 years in Mississippi. The corresponding relationship for the Yazoo Basin watersheds is

$$Q_2 = 66.2 A^{0.88} S_0^{0.51} L^{-0.11}$$
 (4)

where Q_2 is the two-year recurrence interval discharge; A is the drainage area in square miles; and $S_{\rm o}$

is the channel slope in feet per mile defined as the difference in altitude between points located at 10 and 85 percent of the main channel length divided by the channel length between the two points as determined between topographic maps. Similar regionalized discharge relationships are available for most locations in the U.S. (Jennings *et al.*, 1994).

An analysis of data on mainstream length and basin area was performed to verify the applicability of Equation (2) to the Yazoo Basin data. As shown in Figure 4, the Yazoo Basin data closely match Equation (2). Substituting Equation (2) into the USGS Rural Flood Regression Equation for Q_2 (Landers and Wilson, 1991) and converting to a unitless slope yields

$$Q_2 = 5050A^{0.814}S^{0.51} \tag{5}$$

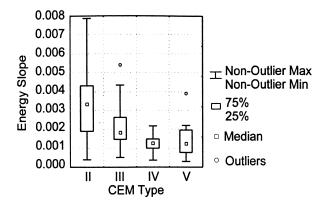
This relationship between the two-year recurrence interval flood and drainage area facilitates comparisons of the traditional slope-area approach with analytical methods based on channel-forming discharge.

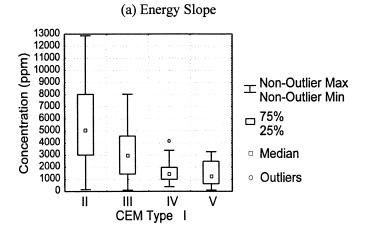
ANALYSIS OF CHANNEL EVOLUTION DATA

Field-based Empirical Data

The sediment transport capacity of each of the 26 Yazoo Basin study reaches was computed by Watson et al. (1996a). These data indicate that sediment transport capacity decreases as the channel evolves to stability. An analogous treatment of these data (Watson et al., 1996b) also indicated that as the channel evolved to a CEM Type V condition, the reach approached the morphology predicted by the minimum stream power solution of the SAM program (Thomas et al., 1994).

Data on hydraulic characteristics and CEM Type from all streams were used to compare trends in slope, sediment transport capacity (shown as concentration), and specific stream power across evolutionary stages for a set of 26 stream reaches. Drainage area for the basins contributing to the 26 stream reaches ranged from 1.3 to 120 square miles, with a median value of 10.2 square miles. For these drainages, the two-year recurrence interval discharge ranged from 6,200 cfs to 269 cfs, with a median discharge of 1,369 cfs. The assumption of uniform flow is made for this investigation, allowing the comparison of bed slopes and computed energy slopes. Box and whisker plots are shown in Figure 5 for: (a) energy slope, (b) the concentration, and (c) the specific stream power for all reaches at the two-year recurrence interval discharge. The concentration was computed in parts per million (ppm) using the Brownlie (1981) sediment transport relationship for the twoyear recurrence interval discharge. The specific stream power was computed as the product of the specific weight of water, the two-year recurrence interval discharge, and the energy slope, divided by the channel width. Units of specific discharge are in Watts per square meter $(14.56 \text{ W/m}^2 = 1 \text{ ft-lb/sec/ft}^2)$. As shown in this figure, each parameter (slope, concentration, specific stream power) decreases in value from the Type II to the Type V reaches. Box and whisker graphs indicate the median value, a box containing all the data between the 25th and 75th percentiles, and whiskers to the maximum and minimum non-outlier values. Outliers were defined as values that are outside the 25th and 75th percentile range by more than 1.5 times the difference between the 25th and 75th percentiles. This analysis represents the largest combined data set of CEM type and hydraulics analyzed to date. Although there is substantial variation in hydraulic attributes within each evolutionary stage, the expected trends are clearly present. Specific stream power appears to be an excellent predictor of channel stability, with most streams attaining relative stability at specific stream power less than 30 W/m² (Figure 5c).





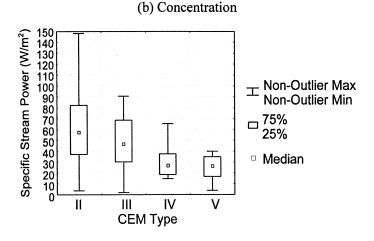


Figure 5. Box and Whisker Plots for All DEC Reaches.

