AN ILLUSTRATION OF GENERATING ROBOTS FROM OPTIMAL FAULT-TOLERANT JACOBIANS

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ABSTRACT

The local kinematic properties of a robotic manipulator's configuration can be described by its corresponding Jacobian matrix. Conversely, one can determine a manipulator that possesses certain desirable kinematic properties by specifying the required Jacobian. In this work, design criteria that require a manipulator to function in a configuration that is optimal under normal operation and after an arbitrary single joint fails and is locked in position is first described. Specifically, the desired Jacobian matrix must be isotropic, i.e., possess all equal singular values prior to a failure, and have equal minimum singular values for every possible single column being removed. Then a simple planar three degree-of-freedom example is used to illustrate how one can identify all of the possible manipulator designs that possess the desired local properties described by the required structure of the Jacobian matrix. This paper concludes by showing that despite having identical local properties, the resulting manipulator designs have significantly different global kinematic properties that can be used to match a design to additional application-specific performance criteria.

KEY WORDS

Robotics, robot design and architecture, fault-tolerant robots, redundant robots.

1. Introduction

The design and operation of fault-tolerant manipulators is critical for applications in remote and/or hazardous environments where routine maintenance and repair are not possible. Example applications include space exploration [1, 2], underwater exploration [3], and nuclear waste remediation [4, 5] where there has been a great deal of research to improve manipulator reliability [6, 7], design fault-tolerant robots [8, 9], and determine mechanisms for analyzing [10], detecting [11, 12], identifying [13–15], and recovering [16–19] from failures. Typical failure modes that have been considered include locked joint failures [20], where a joint is immobilized either due to the failure itself or due to the application of fail-safe brakes, and free-

swinging joint failures [21] where the joint's associated actuator is no longer able to generate a force or torque.

A large body of work on fault-tolerant manipulators has focused on the properties of kinematically redundant robots, both in serial or parallel form [22–28]. These analyses have been performed both on the local properties associated with the manipulator Jacobian [29–32] as well as the global characteristics such as the resulting workspace following a particular failure [33–36]. (Clearly both local and global kinematic properties are related, e.g., workspace boundaries correspond to singularities in the Jacobian.) In this work it is assumed that one is given a set of local performance constraints that require a manipulator to function in a configuration that is optimal under normal operation and after an arbitrary single joint fails and is locked in position. Specifically, the desired Jacobian matrix must be isotropic, i.e., possess all equal singular values prior to a failure, and have equal minimum singular values for every possible single column being removed. However, one can then use global characteristics to distinguish between multiple manipulators that meet the local design constraints.

The remainder of this paper is organized in the following manner. A local definition of failure tolerance centered on desirable properties of the manipulator Jacobian is mathematically defined in the next section. In Section 3, a procedure for computing the kinematic parameters of a manipulator from a given Jacobian is presented. This procedure is then used on a simple example of an optimally fault-tolerant Jacobian in Section 4, to show that there are multiple related manipulator designs that satisfy the mathematical properties of a fault-tolerant Jacobian. The global properties of all the resulting manipulator designs are further analyzed. The conclusions of this work are then presented in Section 5.

2. A Definition of an Optimally Fault-Tolerant Jacobian¹

The dexterity of manipulators is frequently quantified in terms of the properties of the manipulator Jacobian matrix which relates end-effector velocities to joint angle velocities. The Jacobian will be denoted by the $m \times n$ matrix

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¹Portions of this section are summarized from [37].

J where m is the dimension of the task space and n is the number of degrees-of-freedom of the manipulator. For redundant manipulators $n > m$ and the quantity $n - m$ is the degree of redundancy. The manipulator Jacobian can be written as a collection of columns

$$J_{m \times n} = [j_1 \ j_2 \ \cdots \ j_n] \quad (1)$$

where j_i represents the end-effector velocity due to the velocity of joint i . For an arbitrary single joint failure at joint f , assuming that the failed joint can be locked, the resulting m by $n - 1$ Jacobian will be missing the f th column, where f can range from 1 to n . This Jacobian will be denoted by a preceding superscript so that in general

$${}^f J_{m \times n-1} = [j_1 \ j_2 \ \cdots \ j_{f-1} \ j_{f+1} \ \cdots \ j_n]. \quad (2)$$