(c) Specific Stream Power

Multiple regression analyses of CEM Types IV and V were performed using all available Yazoo Basin data, and resulted in the following relationship ($R^2 = 0.96$)

$$S = 0.000273C_s^{0.551}Q^{-0.343}$$
 (6)

where S is the energy slope; C_s is the estimated sediment concentration at two-year discharge (ppm); and Q is the water discharge (cfs). A comparison of energy slopes predicted by Equation (6) versus actual energy slopes for all CEM Type IV and V streams is provided in Figure 6. The close agreement shown in Figure 6 suggests that Equation (6) may be useful in predicting quasi-equilibrium slope for similar streams in the Yazoo Basin.

COMPUTATIONALLY GENERATED DATA

Watson et al. (1998) developed a multi-linear regression equation from minimum stream power channel designs that were generated using the SAM program (Thomas *et al.*, 1994) to predict stable slopes for streams in lower and transitional regimes (concentration generally \leq 1,500 ppm and d₅₀ of 0.2-0.5 mm)

$$S = 0.000112Q^{-0.261}d_{50}^{0.503}B^{0.203}C_{s}^{0.631}$$
 (7)

which may be approximated by

$$\frac{S}{\sqrt{d_{50}}} = 0.000112Q^{-0.261}C_s^{0.631} \tag{8}$$

if bank slopes (1:B) are assumed to approximate 1:1 (H:V) (i.e., B=1).

Sediment samples are available for many of the Yazoo Basin sites. The significance of the bed material term (p = 0.0008) suggests that even within the small range of median bed material sizes occurring in these watersheds (generally 0.2 to 0.5 mm), bed material size is an important determinant of stable channel slope.

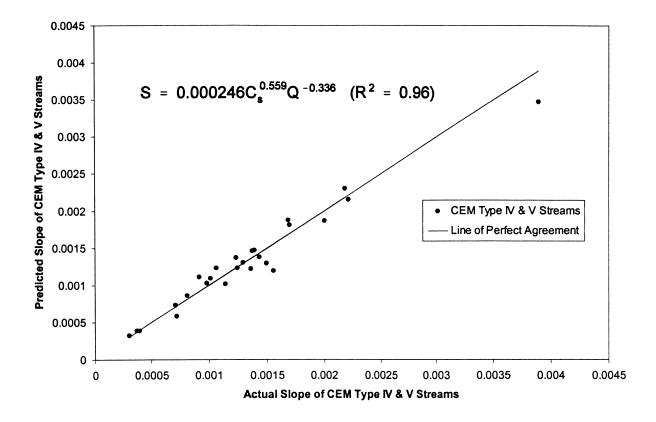


Figure 6. Observed Energy Slope Plotted Against Predicted Energy Slope for All CEM Type IV and Type V Streams in the Yazoo Basin.

COMPARISON AND ANALYSES

To facilitate comparison of actual stream data CEM Type IV and V streams with the relationships derived from SAM (Equations (7) and (8)) by Watson et~al. (1998), slope and d_{50} were combined into a single variable and the hydraulic data from CEM Type IV and V streams were re-analyzed ($R^2 = 0.971$)

$$\frac{S}{\sqrt{d_{50}}} = 0.000256 Q^{-0.260} C_s^{0.554} \tag{9}$$

The application of Equations (8) and (9) in estimating a stable channel slope for a certain discharge and d_{50} requires the selection of a desired sediment transport capacity (C_s). Assuming a constant sediment transport capacity across basin size in SAM results in a slope-area relationship that is much less sensitive to basin size than the empirical slope-area relationship of actual CEM Type IV and V streams (Figure 3). Comparison of computed sediment transport capacity (concentration) of CEM Type IV and V streams suggests that sediment concentration decreases as basin area increases, according to the following relationship ($R^2 = 0.49$)

$$C_s \approx 4670 A^{-0.512}$$
 (10)

The general form of Equation (10) is physically justifiable. It is likely that sediment concentration decreases downstream since sediment yield per unit area decreases, sediment storage increases, and groundwater contributions to streamflow increase (Rubey, 1933; Leopold and Maddock, 1953). Measurements of sediment concentrations throughout a basin during a single flow event are extremely rare, but the occurrence of sediment concentrations decreasing downstream has been documented, especially in basins with relatively high concavity (Xu, 1991).