The properties of a manipulator Jacobian are perhaps best illustrated through the use of the singular value decomposition (SVD), which can be defined as

$$J = UDV^T \quad (3)$$

where U is an m by m orthogonal matrix of the output singular vectors, V is an n by n orthogonal matrix of the input singular vectors, and D is a diagonal matrix of the form

$$D_{m \times n} = \begin{bmatrix} \sigma_1 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \sigma_2 & \ddots & \vdots & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & 0 & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \sigma_m & 0 & \cdots & 0 \end{bmatrix} \quad (4)$$

where the σ_i are the singular values, and are typically ordered from largest to smallest. Most local dexterity measures can be defined in terms of simple combinations of these singular values such as their product (determinant) [38], sum (trace), or ratio (condition number) [39–41]. Task specific dexterity measures also require knowledge of the output singular vectors. The most significant of the singular values is σ_m , the minimum singular value, because it is by definition the measure of proximity to a singularity and tends to dominate the behavior of both the manipulability (determinant) and the condition number. The minimum singular value is also a measure of the worst-case dexterity over all possible end-effector motions.

The definition of failure tolerance used in this work is based on the worst-case dexterity following an arbitrary locked joint failure. Because ${}^f \sigma_m$ denotes the minimum singular value of ${}^f J$, ${}^f \sigma_m$ is a measure of the worst-case dexterity if joint f fails. If all joints are equally likely to fail, then a measure of the worst-case failure tolerance is given by

$$\mathcal{K} = \min_{f=1}^n ({}^f \sigma_m). \quad (5)$$

To insure that manipulator performance is optimal prior to a failure, an optimally failure tolerant Jacobian is further

defined as having all equal singular values due to the desirable properties of isotropic manipulator configurations [39–41]. Under these conditions, to guarantee that the minimum ${}^f \sigma_m$ is as large as possible they should all be equal. It is easy to show that the worst-case dexterity of an isotropic manipulator that experiences a single joint failure is governed by the inequality

$$\min_{f=1}^n ({}^f \sigma_m) \leq \sigma \sqrt{\frac{n-m}{n}} \quad (6)$$

where σ denotes the norm of the original Jacobian. The best case of equality occurs if the manipulator is in an optimally failure tolerant configuration. The above inequality makes sense from a physical point of view because it represents the ratio of the degree of redundancy to the original number of degrees of freedom.

Using the above definition of an optimally failure tolerant configuration one can identify the structure of the Jacobian required to obtain this property [37]. In particular, one can show that the optimally failure tolerant criteria requires that each joint contributes equally to the null space of the Jacobian transformation [31]. Physically, this means that the redundancy of the robot is uniformly distributed among all the joints so that a failure at any joint can be compensated for by the remaining joints. Therefore, in this work an optimally failure tolerant Jacobian is defined as being isotropic, i.e. $\sigma_i = \sigma$ for all i , and having a maximum worst-case dexterity following a failure, i.e. one for which ${}^f \sigma_m = \sigma \sqrt{\frac{n-m}{n}}$ for all f . The second condition is equivalent to having the columns of the Jacobian have equal norms.

The simplest example of an optimally failure tolerant configuration is given by the following Jacobian for a three degree-of-freedom planar manipulator:

$$J = [j_1 \ j_2 \ j_3] = \begin{bmatrix} -\sqrt{\frac{2}{3}} & \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{6}} \\ 0 & -\sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{bmatrix}. \quad (7)$$

The null space at this configuration is given by

$$v_3 = \begin{bmatrix} \sqrt{\frac{1}{3}} \\ \sqrt{\frac{1}{3}} \\ \sqrt{\frac{1}{3}} \end{bmatrix} \quad (8)$$

which illustrates that each joint contributes equally to the null space motion, thus distributing the redundancy proportionally to all degrees of freedom. Geometrically, it is easy to see that the three vectors j_1 , j_2 , and j_3 are all 120° apart which results in a balanced coverage of the planar workspace. If the three possible joint failures are considered, one can show that

$${}^f \sigma_2 = \sqrt{\frac{1}{3}} \quad (9)$$

Figure 1. DH parameters for a joint i of an arbitrary manipulator.

for $f = 1$ to 3, which satisfies the optimally failure tolerant criterion. Given this example of an optimally failure tolerant J , one might be interested in designing the kinematics for a manipulator that would possess these qualities. The following section discusses a procedure for computing the kinematic structure of a robot and its configuration from a given desired J .

3. Computing Denavit and Hartenberg Parameters from a Given Jacobian

3.1 Denavit and Hartenberg Coordinate Frames

The Denavit and Hartenberg (DH) notation for specifying manipulator kinematics is illustrated in Figure 1 with the parameters defined in Table 1 [42]. While typically these are determined for a given manipulator, and then its Jacobian is computed from the DH parameters, here one is interested in computing all of the DH parameters that correspond to a desired Jacobian. It is assumed that J is computed for the hand, and is given with respect to the hand coordinate frame.

Table 1

Definition of the Denavit and Hartenberg parameters

α_i	Link twist: the angle between z_{i-1} and z_i around x_i using the right-hand rule.
a_i	Link length: the shortest distance between the joint axes z_{i-1} and z_i .
θ_i	Joint angle: the angle between x_{i-1} and x_i around z_{i-1} using the right-hand rule.
d_i	Joint offset: the distance from the origin of coordinate frame $i-1$ to coordinate frame i along z_{i-1} .

The i th column of J , that is composed of both the linear partial velocity v_i and rotational partial velocity ω_i , is

closely related to the coordinate axes of the $i-1$ coordinate frame of the DH notation [43], i.e.,

$$j_i = \begin{bmatrix} v_i \\ \omega_i \end{bmatrix} = \begin{bmatrix} \hat{z}_{i-1} \times p_{i-1} \\ \hat{z}_{i-1} \end{bmatrix} \forall i = 1, \dots, n \quad (10)$$

where p_{i-1} is the position vector from the coordinate frame $i-1$ to the hand coordinate frame. Thus one can easily identify the $\hat{z}_{i-1}$ axis by inspection, i.e., it is equal to ω_i , and compute $\hat{x}_i$ from adjacent columns² of J using

$$\hat{x}_i = \pm \frac{\omega_i \times \omega_{i+1}}{\|\omega_i \times \omega_{i+1}\|}. \quad (11)$$

The appropriate sign for $\hat{x}_i$ in (11) is determined based on the convention that $\hat{x}_i$ is pointing away from $\hat{z}_{i-1}$ [43], so that

$$\langle \hat{x}_i, p'_i - p'_{i-1} \rangle > 0 \quad (12)$$

where $\langle \cdot, \cdot \rangle$ denotes the dot product, and p'_{i-1} is defined as

$$p'_{i-1} = \omega_i \times v_i. \quad (13)$$

If $\hat{z}_{i-1}$ and $\hat{z}_i$ are parallel, then $\hat{x}_i$ is given by:

$$\hat{x}_i = \frac{p'_i - p'_{i-1}}{\|p'_i - p'_{i-1}\|}. \quad (14)$$

Finally, the $\hat{y}_i$ axis can be obtained by taking the cross product of $\hat{z}_i$ and $\hat{x}_i$.

The procedure presented here assumes that J is specified with respect to the hand coordinate frame. If the desired J is given with respect to the base coordinate frame, it is easy to transform it to the hand frame by performing the appropriate rotation. To transform J to the hand coordinate frame, the following rotation matrix is used:

$${}^n R_0 = [\hat{x}_n \quad \hat{y}_n \quad \hat{z}_n] \quad (15)$$

where

$$\hat{z}_n = \omega_n, \quad (16)$$

$$\hat{x}_n = \frac{v_n \times \omega_n}{\|v_n \times \omega_n\|}, \quad (17)$$

and

$$\hat{y}_n = \frac{\hat{z}_n \times \hat{x}_n}{\|\hat{z}_n \times \hat{x}_n\|} \quad (18)$$

Hence, the Jacobian with respect to the hand coordinate frame can be obtained by multiplying each of the v_i 's and ω_i 's by ${}^n R_0$.

3.2 Computing the Denavit and Hartenberg Parameters

After defining the coordinate frames for each joint, the four DH parameters can be derived using their definitions in Table 1.

²To compute the n th coordinate frame assume that $v_{n+1} = [0 \ 0 \ 0]^T$ and $\omega_{n+1} = [0 \ 0 \ 1]^T$.

3.2.1 Link Twist α

The value of α_i can be computed using:

$$\alpha_i = \pm \text{acos}(\langle \hat{z}_{i-1}, \hat{z}_i \rangle) \quad (19)$$

where acos is the inverse of cosine that returns a value between 0 and π . The sign of α should be the same as the sign of $\langle \hat{x}_i, \hat{z}_{i-1} \times \hat{z}_i \rangle$.

3.2.2 Link Length a

The value of a_i can be computed using:

$$a_i = \langle \hat{x}_i, p'_i - p'_{i-1} \rangle. \quad (20)$$

Because $\hat{x}_i$ is defined to point away from $\hat{z}_{i-1}$, the value of a_i is always positive.

3.2.3 Joint Angle θ

The value of θ_i can be computed using

$$\theta_i = \pm \text{acos}(\langle \hat{x}_{i-1}, \hat{x}_i \rangle). \quad (21)$$

The sign of θ should be the same as the sign of $\langle \hat{z}_{i-1}, \hat{x}_{i-1} \times \hat{x}_i \rangle$. The value of θ_1 is arbitrary because $\hat{x}_0$ is not defined and can be assumed to be zero.