The relationship between SAM results and the empirical slope-area relationship was further examined by substituting Equations (5) and (10) into the relationship describing minimum stream power solutions based on SAM (Equation 8). The resulting relationship

$$S = 0.00505 \left(\frac{d_{50}}{A}\right)^{0.44} \tag{11}$$

is the same form of the relationship proposed by Hack (1957) for relating slope to drainage area and bed material in streams in Virginia and Maryland. Hack's relationships and Equation (11) essentially represent

more generalized forms of the slope-area and slopedischarge approaches for predicting slopes. If the value of d_{50} in Equation (11) is assumed to equal 0.3 mm, which is approximately equal to the median bed material size for Yazoo Basin study sites, Equation (11) reduces to the following relationship

$$S = 0.003A^{-0.44} \tag{12}$$

This result indicates that the multi-linear relationship developed from minimum stream power solutions in SAM (Equation 8) is in close accordance with the slope-area relationship derived from CEM Type IV and V streams and the relationship of Hack (1957) if sediment concentration is decreased in a downstream direction according to Equation (10).

SUMMARY AND CONCLUSIONS

Hydraulic characteristics of stable incised channels evolve with decreasing slope and stream power and follow the sequence predicted by previous qualitative models of incised channel response. Comparison of field-based empirical data with minimum slope solutions computed using the SAM program (Thomas et al., 1994) indicates that the minimum slope solutions are compatible with observed quasi-equilibrium streams. Use of the SAM-developed relationships (Equations 7 and 8), or directly applying SAM for the minimum slope solution provides a useful computational procedure that explicitly includes sediment transport capacity, and an estimate of channel-forming discharge. Thus, simple but more robust computational procedures can be developed to expand the concepts recognized in the slope area diagram.

By including sediment size, discharge, and sediment concentration specifically into the computational procedure, a wide variety of channel rehabilitation measures can be evaluated during the preliminary design phase. Although the approach provides a reasonable basis for preliminary design of rehabilitation measures for incised channels, consideration of risk and uncertainty is an essential step that should be included in any design process. Following the preliminary design, a thorough analysis of sediment continuity in the system for a range of discharges should be performed using a sediment routing model, or sediment yield balance.

ACKNOWLEDGMENTS

The research presented herein was funded by the DEC Project, USACE, Vicksburg District. Their assistance is greatly appreciated.