3.2.4 Link Offset d

The last parameter, d_i , requires that one determine the origins of the coordinate frames $i-1$ and i with respect to the hand coordinate frame, denoted ${}^0O_{i-1}$ and 0O_i respectively. One can then apply the definition in Table 1 to compute the distance between these origins along the $\hat{z}_{i-1}$ using:

$$d_i = \langle {}^0O_i - {}^0O_{i-1}, \hat{z}_{i-1} \rangle. \quad (22)$$

To determine the origin of any coordinate frame i , one can use the fact that the line L_i parameterized by t_i given by

$$L_i(t_i) = p'_i + t_i \hat{z}_i \quad (23)$$

is known because $\hat{z}_i$ and p'_i are known. One can then use the known value of a_i to determine a linear set of equations relating L_i and L_{i-1} that can be easily solved. That is, the relationship $L_i(t_i) = a_i \hat{x}_i + L_{i-1}(t_{i-1})$ has a unique solution for t_i and t_{i-1} unless the lines L_i and L_{i-1} are parallel.

4. Examples of Manipulators from Optimal Fault-Tolerant Jacobians

The preceding section presented a procedure for computing a set of DH parameters that define a robot in a configuration that has a specified J . This section uses this procedure to compute the DH parameters of robots that are optimal in

Figure 2. A simple three degree-of-freedom planar robot that corresponds to the optimal fault-tolerant Jacobian given by (24) is shown in (a). Changing the sign of the second column of J and recomputing the DH parameters for this new Jacobian, that has the same optimal properties as the original one, results in the robot shown in (b).

terms of the fault-tolerant measures presented in section 2.. It also illustrates that there are multiple related Jacobians that have the same optimality property that result in different DH parameters, and thus physically different robots. While all of these robots have the same properties of the Jacobian at the given configuration, it will be shown that the fault-tolerant properties elsewhere in the workspace can be quite different.

Consider the simple planar three degree-of-freedom optimal fault-tolerant Jacobian given by (7). The fully three-dimensional representation of this Jacobian is given by a six by three matrix

$$J = \begin{bmatrix} -\sqrt{\frac{2}{3}} & \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{6}} \\ 0 & -\sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}. \quad (24)$$

The robot whose DH parameters correspond to this Jacobian are shown in Table 2 with a planar depiction of the corresponding robot in Figure 2(a).

Table 2
DH parameters of J of (24)

i	α_i	θ_i	a_i	d_i
1	0	60°	$\sqrt{2}$	0
2	0	120°	$\sqrt{2}$	0
3	0	150°	$\sqrt{\frac{2}{3}}$	0

The isotropic and fault-tolerant properties of J are not affected by either permuting the columns or changing the sign of the columns. For a three column J there are six possible permutations and eight combinations of different signs, for a total of 48 possible Jacobian matrices with the desired isotropy and optimal fault-tolerance property. The notation J_{123} will be used to indicate the Jacobian in (7) that is in standard form and indicate column permutations or changes in sign on the subscripts, e.g., a permutation

Figure 3. All the robots that are obtained by performing all possible sign combinations and permutations of the 2nd and 3rd columns of the J when j_1 is the first column.

Figure 4. All the robots that are obtained by performing all possible sign combinations and permutations of the 2nd and 3rd columns of the J when j_2 is the first column.

of the second and third columns as J_{132} and a change in the sign of column three as $J_{12(-)3}$. As an example, if the sign of the second column is changed, i.e., $J_{1(-)23}$, and one computes the DH parameters for this Jacobian, one would obtain the robot shown in Figure 2(b). Figures 3, 4, and 5 illustrate all of the possible robots for all 48 permutations and combinations.

While there are 48 possible permutations and combinations, it is obvious that they do not result in different physical robots. In particular, consider the robots indicated with bold lines in Figure 3. These correspond to all eight combinations of sign changes for the Jacobian with the per-

Figure 5. All the robots that are obtained by performing all possible sign combinations and permutations of the 2nd and 3rd columns of the J when j_3 is the first column.

mutation J_{123} . It is easy to see that the four different cases indicated by (a), (b), (c) and (d), represent a single robot each, because the robot below the x -axis is the same as the one above with its first joint angle rotated by 180° . In fact, this is true for all six permutations of the columns so that there are really only four unique robots with a Jacobian that has the desired properties. Note that in each of the Figures 3 to 5, there are two different permutations of the columns that result in mirror versions of the same robot. The DH parameters for the four different robots are given in Table 3 where $L_s = \sqrt{\frac{2}{3}}$ and $L_l = \sqrt{2}$. Note that the value of θ_1 in all cases is arbitrary because the properties of the Jacobian are independent of θ_1 .