LITERATURE CITED

- Bradford, M. J. and R. F. Piest, 1980. Erosional Development in Valley-Bottom Gullies in the Upper Midwestern United States. *In:* Thresholds in Geomorphology, D. R. Coates and J. D. Vitak (Editors), pp. 75-101.
- Brownlie, W. R., 1981. Prediction of Flow Depth and Sediment Discharge in Open Channels. Report No. KH-R-43A, W. M. Keck Laboratory, California Institute of Technology, Pasadena, California.
- Daniels, R. B., 1960. Entrenchment of the Willow Drainage Ditch, Harrison County, Iowa. American Journal of Science 258:161-176.
- Davis, W. M., 1902. Baselevel, Grade, and Peneplain. Journal of Geology 10:77-111.
- Elliot, J. G., 1979. Evolution of Large Arroyos, the Rio Puerco of New Mexico. Unpublished MSc Thesis, Colorado State University, Fort Collins, Colorado.
- Emerson, J. W., 1971. Channelization: A Case Study. Science 172:325-326.
- Hack, J. T., 1957. Studies of Longitudinal Stream Profiles in Virginia and Maryland. U.S. Geological Survey Professional Paper 294-B, 97 pp.
- Harvey, M. D. and C. C. Watson, 1986. Fluvial Processes and Morphological Thresholds in Incised Channel Restoration. Water Res. Bulletin 3(3):359-368.
- Ijjasz-Vasquez, E. J., R. L. Bras, and I. Rodriguez-Iturbe, 1993. Hack's Relation and Optimal Channel Networks: The Elongation of River Basins as a Consequence of Energy Minimization. Geophysical Research Letters 20:1583-1586.
- Ireland, H. A., C. F. Sharpe, and D. H. Eargle, 1939. Principles of Gully Erosion in the Piedmont of South Carolina. U.S. Department of Agriculture Technical Bulletin 633, 142 pp.
- Jennings, M. E., W. O. Thomas, Jr., and H. C. Riggs, 1994. Nation-wide Summary of U.S. Geological Survey Regional Regression Equations for Estimating Magnitude and Frequency of Floods for Ungaged Sites, 1993. U.S. Geological Survey, Water-Resources Investigations Report 94-4002.
- Keller, E. A., 1972. Development of Alluvial Stream Channels: A Five-Stage Model. Bulletin of the Geological Society of America 83:1531-1536.
- Landers, M. N. and K. V. Wilson, Jr., 1991. Flood Characteristics of Mississippi Streams. U.S. Geological Survey Water-Resources Investigations Report 91-4037, 82 pp.
- Leopold, L. B and T. Maddock, Jr., 1953. The Hydraulic Geometry of Stream Channels and Some Physiographic Implications. U.S. Geological Survey Professional Paper 252, 53 pp.
- Rubey, W. W., 1933. Equilibrium Conditions in Debris-Laden Streams. American Geophysical Union Transactions 497-505.
- Schröder, R., 1991. Test of Hack's Slope to Bed Material Relationship in the South-Eifel Uplands, Germany. Earth Surface Processes Landforms 16:731-736.
- Schumm, S. A. and R. F. Hadley, 1957. Arroyos and the Semiarid Cycle of Erosion. American Journal of Science 225:161-174.
- Schumm, S. A., M. D. Harvey, and C. C. Watson, 1984. Incised Channels: Morphology, Dynamics and Control. Water Resources Publications, Littleton, Colorado.
- Shields, Jr., F. D., S. S. Knight, and C. M. Cooper, 1994. Effects of Channel Incision on Base Flow Stream Habitats and Fishes. Environmental Management 18(1):43-57.
- Simon, A., 1989. A Model of Channel Response in Disturbed Alluvial Channels. Earth Surface Processes and Landforms 14:11-26.
- Thomas, W. A., R. R. Copeland, N. K. Raphelt, and D. N. McComas, 1994. User's Manual for the Hydraulic Design Package for Channels (SAM). Waterways Experiment Station, U.S. Army Corps of Engineers.

- USACE (U.S. Army Corps of Engineers), 1990. Yazoo Basin, Mississippi, Demonstration Erosion Control Project. U.S. Army Corp of Engineers, Soil Conservation Service, Vicksburg, Mississippi, General Design Memorandum No. 54.
- Watson, C. C. and B. P. Bledsoe, 1999. Demonstration Erosion Control Monitoring Sites, 1998 Evaluation. Report for U.S. Army Corps of Engineers, Waterways Experiment Station, Vicksburg, Mississippi, 157 pp.
- Watson, C. C., M. D. Harvey, D. S. Biedenharn, and P. G. Combs, 1988. Geotechnical and Hydraulic Stability Numbers for Channel Rehabilitation, Part I: The Approach. *In:* Proc. ASCE Hydraulics Division 1988 National Conference, S. R. Abt and J. Gessler (Editors), pp. 120-125.
- Watson, C. C., D. Gessler, S. R. Abt, C. I. Thornton, and P. Kozinski, 1996a. Demonstration Erosion Control Monitoring Sites, 1995 Evaluation. Annual Report DACW39-92-K-0003, Colorado State University, Fort Collins, Colorado.
- Watson, C. C., S. R. Abt, D. Gessler, and N. K. Raphelt, 1996b. Validation of Extremal Hypotheses for Stable Channel Design. Proc. of the Sixth Federal Interagency Sedimentation Conference, Las Vegas, Nevada, pp. II-49 to II-55.
- Watson, C. C., C. I. Thornton, P. Kozinski, B. P. Bledsoe, A. L. Firenzi, and A. M. Haas, 1998. Demonstration Erosion Control Monitoring Sites 1997 Evaluation. Report for U.S. Army Corps of Engineers, Waterways Experiment Station, Vicksburg, Mississippi.
- Xu, J., 1991. Study of Overall Longitudinal Profile Shape of Alluvial Rivers: An Example From Rivers in the Huanghuaihai Plain, China. Zeitschrift für Geomorphologie 35:479-490.