Table 3
DH parameters of the four generated robots

	J_{123}		$J_{1(-)23}$		$J_{1(-)2(-)3}$		$J_{12(-)3}$	
i	θ_i	a_i	θ_i	a_i	θ_i	a_i	θ_i	a_i
1	60	L_l	150	L_s	150	L_s	60	L_l
2	120	L_l	-60	L_s	-150	L_l	-150	L_s
3	150	L_s	-120	L_s	150	L_s	-120	L_s

While all four robots in Table 3 have the same desired local behavior at the given configuration, they are quite different in terms of their global properties. For example, even when joint limits are not considered, the workspaces are quite different, e.g., the maximum reach will be either $3L_s$, $2L_s + L_l$, or $L_s + 2L_l$. More importantly, if one is concerned with fault-tolerance, the values of the proposed fault-tolerance measure vary significantly for these four robot designs.

To determine how the fault tolerance measure $\mathcal{K}$ varies as a robot moves away from the configuration that has the optimal Jacobian, the optimal value of $\mathcal{K}$ were computed for every location within each of the four robot's

Figure 6. The relationship between $\mathcal{K}$ and the distance from the base for the (L_l, L_l, L_s) robot.

workspaces. Because $\mathcal{K}$ is not a function of θ_1 , it is sufficient to compute its maximum value as a function of distance from the base of the manipulator. The maximum value of $\mathcal{K}$ is determined by computing all possible robot configurations for each distance, and calculating $\mathcal{K}$ for the Jacobian at that configuration. The results for each of the four robots is shown in Figures 6 to 9 where all three values of $f\sigma_2$ for the configuration with maximum $\mathcal{K}$ are also plotted.

The first interesting point to note is that the manipulator with link lengths (L_l, L_l, L_s) in Figure 6 actually has a configuration with a larger value of $\mathcal{K}$ at the design point that is a distance of $\sqrt{2/3}$ from the base than that of the optimal value of $\mathcal{K} = \sqrt{1/3}$. This is possible because at this configuration the Jacobian is no longer isotropic, however, its non-isotropy is due to a larger maximum singular value, and so may not be considered undesirable. In addition, the value of $\mathcal{K}$ is significantly higher than the optimal value for a significant portion of this manipulator's workspace, making it particularly well suited for applications that require failure tolerance.

In contrast, consider the manipulator with link lengths (L_s, L_s, L_s) in Figure 7. It has a value of $\mathcal{K} = \sqrt{1/3}$ at the the optimal distance as designed, however, this is its peak value of $\mathcal{K}$, and $\mathcal{K}$ is monotonically decreasing away from this point. Thus, in addition to having the smallest workspace, this manipulator has a significantly smaller tolerance to joint failures throughout its workspace.

The characteristics of the two medium length robots fall somewhat inbetween the two extremes just described. The (L_s, L_l, L_s) robot shown in Figure 8 has a flat region of $\mathcal{K}$ that starts at the optimal point and continues for approximately 1.0 units, i.e., approximately 33% of its total reach. The (L_l, L_s, L_s) robot shown in Figure 9 has comparable magnitudes of $\mathcal{K}$, however, it is unique in that the value of $\mathcal{K}$ has multiple local maxima. In summary, even though all four robots are derived from the same optimally fault-tolerant Jacobian, their global properties are

Figure 7. The relationship between $\mathcal{K}$ and the distance from the base for the (L_s, L_s, L_s) robot.

Figure 8. The relationship between $\mathcal{K}$ and the distance from the base for the (L_s, L_l, L_s) robot.

quite different, both prior to a failure and afterward. The (L_l, L_l, L_s) robot is arguably the most preferable due to its larger pre-failure workspace and the large value of fault tolerance over a substantial portion of its workspace.

5. Conclusion and Future Work

This work has presented preliminary results that illustrate how multiple different manipulators can be designed from a Jacobian that has been selected to have desirable failure tolerance properties. It has been shown that even though these manipulators all have the same local properties, their global properties can differ significantly, both in terms of pre-failure kinematics as well as post-failure performance. This can provide robot system designers with a great deal of flexibility when considering the different constraints that arise from different applications.

Figure 9. The relationship between $\mathcal{K}$ and the distance from the base for the (L_l, L_s, L_s) robot.

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